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Behaviour of Supercritical Nozzles under Three-Dimensional Oscillatory Conditions

by L. Crocco and W. A. Sirignano

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1967

NORTH ATLANTIC TREATY ORGANIZATION

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ADVISORY GROUP FOR AEROSPACE RESEARCH AND DEVELOPMENT

(ORGANISATION DU TRAITE DE L'ATLANTIQUE NORD)

BEHAVIOR OF SUPERCRITICAL NOZZLES UNDER THREE-DIMENSIONAL OSCILLATORY CONDITIONS

by

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1967

This AGARDograph was prepared at the request of the
Propulsion and Energetics Panel of AGARD

SUMMARY

A linearized treatment of three-dimensional oscillatory flow in supercritical nozzles has been performed for the two cases where the steady-state flow is axisymmetric or two-dimensional. In the axisymmetric case, perturbation series have been employed to study the nonlinear oscillations. In these analyses, variables have been separated, reducing the system of partial differential equations to a system of ordinary differential equations. The variables describing the transverse dependencies of the flow properties are governed by well-known differential equations (for example, Bessel's equation is obtained). The axial dependencies, on the other hand, are governed by differential equations which must be solved by numerical means. The nozzle admittance coefficients are related to the axial dependencies of the flow properties. Certain techniques have been applied to reduce the order of the differential equations for the purpose of easier calculation of the admittance coefficients.

These admittance coefficients are to be used in the boundary condition applied at the exit of the chamber joined to the nozzle and their calculation is the most important result of this research effort. The calculations have been performed for conical nozzles and are presented in tabular form. Examples are presented which demonstrate the use of the tables in typical problems. Oscillatory pressures and velocities are also calculated in a limited number of cases in order to provide physical insight to the oscillation. An asymptotic analysis has been performed which is valid in the low entrance-Mach-number regime. By predicting the admittance coefficients and flow properties through closed-form solutions, the asymptotic analysis is an asset in the interpretation of results.

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SOMMAIRE

Un traitement linéarisé d'un écoulement oscillatoire tridimensionnel dans les tuyères supercritiques a été effectué pour les deux cas où l'écoulement permanent est axisymétrique ou plan. Dans le cas axisymétrique des séries perturbatrices ont été utilisées pour l'étude des oscillations non linéaires. Dans les analyses effectuées on a séparé les variables en réduisant à un système d'équations différentielles ordinaires le système d'équations différentielles partielles. Les variables qui décrivent les dépendances transversales des propriétés de l'écoulement sont régies par des équations différentielles connues (par exemple, on obtient l'équation de Bessel). Par contre, les dépendances axiales sont décrites par des équations différentielles qu'il faut résoudre par des moyens numériques. Les coefficients d'entrée de tuyère sont liés aux dépendances axiales des propriétes de l'écoulement. Certaines techniques ont été appliquées pour réduire l'ordre des équations différentielles en vue de faciliter le calcul des coefficients d'entrée.

Ces coefficients d'entrée seront employés pour la condition limite appliquée à la sortie de la chambre reliée à la tuyère et leur calcul représente le résultat le plus important de ces efforts de recherches. Les calculs ont été effectués pour les tuyères coniques et sont présentés sous forme de tableaux. Des exemples sont cités pour démontrer l'utilisation de ces tableaux pour la solution de problèmes types. Les pressions et les vitesses oscillatoires sont également calculées dans un nombre limité de cas pour obtenir une connaissance physique de l'oscillation. Une analyse asymptotique a été réalisée qui est valable dans les conditions de faible nombre de Mach d'entrée. Elle s'avère très avantageuse pour l'interprétation des résultats en permettant de prévoir les coefficients d'entrée au moyen de solutions de forme fermée.

FOREWORD

The theory, on which the results presented in this monograph are based, was developed by the senior author over a decade ago and presented at a meeting at the University of Maryland. At that time, the complete formulation of the linear admittance coefficient for a supercritical nozzle had been derived in the general case where both vorticity and entropy oscillations exist at the nozzle entrance. In the following years, a small number of calculations were made with the purpose of providing the necessary data for the determination of the combustion instability limits in a particular experimental rocket[11]. However, the publication of the theory was postponed until more complete calculations and interpretations would become available.

In recent years, the project became a cooperative venture and, primarily through the efforts of the junior author, the computer calculations and other related analyses were accomplished. Meanwhile, the need for the public availability has become pressing, due to the problem of combustion instability in rocket motors. One of the leading rocket manufacturers in the United States has found it necessary to establish its own computer program based on the above-mentioned theoretical developments[6]. Another scientist has decided to attack the problem independently in a somewhat-different manner[7]. We welcomed therefore the opportunity (made possible by AGARD) to publish the theory, some of its extensions, the methods of calculation, and the numerical results. It has been felt that they would fill a well-defined void in the existing technical literature and play a useful role.

In Part I, we present the theoretical background. The original theory of the senior author is contained in Sections 1 through 12. Section 13 contains the discussion of the extension of the theory to the nonlinear shockless case performed by B.T. Zinn, under the supervision of the authors, as part of his Ph.D. thesis[8]. Part II contains the discussion of the calculations, related analyses, interpretation of the results, and examples of applications and is mostly due to the junior author. Sections 14, 15, and 16 present the method of calculation and a discussion of the results. Sections 17 and 18 present an asymptotic theory useful for the purpose of interpretation. Finally, Section 19 contains a few sample applications of the results.

We wish to recognize the major technical support provided by the staff of the Guggenheim Laboratories Computing Group at Princeton University. Also, we wish to recognize the major financial support provided by the National Aeronautics and Space Administration. Additional support for the calculations was provided by the William B. Reed, Jr. Fund for Engineering Research of Princeton University and (indirectly through the support of the Princeton University Computing Center) by the National Science Foundation.

Luigi Crocco
William A. Sirignano

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NOTATION

A	pressure admittance coefficient defined in (122) for axisymmetric nozzle
A_1	pressure admittance coefficient for bidimensional nozzle
$\mathcal{A}$	pressure admittance coefficient defined after (161) for axisymmetric nozzle
$A, A_n, A_{n,m\nu,q}$	function defined in (130), its Fourier series coefficients, and its eigenfunction series coefficients, respectively
A, B, C, D	coefficients in (179)
$\bar{A}, \bar{B}$	constants in (186)
a	constant defined after (177)
B	radial velocity admittance coefficient defined in (123) for axisymmetric nozzle
B_1	transverse velocity admittance coefficient for bidimensional nozzle
$\mathcal{B}$	radial velocity admittance coefficient defined after (161) for axisymmetric nozzle
$B, B_n, B_{n,m\nu,q}$	function defined in (131), its Fourier series coefficients, and its eigenfunction series coefficients, respectively
B_r, B_i, C_r, C_i	parameters in initial conditions (174)
b, g, h, j, k	coefficients in (163)
b, c, d	integration constants in (184)
b	nondimensional width of the bidimensional nozzle
$\tilde{b}$	coefficient in power series expansion
C	entropy admittance coefficient defined in (124) for axisymmetric nozzle
C_1	entropy admittance coefficient for bidimensional nozzle or integration constant
$\mathcal{C}$	entropy admittance coefficient defined after (161) for axisymmetric nozzle

C_0, C_2, C_3	integration constants
C_ψ, C_y	separation constants
$C, C_n, C_{n,m\nu,q}$	function defined in (132), its Fourier series coefficients, and its eigenfunction series coefficients, respectively
$C_{1n,m\nu,q}, C_{2n,m\nu,q}$	integration constants
c	speed of sound
c_p	specific heat at constant pressure
D_1	spanwise velocity admittance coefficient for bidimensional nozzle
$D, D_n, D_{n,m\nu,q}$	function defined in (133), its Fourier series coefficients, and its eigenfunction series coefficients, respectively
$\mathcal{E}$	combined admittance coefficient for axisymmetric nozzles
$E, E_n, E_{n,m\nu,q}$	function defined in (134), its Fourier series coefficients, and its eigenfunction series coefficients, respectively
e	unit vector
$F^{(j)}$	functions defined in (89)
$F_1^{(j)}$	functions defined in (97)
$F_{n,m\nu,q}^{(j)}$	functions defined after (145)
$F, F_n, F_{n,m\nu,q}$	function defined in (135), its Fourier series coefficients, and its eigenfunction series coefficients, respectively
f_0	function defined in (77)
f_1	function defined in (81)
f_2	function defined in (83)
f_3	function defined in (85)
f_{0n}	functions defined after (139)
f_{1n}, f_{2n}	functions defined after (150)
f_{3n}	functions defined after (135)
G, H	functions defined after (172)

G_n	functions defined after (139)
H_n	functions defined after (141)
$h_\phi, h_\psi, h_\theta, y$	defined in (23) and after (34)
I_n	inhomogeneous part defined in (141) or function defined by (188) and (190)
$I_{n,m\nu,q}$	inhomogeneous part defined in (144)
I_1, I_2, I_3, I_4	parameters defined after (174)
i	imaginary unit
J_ν, Y_ν	Bessel functions of order ν of the first kind and second kind, respectively
$\mathcal{J}_n$	integral defined after (193)
$K^{(j)}_{n,m\nu,q}$	functions defined after (155)
K_1, K_2, K_3	constants defined after (183)
k	related to separation constant in (54) or constant in (189)
L^*	characteristic length in nondimensional scheme
L	combustion chamber length
ℓ	length ED of cylinder in Figure 17
M, N	integers describing mode of transverse oscillation in bidimensional nozzle
M	integer in (190)
$M^{(i)}$	functions defined after (191)
m	integer in Section 13, or parameter defined after (193)
$N_{n,m\nu,q}$	functions defined after (145)
n	integer subscript
P, P_1	separated pressure variables defined in (44) and (61)
$P_{n,m\nu,q}$	coefficient in eigenfunction series for pressure
$\mathcal{P}$	separated pressure variable defined in (160)

p	pressure
$Q_{\nu h}$	separated energy release per unit volume in combustion chamber
q	velocity vector
R, R_1	separated density variables defined in (44) and (61)
R, R_2	nozzle wall radii of curvature at throat and entrance, respectively
$R_{n,m\nu,q}$	coefficient in eigenfunction series for density
$\mathcal{R}$	separated density variable defined in Section 15
r	radial position or local wall radius in Section 14
S, S_1	separated entropy variables defined in (44) and (61)
$S_{n,m\nu,q}$	coefficient in eigenfunction series for entropy
$\mathcal{S}$	separated entropy variable defined in (160)
s	entropy
$s_{\nu h}, s_{n\nu,q}$	eigenvalue corresponding to roots of the derivative of the Bessel function
s_M	parameter defined in (73)
t	time or variable in (190)
U, U_1	separated axial velocity variables defined in (44) and (61)
$U_{n,m\nu,q}$	coefficient in eigenfunction series for axial velocity
$\mathcal{U}$	separated axial velocity variable defined in (160)
u, v, w	ϕ, ψ, and θ or ϕ, ψ, and y components of velocity, respectively
V, V_1	separated radial velocity variables defined in (44) and (61)
$V_{n,m\nu,q}$	coefficient in eigenfunction series for radial velocity
$\mathcal{V}$	separated radial velocity variable defined in (160)
W, W_1	separated azimuthal velocity variables defined in (44) and (61)
$W_{n,m\nu,q}$	coefficient in eigenfunction series for azimuthal velocity

$\mathfrak{W}$	separated azimuthal velocity variable defined in Section 15
x	coordinate for bidimensional nozzle
Y	separated variable defined in (61)
y	spanwise coordinate for bidimensional nozzle or normalized time in Section 13 or transformed velocity variable defined in (178)
y_1, y_2, y_3, y_4	functions defined before (173)
z	physical axial coordinate
α	phase constant in (56) or admittance coefficient defined in (105) or (107)
α_1, δ_1	coefficients in (157)
$\alpha_0, \beta_0, \alpha_1, \beta_1$	parameters appearing in initial conditions for (163)
$\alpha, \beta, \delta, \epsilon$	constants in (170)
β	function defined in (110) or scaling factor discussed in Section 12
γ	ratio of specific heats
Δ	phase angle in Section 16
δl	elementary length
δn	elementary length in streamwise direction
δs	elementary length normal to streamline
ϵ	perturbation parameter
ζ, ζ_1	functions defined in (103) and (117), respectively
ζ_n	functions defined in (136)
$\zeta_{n,m\nu,q}$	functions defined in (154)
η	constant in (190)
η_n	functions defined in (136)
$\bar{\eta}_n$	functions defined after (141)
Θ	separated variable defined in (44) and (61)

θ	azimuthal position
θ_1	semi-angle of convergent portion of conical nozzle
κ	transformation constant discussed in Section 14
κ_n	functions defined in (136)
λ_1,λ_2	constants defined after (170) or constants of integration in (192)
$\lambda_3,\lambda_4,\lambda_5$	constants defined in (179) and (200)
ν	separation constant defined in (53)
$\xi^{(j)},\xi_1^{(j)}$	functions defined in (103) and (117), respectively
ξ_n	functions defined in (136)
$\xi_{n,m\nu,q}$	functions defined in (155)
π_n	functions defined in (136)
ρ	density
σ	integration constant related to initial entropy
σ_N	parameter defined in (73)
σ_n	functions defined in (136)
$\sigma_{n,m\nu,q}$	integration constants
Φ,Φ_1	functions defined in (79) and (92), respectively
$\Phi^{(j)},\Phi_1^{(j)}$	particular solutions of (99) and (114), respectively
Φ_n	functions defined in (137)
$\Phi_{n,m\nu,q}$	functions defined in (143a) or (143b)
ϕ	velocity potential function
Ψ,Ψ_1	separated variables defined in (44) and (61), respectively
ψ	stream function or transformed variable in (180)
ω	angular frequency nondimensionalized with respect to throat radius
ω_c	angular frequency nondimensionalized with respect to chamber radius

Superscripts

star (*)	dimensional quantity
bar	steady-state variable or admittance coefficient at end of cylinder in Section 15
caret	transformed variable discussed in Section 14
prime	perturbation quantity
(t)	quantity pertaining to traveling wave
(s)	quantity pertaining to standing wave
o	stagnation quantity
(0),(1),(2), etc.	coefficient in perturbation series in Section 13

Subscripts

arg	argument of a complex number
e	quantity at nozzle entrance
h	homogeneous solution
i	imaginary part of quantity
mod	modulus of a complex number
p	particular solution
r	real part of quantity
ref	quantity pertaining to reference nozzle
th	quantity at nozzle throat
w	quantity at nozzle wall

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BEHAVIOR OF SUPERCRITICAL NOZZLES UNDER THREE-DIMENSIONAL OSCILLATORY CONDITIONS

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PART I. THEORY

1. INTRODUCTION

Many propulsive devices are terminated by a nozzle through which the propulsive gases are discharged. Very often the nozzle operates in the supercritical range and is shaped as a classical Laval nozzle, converging up to a throat (where, in steady operation, the sonic velocity is achieved) and diverging thereafter.

Unsteady conditions are present in the nozzle when the operation in the propulsive device is unsteady. A particular type of unsteady operation which has great importance in practice results from combustion instability in the propulsive device. In this case the operation is oscillatory, characterized by periodic variations of the flow parameters both in the combustion chamber and in the nozzle which follows.

The study of combustion instability is of great importance for the safety of operation of combustion devices, and has been already the subject of a great deal of theoretical and experimental research. It is important in these studies to know the behavior of the nozzle under oscillatory conditions. In particular, it is necessary to find out how a wave generated in the combustion chamber is partially reflected and partially transmitted at the entrance of the nozzle. Mathematically, this is equivalent to saying that it is necessary to know the boundary conditions created by the nozzle to the oscillatory flow present in the combustion chamber. For instance, on any solid wall of the chamber, the boundary condition is that the velocity component of the gases normal to the wall must be zero at every instant, and so must be, therefore, the corresponding velocity perturbation, no matter what are the perturbations of the tangential velocities, pressure, entropy, etc. Obviously if the entrance of the nozzle is considered to represent one of the boundaries of the combustion chamber, the corresponding boundary conditions are more involved. No single perturbation can be assumed to vanish, and the boundary condition may be expected to be expressed by a

relation between the various perturbations which shall be called the admittance condition*. In particular, if the perturbations are assumed to be of sufficiently small amplitude, so that the problem can be linearized, the aforesaid relation expressing the boundary condition must be linear. Observe that a linearized treatment can be only applied to the study of incipient instability, that is to the study of the combustion stability limits of a given propulsive device. When the instability is fully developed, nonlinear effects become essential and must be taken into consideration.

If the unperturbed flow in the nozzle can be assumed to be one-dimensional, a particularly simple case is obtained when also the perturbations are taken one-dimensional, that is the perturbations are assumed to be uniform, at each instant, on each section of the nozzle. Obviously in this case the conditions in the combustion chamber are also one-dimensional, and any wave present in the chamber or in the nozzle is of an "axial" type. The behavior of the nozzle in the presence of axial waves has been analyzed by Tsien[1] in a few simple cases, and by Crocco[2,3] for the most general type of linear axial oscillation, in particular when the entropy is not constant. The variability of the entropy is in fact an unavoidable consequence of the combustion taking place under oscillatory conditions, and may produce interesting effects on the combustion instability[3]. Experiments carried out some time ago in nearly isentropic conditions show satisfactory agreement with the theoretical predictions[4].

However, the axial type of oscillations is only a particular case, and in actual conditions transverse perturbations are often present, this being particularly true in large propulsive devices. The problem is now complicated by the increased number of degrees of freedom, but under the same assumptions that the perturbations are small and that the unperturbed flow in the nozzle is one-dimensional (hence, irrotational), isoenergetic and isentropic, it is amenable to a relatively simple analytical treatment. It is the purpose of this monograph to show how, in this simpler case, the admittance condition at the nozzle entrance can be expressed and to discuss the corresponding numerical results. We shall treat both the case of axisymmetric nozzle (the most common in practical devices) and that of two-dimensional nozzle. We shall also briefly discuss the nonlinear treatment of the nozzle admittance problem in the absence of shock waves.

Observe that the results of the present study are applicable to any type of device, propulsive or not, involving combustion processes or not.

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* The following considerations may help in understanding the nature of the admittance condition. It is clear that, if the flow in the nozzle is supercritical, for sufficiently small oscillations the supersonic portion of the nozzle has no effect on the chamber conditions, because downstream of the throat the oscillations, no matter how distorted, must always propagate downstream and cannot interfere with the upstream flow. Hence, the logical choice for the surface on which boundary conditions must be prescribed would be the surface where the sonic velocity is achieved, or, for small oscillations around an approximately one-dimensional flow, the throat itself. It has been shown[2,3] that the proper boundary condition at the throat is that the solution remains regular here (where, indeed, a singularity tends to result from the inability of the disturbances to propagate upstream from the supersonic into the subsonic region). In practice, however, it is useful to divide the whole of the chamber plus the nozzle in two parts: the combustion chamber extending down to the nozzle entrance where the processes of combustion are taking place but the mean flow Mach number is relatively low; and the nozzle where no combustion is assumed to take place but the mean Mach number grows up to unity. The result of this subdivision is to move the boundary of the combustion chamber from the throat up to the nozzle entrance, where the appropriate boundary conditions can be obtained by studying the oscillatory behavior of the nozzle *per se* and obtaining the proper relation between the perturbations at the nozzle entrance (the admittance condition) from the condition of non-singularity at the throat.

2. THE EQUATIONS

Using stars to denote dimensional quantities and operators, the equations of motion for an inviscid, non-heat-conducting gas are the following.

Continuity:

$$\frac{\partial \rho^*}{\partial t^*} + \nabla^* . (\rho^* q^*) = 0 .$$

Momentum:

$$\frac{\partial q^*}{\partial t^*} + \frac{1}{2} \nabla(q^{*2}) + (\nabla^* \times q^*) \times q^* = -\frac{1}{\rho^*} \nabla^* p^* ,$$

where t^* represents the time, ρ^* and p^* the density and pressure, and q^* the velocity vector.

When viscosity and heat conductivity are disregarded the energy equation in its simplest form expresses the constancy of the entropy for a fluid particle after it enters the nozzle:

$$\frac{\partial s^*}{\partial t^*} + q^* . \nabla^* s^* = 0 ,$$

where
$$s^* = c_p \left(\frac{1}{\gamma} \log_e p^* - \log_e \rho^* \right) + \text{constant} \qquad (1)$$

represents the entropy. The specific heat c_p is assumed to be constant. With this particular form of the energy equation it is not necessary to introduce the equation of state, which is implicitly taken into account in Equation (1). and the four equations just written are complete in the unknowns q^* , p^* , ρ^* , s^* .

These equations can be nondimensionalized using appropriate reference values. Assuming the gas entering the nozzle in the unperturbed flow to be isoenergetic and isentropic (as well as irrotational), and hence to stay such in the following expansion through the nozzle, the corresponding stagnation quantities remain constant throughout the unperturbed flow and hence are suitable reference quantities. Hence we define

$$q = \frac{q^*}{\overline{c^o}^*} ; \qquad p = \frac{p^*}{\overline{p^o}^*} ; \qquad \rho = \frac{\rho^*}{\overline{\rho^o}^*} , \qquad (2)$$

where c denotes the sonic velocity, the superscript ^{o} indicates stagnation values and the superposed bar unperturbed (steady) values. Observe that

$$\overline{c^o}^* = \left(\gamma \frac{\overline{p^o}^*}{\overline{\rho^o}^*} \right)^{\frac{1}{2}} . \qquad (3)$$

The lengths can be nondimensionalized using a suitable characteristic length L^* to be further defined, and a nondimensional time is immediately obtained as

$$t = \frac{\overline{c^o}^*}{L^*} t^* \quad . \tag{4}$$

Transforming also the operators to nondimensional coordinates we obtain the equations of motion in the form

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{q}) = 0 \tag{5}$$

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{1}{2}\nabla(q^2) + (\nabla \times \mathbf{q}) \times \mathbf{q} = -\frac{1}{\gamma\rho}\nabla p \tag{6}$$

$$\frac{\partial s}{\partial t} + \mathbf{q}.\nabla s = 0 \ , \tag{7}$$

with

$$s = \frac{s^*}{c_p} = \frac{1}{\gamma}\log_e p - \log_e \rho + \text{constant} \tag{8}$$

representing the nondimensional entropy. Equation (3) is replaced by

$$c = \frac{c^*}{\overline{c^o}^*} = \left(\frac{p}{\rho}\right)^{\frac{1}{2}} \ , \tag{9}$$

c representing the nondimensional sonic velocity.

3. LINEARIZATION OF THE EQUATIONS

We now introduce the ordinary assumption that the (unsteady) perturbations around the (steady) unperturbed quantities are of such small amplitude that only linear terms in the perturbations need be considered. Hence we introduce

$$q = \bar{q} + q' \ ; \qquad p = \bar{p} + p' \ ; \qquad \rho = \bar{\rho} + \rho' \ ; \qquad s = \bar{s} + s' \tag{10}$$

in Equations (5) to (8) and, after neglecting all terms of order higher than first in the primed quantities, we separate each equation into its unperturbed portion (containing only the barred quantities) from the perturbation portion (linear in the primed quantities). Clearly each portion of the equations must be satisfied separately. The equations for the unperturbed flow are

$$\left.\text{div}\,(\bar{\rho}\bar{q}) = 0 \ ; \qquad \text{grad}\,\frac{\overline{q^2}}{2} + (\nabla \times \bar{q}) \times \bar{q} = -\frac{1}{\gamma\bar{\rho}}\,\text{grad}\,\bar{p} \ \right\} \tag{11}$$

$$\left.\bar{q}\cdot\text{grad}\,\bar{s} = 0\,; \qquad \bar{s} = \frac{1}{\gamma}\log_e \bar{p} - \log_e \bar{\rho} + \text{constant}\,.\right\} \tag{11}$$

These equations can be replaced by the following simpler ones, obtained from (11) in a standard fashion when the flow is irrotational,

$$\text{div}(\bar{\rho}\bar{q}) = 0\,; \qquad \bar{p} = \bar{\rho}^{\gamma}\,; \qquad \frac{\bar{p}}{\bar{\rho}} = 1 - \frac{\gamma - 1}{2}\overline{q^2}\,. \tag{12}$$

The second expresses the constancy of the entropy $\bar{s}$, the third that of the stagnation temperature. Comparing this last equation to (9) we obtain the unperturbed sonic velocity

$$\overline{c^2} = 1 - \frac{\gamma - 1}{2}\overline{q^2} = \bar{\rho}^{(\gamma-1)}\,, \tag{13}$$

the last term, resulting from (12), providing the unperturbed density. When the nozzle is axisymmetric or two-dimensional the first (12) can be used to define a stream function. We may write indeed

$$r\bar{\rho}\bar{q} = e_\theta \times \nabla\psi \tag{14}$$

in the axisymmetric case (r representing the nondimensional distance from the axis of symmetry and e_θ the unit vector in the tangential direction) and

$$b\bar{\rho}\bar{q} = \mathbf{e}_y \times \nabla\psi \tag{15}$$

in the two-dimensional case (b representing the nondimensional width of the nozzle, which must be assumed to be finite if three-dimensional oscillations are to be considered, and $\mathbf{e}_y$ the spanwise unit vector.)

In the present assumption of irrotationality of the unperturbed flow, a potential function can also be defined as

$$\bar{q} = \nabla\phi\,. \tag{16}$$

The stream and potential functions introduced are nondimensional.

In what follows the unperturbed flow will be assumed to be in the meridional plane (axisymmetric case) or purely two-dimensional (two-dimensional nozzle).

The perturbation equations can be written as

$$\frac{\partial\rho'}{\partial t} + \nabla\cdot(\bar{q}\rho' + \bar{\rho}\mathbf{q}') = 0 \tag{17}$$

$$\frac{\partial\mathbf{q}'}{\partial t} + \nabla(\bar{q}\cdot\mathbf{q}') + (\nabla\times\mathbf{q}')\times\bar{q} = -\frac{1}{\gamma\bar{\rho}}\nabla p' + \frac{\rho'}{\gamma\bar{\rho}^2}\nabla\bar{p} \tag{18}$$

$$\frac{\partial s'}{\partial t} + \bar{q}.\nabla s' = 0 \tag{19}$$

$$s' = \frac{p'}{\gamma\bar{p}} - \frac{\rho'}{\bar{\rho}} . \tag{20}$$

Equations (18) to (20) represent the system of equations to be solved. It is linear in the perturbations, and has coefficients depending on the solution of the unperturbed Equations (12).

It is useful to rewrite the second member of (18) in a different form, using the following transformations:

$$\left.\begin{aligned} -\frac{1}{\gamma\bar{\rho}}\nabla p' + \frac{\rho'}{\gamma\bar{\rho}^2}\nabla\bar{p} &= -\nabla\left(\frac{p'}{\gamma\bar{\rho}}\right) - \frac{p'}{\gamma\bar{\rho}^2}\nabla\bar{\rho} + \frac{\rho'}{\gamma\bar{\rho}^2}\nabla\bar{p} \\ &= -\nabla\left(\frac{p'}{\gamma\bar{\rho}}\right) - \frac{\nabla\bar{p}}{\gamma\bar{\rho}}\left(\frac{p'}{\gamma\bar{p}} - \frac{\rho'}{\bar{\rho}}\right) \\ &= -\nabla\left(\frac{p'}{\gamma\bar{\rho}}\right) + \tfrac{1}{2}(\nabla\bar{q}^2)s' , \end{aligned}\right\} \tag{21}$$

where use has been made of the second (12), of the second (11) and of (20).

4. CHOICE OF THE INDEPENDENT VARIABLES: A''ISYMMETRIC NOZZLE

Abandoning the vectorial representation, it is useful to choose the independent spatial variables in a way appropriate to the introduction of the boundary conditions at the nozzle walls. In the case of axial symmetry a suitable choice is to take the steady-state potential function to replace the axial variable, and the steady-state stream function to replace the radial variable. Indicating with δs and δn elementary (nondimensional) lengths in the direction of the unperturbed streamlines and of their normal in the meridional plane of Figure 1, Equations (14) and (16) can be written

$$\bar{q} = \frac{d\phi}{\delta s}; \qquad r\bar{\rho}.\bar{q} = \frac{d\psi}{\delta n}. \tag{22}$$

Hence the s...re of the elementary length dl can be written, in terms of $d\phi$, $d\psi$, and $d\theta$ (θ representing the azimuthal variable) as

$$dl^2 = h_\phi^2 d\phi^2 + h_\psi^2 d\psi^2 + h_\theta^2 d\theta^2 = \frac{1}{\bar{q}^2} d\phi^2 + \left(\frac{1}{r\bar{\rho}\bar{q}}\right)^2 d\psi^2 + r^2\, d\theta^2 ,$$

from which we get

$$h_\phi = \frac{1}{\bar{q}}; \qquad h_\psi = \frac{1}{r\bar{\rho}\bar{q}}; \qquad h_\theta = r . \tag{23}$$

These quantities are used to calculate the divergences, gradients, and rotors appearing in the equations. If f is any scalar and $F = F_\phi e_\phi + F_\psi e_\psi + F_\theta e_\theta$ is any vector, e_ϕ , e_ψ , e_θ being unit vectors normal to the surfaces ϕ = constant , ψ = constant , θ = constant , we have

$$\nabla f = \left(\frac{1}{h_\phi}\frac{\partial f}{\partial \phi}\right) e_\phi + \left(\frac{1}{h_\psi}\frac{\partial f}{\partial \psi}\right) e_\psi + \left(\frac{1}{h_\theta}\frac{\partial f}{\partial \theta}\right) e_\theta \tag{24}$$

$$= \left(\bar{q}\frac{\partial h}{\partial \phi}\right) e_\phi + \left(r\bar{\rho}\bar{q}\,\frac{\partial f}{\partial \psi}\right) e_\psi + \left(\frac{1}{r}\frac{\partial f}{\partial \theta}\right) e_\theta$$

$$\nabla . F = \frac{1}{h_\phi h_\psi h_\theta}\left[\frac{\partial}{\partial \phi}\,(h_\psi h_\theta F_\phi) + \frac{\partial}{\partial \psi}\,(h_\theta h_\phi F_\psi) + \frac{\partial}{\partial \theta}\,(h_\phi h_\psi F_\theta)\right] \tag{25}$$

$$= \bar{\rho}\bar{q}^2\left[\frac{\partial}{\partial \phi}\left(\frac{F_\phi}{\bar{\rho}\bar{q}}\right) + \frac{\partial}{\partial \psi}\left(\frac{rF_\psi}{\bar{q}}\right) + \frac{\partial}{\partial \theta}\left(\frac{F_\theta}{r\bar{\rho}\bar{q}^2}\right)\right]$$

$$\nabla \times F = \frac{1}{h_\phi h_\psi h_\theta}\begin{vmatrix} h_\phi e_\phi & h_\psi e_\psi & h_\theta e_\theta \\ \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \psi} & \frac{\partial}{\partial \theta} \\ h_\phi F_\phi & h_\psi F_\psi & h_\theta F_\theta \end{vmatrix} \tag{26}$$

$$= \bar{\rho}\bar{q}\left[\frac{\partial}{\partial \psi}\,(rF_\theta) - \frac{\partial}{\partial \theta}\left(\frac{F_\psi}{r\bar{\rho}\bar{q}}\right)\right] e_\phi + \frac{\bar{q}}{r}\left[\frac{\partial}{\partial \theta}\left(\frac{F_\phi}{\bar{q}}\right) - \frac{\partial}{\partial \phi}\,(rF_\theta)\right] e_\psi +$$

$$+ r\bar{\rho}\bar{q}^2\left[\frac{\partial}{\partial \phi}\left(\frac{F_\psi}{r\bar{\rho}\bar{q}}\right) - \frac{\partial}{\partial \psi}\left(\frac{F_\phi}{\bar{q}}\right)\right] e_\theta \quad .$$

Using these relations, the conservation equations can be written explicitly, noticing that $\bar{\mathbf{q}} = \bar{q} e_\phi$ and defining the components of the velocity perturbation

$$q' = u' e_\phi + v' e_\psi + w' e_\theta \ . \tag{27}$$

We obtain:

Continuity (Equation (17) divided by $\bar{\rho}$):

$$\frac{\partial}{\partial t}\left(\frac{\rho'}{\bar{\rho}}\right) + \bar{q}^2\frac{\partial}{\partial \phi}\left(\frac{\rho'}{\bar{\rho}} + \frac{u'}{\bar{q}}\right) + \bar{q}^2\frac{\partial}{\partial \psi}\left(r^2\rho^2\,\frac{v'}{r\bar{\rho}\bar{q}}\right) + \frac{1}{r^2}\,\frac{\partial (r w')}{\partial \theta} = 0 \quad . \tag{28}$$

Momentum Equation (18), taking into account (21):

ϕ-component (divided by $\bar{q}$):

$$\frac{\partial}{\partial t}\left(\frac{u'}{\bar{q}}\right) + \frac{\partial}{\partial \phi}\left(\overline{q}^2 \frac{u'}{\bar{q}}\right) + \frac{\partial}{\partial \phi}\left(\frac{p'}{\gamma\bar{p}}\right) = \frac{1}{2}\frac{\partial \overline{q^2}}{\partial \phi} s' . \tag{29}$$

ψ-component (divided by $r\bar{\rho}\bar{q}$):

$$\frac{\partial}{\partial t}\left(\frac{v'}{r\bar{\rho}\bar{q}}\right) + \overline{q}^2 \frac{\partial}{\partial \phi}\left(\frac{v'}{r\bar{\rho}\bar{q}}\right) + \frac{\partial \overline{q^2}}{\partial \psi}\frac{u'}{\bar{q}} + \frac{\partial}{\partial \psi}\left(\frac{p'}{\gamma\bar{p}}\right) = \frac{1}{2}\frac{\partial \overline{q^2}}{\partial \psi} s' . \tag{30}$$

θ-component (multiplied by r) taking into account that, in view of its axial symmetry, the unperturbed flow does not depend on θ :

$$\frac{\partial}{\partial t}(rw') + \overline{q}^2 \frac{\partial}{\partial \phi}(rw') + \frac{\partial}{\partial \theta}\left(\frac{p'}{\gamma\bar{p}}\right) = 0 . \tag{31}$$

Entropy, Equation (19):

$$\frac{\partial s'}{\partial t} + \overline{q}^2 \frac{\partial s'}{\partial \phi} = 0 . \tag{32}$$

In view of (9), Equation (20) can be rewritten in the form

$$s' = \frac{1}{\overline{c}^2}\frac{p'}{\gamma\bar{p}} - \frac{\rho'}{\bar{\rho}} . \tag{33}$$

The preceding equations have been purposely written in a way that shows that, instead of just the perturbations, it is convenient to choose the dependent variables to be the combinations $\rho'/\bar{\rho}$, $u'/\bar{q}$, $v'/r\bar{\rho}\bar{q}$, rw' , $p'/\gamma\bar{p}$, and s' .

We see at once that, from the preceding six equations in these six dependent variables, one can obtain a system of four equations in the first four of the dependent variables just listed. In fact, s' can be obtained from Equation (32) independently of the other equations once the corresponding s'-distribution is prescribed at the nozzle entrance as a function of time, after which $p'/\gamma\bar{p}$ can be obtained from (33) in terms of $\rho'/\bar{\rho}$. However, the resulting system is still way too complicated to be amenable to solution. A major step in the way to a solution would be accomplished if the variables were separable, which they are not in the system as it is.

5. CHOICE OF THE INDEPENDENT VARIABLES: BIDIMENSIONAL NOZZLE

If the nozzle, and the corresponding unperturbed flow, are two-dimensional, the independent variables are chosen in a similar way. According to Equations (15) and (16), Equations (22) can be replaced by

$$\bar{q} = \frac{d\phi}{\delta s}; \qquad b\bar{\rho}\bar{q} = \frac{d\psi}{\delta n} . \tag{34}$$

δs and δn are contained in the plane of the unperturbed flow and the square of the elementary length is given by

$$dl^2 = \frac{1}{\bar{q}^2} d\phi^2 + \left(\frac{1}{b\bar{\rho}\bar{q}}\right)^2 d\psi^2 + dy^2 ,$$

where dy represents the (nondimensional) spanwise coordinate. Hence we obtain, instead of (23) (replacing the θ-variable with the z-variable),

$$h_\phi = \frac{1}{\bar{q}}; \qquad h_\psi = \frac{1}{b\bar{\rho}\bar{q}}; \qquad h_y = 1$$

and, instead of (24), (25), and (26),

$$\nabla f = \left(\bar{q}\frac{\partial f}{\partial \phi}\right) \mathbf{e}_\phi + \left(b\bar{\rho}\bar{q}\frac{\partial f}{\partial \psi}\right) \mathbf{e}_\psi + \frac{\partial f}{\partial y} \mathbf{e}_y \tag{35}$$

$$\nabla \cdot \mathbf{F} = b\bar{\rho}\bar{q}^2 \left[\frac{\partial}{\partial \phi}\left(\frac{F_\phi}{b\bar{\rho}\bar{q}}\right) + \frac{\partial}{\partial \psi}\left(\frac{F_\psi}{\bar{q}}\right) + \frac{\partial}{\partial y}\left(\frac{F_y}{b\bar{\rho}\bar{q}^2}\right)\right] \tag{36}$$

$$\nabla \times \mathbf{F} = b\bar{\rho}\bar{q}\left[\frac{\partial F_y}{\partial \psi} - \frac{\partial}{\partial y}\left(\frac{F_\psi}{b\bar{\rho}\bar{q}}\right)\right] \mathbf{e}_\phi + \bar{q}\left[\frac{\partial}{\partial y}\left(\frac{F_\phi}{\bar{q}}\right) - \frac{\partial F_y}{\partial \phi}\right] \mathbf{e}_\psi + \\ + b\bar{\rho}\bar{q}^2\left[\frac{\partial}{\partial \phi}\left(\frac{F_\psi}{b\bar{\rho}\bar{q}}\right) - \frac{\partial}{\partial \psi}\left(\frac{F_\phi}{\bar{q}}\right)\right] \mathbf{e}_y . \tag{37}$$

Applying these relations, noticing again that $\bar{\mathbf{q}} = \bar{q}\mathbf{e}_\phi$ and defining

$$\mathbf{q}' = u'\mathbf{e}_\phi + v'\mathbf{e}_\psi + w'\mathbf{e}_y ,$$

Equations (17) to (20) can be explicitly expressed as follows.

Continuity (divided by $\bar{\rho}$):

$$\frac{\partial}{\partial t}\left(\frac{\rho'}{\bar{\rho}}\right) + \bar{q}^2\frac{\partial}{\partial \phi}\left(\frac{\rho'}{\bar{\rho}} + \frac{u'}{\bar{q}}\right) + b^2\bar{\rho}^2\bar{q}^2\frac{\partial}{\partial \psi}\left(\frac{v'}{b\bar{\rho}\bar{q}}\right) + \frac{\partial w'}{\partial y} = 0 . \tag{38}$$

Momentum:

ϕ-component (divided by $\bar{q}$):

$$\frac{\partial}{\partial t}\left(\frac{u'}{\bar{q}}\right) + \frac{\partial}{\partial \phi}\left(\bar{q}^2\frac{u'}{\bar{q}}\right) + \frac{\partial}{\partial \phi}\left(\frac{p'}{\gamma\bar{p}}\right) = \frac{1}{2}\frac{\partial \bar{q}^2}{\partial \phi} s' . \tag{39}$$

ψ-component (divided by $b\bar{\rho}\bar{q}$):

$$\frac{\partial}{\partial t}\left(\frac{v'}{b\bar{\rho}\bar{q}}\right) + \overline{q}^2 \frac{\partial}{\partial \psi}\left(\frac{v'}{b\bar{\rho}\bar{q}}\right) + \frac{\partial \overline{q}^2}{\partial \psi}\frac{u'}{\bar{q}} + \frac{\partial}{\partial \psi}\left(\frac{p'}{\gamma\bar{\rho}}\right) = \frac{1}{2}\frac{\partial \overline{q}^2}{\partial \psi} s' \quad . \tag{40}$$

y-component, taking into account that the unperturbed flow does not depend on y :

$$\frac{\partial w'}{\partial t} + \overline{q}^2 \frac{\partial w'}{\partial \phi} + \frac{\partial}{\partial y}\left(\frac{p'}{\gamma\bar{\rho}}\right) = 0 \quad . \tag{41}$$

Entropy:

$$\frac{\partial s'}{\partial t} + q^2 \frac{\partial s'}{\partial \phi} = 0 \; ; \qquad s' = \frac{1}{\bar{c}^2}\frac{p'}{\gamma\bar{\rho}} - \frac{\rho'}{\bar{\rho}} \quad . \tag{42}$$

These last two equations are the same as for the axisymmetric nozzle, and can be used to reduce to four the number of dependent variables. However the same difficulty as before is found, due to the complexity of the equations which prevents the separation of the variables.

6. SEPARATION OF THE VARIABLES FOR ONE-DIMENSIONAL UNPERTURBED FLOW: AXISYMMETRIC NOZZLE

The chief obstacle that prevents the separation of the variables in Equations (28) to (33) is the fact that $\bar{q}$ and $\bar{\rho}$ depend on both ϕ and ψ in a way determined by the solution of (12). An additional obstacle is due to the presence of the factors r^2 and r^{-2} appearing in the two last terms of Equation (28). These factors should be expressed in terms of the independent variables ϕ, ψ in a way depending again on the solution of (12).

Both obstacles are removed if the unperturbed flow is one-dimensional. This means that the dependence of $\bar{q}$ and $\bar{\rho}$ on ψ can be practically disregarded, so that they can be considered practically uniform on each surface ϕ = constant . It means also that the angle of obliquity of the streamlines with respect to the axis of symmetry is sufficiently small so that its cosine is practically 1 and the element of normal δn along the surface ϕ = constant can be identified with dr . Hence Equation (22) can be integrated, providing

$$\psi = \bar{\rho}(\phi)\bar{q}(\phi)\frac{r^2}{2}$$

or

$$r^2 = \frac{2\psi}{\bar{\rho}\bar{q}} , \tag{43}$$

which provides a simple expression for r^2 , to be introduced in (28). An additional advantage of dropping the dependence of $\bar{q}$ on ψ is that two terms of Equation (30) vanish identically.

Without writing down explicitly the equations obtained assuming $\bar{q} = \bar{q}(\phi)$, $\bar{\rho} = \bar{\rho}(\phi)$ and introducing (43), we observe that the variables are now separable. In fact, if we assume a harmonic time dependence, expressed in the complex form, and express the dependent variables as follows*:

$$\left.\begin{aligned}
\frac{\rho'}{\bar{\rho}} &= R(\phi)\Psi(\psi)\Theta(\theta)e^{i\omega t} \\
\frac{u'}{\bar{q}} &= U(\phi)\Psi(\psi)\Theta(\theta)e^{i\omega t} \\
\frac{v'}{r\bar{\rho}\,\bar{q}} &= V(\phi)\Psi'(\psi)\Theta(\theta)e^{i\omega t} \\
rw' &= W(\phi)\Psi(\psi)\Theta'(\theta)e^{i\omega t} \\
\frac{p'}{\gamma\bar{p}} &= P(\phi)\Psi(\psi)\Theta(\theta)e^{i\omega t} \\
s' &= S(\phi)\Psi(\psi)\Theta(\theta)e^{i\omega t} \ ,
\end{aligned}\right\} \tag{44}$$

with $\Psi'(\psi) = d\Psi/d\psi$, $\Theta'(\theta) = d\Theta/d\theta$, and ω representing the nondimensional angular frequency, related to the dimensional ω^* by

$$\omega = \frac{L^*}{\bar{c}^{o*}}\omega^* \ . \tag{45}$$

The equations, divided by $\Psi\Theta \exp(i\omega t)$, take the following form:

$$i\omega R + \overline{q^2}R' + \overline{q^2}U' + 2\bar{\rho}\bar{q}V\left(\psi\frac{\Psi''}{\Psi} + \frac{\Psi'}{\Psi}\right) + \frac{\bar{\rho}\bar{p}W}{2\psi}\frac{\Theta''}{\Theta} = 0 \tag{46}$$

$$i\omega U + \frac{d}{d\phi}(\overline{q^2}U) + P' - \frac{1}{2}\frac{d\overline{q^2}}{d\phi}S = 0 \tag{47}$$

$$(i\omega V + \overline{q^2}V' + P)\frac{\Psi'}{\Psi} = 0 \tag{48}$$

$$(i\omega W + \overline{q^2}W' + P)\frac{\Theta'}{\Theta} = 0 \tag{49}$$

* The various functions appearing in (44) may be complex functions of the corresponding variable.

$$i\omega S + \overline{q^2} S' = 0 \tag{50}$$

$$S = \frac{1}{c^2} P - R \ , \tag{51}$$

where, again, primes represent differentiations with respect to the corresponding independent variable.

It is interesting to express the components of the flow vorticity using (26) and (44). The result is

$$\nabla \times \mathbf{q} = \bar{\rho}\bar{q}(W - V)\Psi'\Theta' e^{i\omega t} \mathbf{e}_\phi + \frac{\bar{q}}{r}(U - W')\Psi\Theta' e^{i\omega t}\mathbf{e}_\psi + r\overline{\rho}\overline{q^2}(V' - U)\Psi'\Theta e^{i\omega t}\mathbf{e}_\theta \ . \tag{52}$$

We see that the variables are separated in Equations (47) to (51), which are made up of factors depending on a single independent variable. Thus the only requirement for complete separation is that the variables ψ and θ disappear from (46). For θ to disappear, Θ''/Θ must be a constant; more precisely, in view of the necessary periodicity of Θ, we must have

$$\frac{\Theta''}{\Theta} = -\nu^2 \ . \tag{53}$$

ν representing an integer.

Introducing (53) in Equation (46), it appears that the dependence on ψ disappears only if $V = W$ and $\Psi(\psi)$ satisfies the equation

$$\psi\frac{\Psi''}{\Psi} + \frac{\Psi'}{\Psi} - \frac{\nu^2}{4\psi} = \text{constant} \ , \tag{54}$$

with the value of the constant to be determined from the boundary conditions.

This result shows that the requirement for the separation of the variables introduces a certain restriction on the solution of our equations. In fact, the condition

$$V = W \tag{55}$$

is equivalent, as (52) shows, to the condition that the vorticity component of the perturbed flow along the streamlines must be identically zero. The resulting loss of generality does not appear too important, since in most practical cases no mechanism is present to generate axial vorticity in combustion chambers.

The general solution of (53) is*

$$\Theta = \cos\nu(\theta - \alpha) \ ; \qquad \Theta = e^{\pm i\nu(\theta-\alpha)} \ , \tag{56}$$

where α represents a constant, and an unessential multiplicative constant has been taken to be unity.

* The following discussion concerning the Θ and Ψ functions proceeds along the well known lines used for acoustic oscillations in cylinders[5]. We repeat it here for the purpose of completeness.

The first (56) leads to a standing mode of oscillation, as clearly seen from the fact that the amplitude of each of the perturbations defined by (44) vanishes on ν fixed diametrical planes of the nozzle. The second (56) leads to a rotating mode of oscillations, where the amplitude of the perturbations remains constant for fixed values of $\pm\nu\theta + \omega t$. The angular speed of the rotating modes is $d\theta/dt = \mp\omega/\nu$. In both cases ν represents the number of the diametral nodal planes (standing or rotating) for the mode under consideration.

Next, consider (54). Taking $(\psi)^{\frac{1}{2}}$ as independent variable, this is transformed into a Bessel equation of order ν . The general solution, if the constant appearing on the right-hand side is negative*, and set equal to $-k^2/4$, is given by

$$\Psi = C_3 J_\nu(k[\psi]^{\frac{1}{2}}) + C_4 Y_\nu(k[\psi]^{\frac{1}{2}}) ,$$

where J_ν and Y_ν are Bessel functions of the first and second kind, respectively. Since Y_ν does not remain finite as ψ approaches zero, C_4 must vanish identically if the perturbations are to be finite on the nozzle axis. Hence, setting the unessential multiplicative constant C_3 to be unity, we have

$$\Psi(\psi) = J_\nu(k[\psi]^{\frac{1}{2}}) . \tag{57}$$

The constant k is determined by considering the boundary condition $v' = 0$ at the nozzle wall, which is a $\psi =$ constant surface. Calling ψ_w the value of the stream function at the wall†, and recalling (43), the condition $v' = 0$ at the wall is satisfied if

$$J_\nu'(k[\psi]^{\frac{1}{2}}) = 0 . \tag{58}$$

The prime here indicates differentiation with respect to the argument. Hence, if $s_{\nu h}$ is the h^{th} zero of $J_\nu'(x)$, in order to satisfy (58) we must have $k = s_{\nu h}/(\psi_w)^{\frac{1}{2}}$ and (57) becomes**:

$$\Psi(\psi) = J_\nu(s_{\nu h}[\psi/\psi_w]^{\frac{1}{2}}) = J_\nu\left(s_{\nu h}\frac{r}{r_w}\right) . \tag{59}$$

the last term being obtained from (43). Observe that the last form of $\Psi(\psi)$ and the forms (56) of $\Theta(\theta)$ are exactly the same as for the acoustic oscillations of a gas in a cylindrical chamber, the only difference being that in (59) r_w is the variable radius, $r_w(\phi)$, of the nozzle sections, instead of a constant. Hence within the validity of the one-dimensional assumption for the unperturbed flow, the perturbations are distributed on each section of the nozzle in the same way as they are in the acoustic oscillations in a cylindrical chamber. Corresponding to the lowest values of ν and h we have for $s_{\nu h}$ the values given in Table I on p.84 (Ref.5)††.

* Positive values of the constant, leading to the Bessel functions of imaginary argument, would not allow the boundary condition at the nozzle wall to be satisfied.

† The value of ψ_w is immediately obtained from (43) evaluated at the throat or at the nozzle entrance, once the reference length L^* has been chosen. Appropriate values for L^* are either the radius of the throat section or the radius of the entrance section of the nozzle.

** The index h has a meaning similar to that of ν ; $h - 1$ represents the number of nodal circles for the pressure perturbations, that is, the number of circles on each section on which the pressure does not oscillate.

†† The argument of the Bessel function in Reference 5 is $\pi s_{\nu h} r/r_w$ so that their eigenvalues must be multiplied by π in order to obtain the values in Table I.

Inserting in Equation (46) the relations (55), (53), and (54) with the value of the constant in the last relation given by $-k^2/4 = -s_{\nu h}^2/4\psi_w$, we obtain

$$i\omega R + \overline{q^2}R' + \overline{q^2}U' - \frac{s_{\nu h}^2}{2\psi_w}\overline{\rho}\overline{q}V = 0 . \tag{60}$$

The separation of the variables is now complete.

7. SEPARATION OF THE VARIABLES FOR ONE-DIMENSIONAL UNPERTURBED FLOW: BIDIMENSIONAL NOZZLE

When $\overline{q}$ and $\overline{\rho}$ are taken to be functions only of ϕ , Equations (38) to (42) become separable too. We take

$$\left.\begin{aligned}
\frac{\rho'}{\overline{\rho}} &= R_1(\phi)\Psi_1(\psi)Y(y)e^{i\omega t} \\
\frac{u'}{\overline{q}} &= U_1(\phi)\Psi_1(\psi)Y(y)e^{i\omega t} \\
\frac{v'}{b\overline{\rho}\overline{q}} &= V_1(\phi)\Psi_1'(\psi)Y(y)e^{i\omega t} \\
w' &= W_1(\phi)\Psi_1(\psi)Y'(y)e^{i\omega t} \\
\frac{p'}{\gamma\overline{p}} &= P_1(\phi)\Psi_1(\psi)Y(y)e^{i\omega t} \\
s' &= S_1(\phi)\Psi_1(v)Y(y)e^{i\omega t} ,
\end{aligned}\right\} \tag{61}$$

and replace these expressions in the equations. Dividing all equations by $\Psi_1(\psi)Z(z)\exp(i\omega t)$, we obtain

$$i\omega R_1 + \overline{q^2}R_1' + \overline{q^2}U_1' + b^2\overline{\rho^2}\overline{q^2}V_1\frac{\Psi_1''}{\Psi_1} + W_1\frac{Y''}{Y} = 0 \tag{62}$$

$$i\omega U_1 + \overline{q^2}U_1' + \frac{d\overline{q^2}}{d\phi}U_1 + P_1' - \frac{1}{2}\frac{d\overline{q^2}}{d\phi}S_1 = 0 \tag{63}$$

$$(i\omega V_1 + \overline{q^2}V_1' + P_1)\frac{\Psi_1'}{\Psi} = 0 \tag{64}$$

$$(i\omega W_1 + \overline{q^2}W_1' + P_1)\frac{Y'}{Y} = 0 \tag{65}$$

$$i\omega S_1 + \bar{q}^2 S_1' = 0 \tag{66}$$

$$S_1 = \frac{1}{\bar{c}^2} P_1 - R_1 . \tag{67}$$

The flow vorticity is given by (37) after substitution of (61):

$$\nabla \times q = b\bar{\rho}\bar{q}(W_1 - V_1)\Psi_1' Y' e^{i\omega t} e_\phi + q(U_1 - W_1')\Psi_1 Y' e^{i\omega t} e_\psi +$$

$$+ b\bar{\rho}\bar{q}^2(V_1' - U_1)\Psi_1' Y e^{i\omega t} e_z . \tag{68}$$

We see that in Equations (63) to (67) the variables are separated. In order that the separation may be complete we must have

$$\frac{\Psi_1''}{\Psi_1} = C_\psi ; \qquad \frac{Y''}{Y} = C_y , \tag{69}$$

C_ψ and C_y representing constants to be determined through the boundary conditions.

We notice that, contrary to the axisymmetric nozzle, in the two-dimensional case no restriction is placed on the solution by the requirement of variable separation, and that, according to (68), the vorticity of the flow may have non-vanishing components in any direction. The values of C_ψ and C_y are determined by the boundary conditions of vanishing normal velocity at the nozzle walls, that is $v' = 0$ at $\psi = 0$ and $\psi = \psi_w$, and $w' = 0$ at $y = 0$ and $y = b$ *. We observe that, since in the assumption of small obliquity δn may be identified with dx in the second Equation (34), x representing the nondimensional coordinate along the nozzle height, we obtain

$$\psi = b\bar{\rho}\bar{q}x , \qquad \psi_w = b\bar{\rho}\bar{q}a , \tag{70}$$

where $a = a(\phi)$ is the nondimensional height of the nozzle section under consideration. The value of ψ_w (corresponding, as for the axisymmetric nozzle, to the total flux of mass through the nozzle) can be determined once the reference length L^* has been chosen. According to (61) the boundary conditions can be written

$$\Psi_1'(0) = \Psi_1'(\psi_w) = 0 ; \qquad Y'(0) = Y'(b) = 0 . \tag{71}$$

The proper solutions of (69) satisfying (71) are

$$\left.\begin{aligned} \Psi_1(\psi) &= \cos\left(M\pi \frac{\psi}{\psi_w}\right) = \cos\left(M\pi \frac{x}{a}\right) \\ Y(y) &= \cos\left(N\pi \frac{y}{b}\right) , \end{aligned}\right\} \tag{72}$$

* For the sake of symmetry, the "axis" of the two-dimensional nozzle should be taken as the line $\psi = 0$, $y = 0$. It is more convenient, however, to take $\psi = 0$ on one of the curved walls of the nozzle, and $y = 0$ on one of the plane walls. The other curved wall is then a ψ = constant = ψ_w surface, and the other plane wall is the $y = b$ surface.

M and N representing integral numbers defining the mode of transverse oscillation. Their meaning is immediately found to be the number of nodal surfaces for the pressure perturbation in the corresponding direction. Equations (72) are the same as for the acoustic oscillations of a gas in a chamber of rectangular cross-section, so that again we find that under the assumption of one-dimensional unperturbed flow the perturbations are distributed in a similar fashion on each cross-section of the actual duct as they are in the acoustic oscillations in a rectangular chamber.

The comparison of (72) and (69) gives

$$\frac{\Psi_1''}{\Psi_1} = C_\psi = -\frac{M^2\pi^2}{\psi_w^2} ; \qquad \frac{Y''}{Y} = C_y = -\frac{N^2\pi^2}{b^2} .$$

Replacing these values in (62), and writing for brevity

$$s_M = \frac{M\pi b}{\psi_w} ; \qquad \sigma_N = \frac{N\pi}{b} , \tag{73}$$

we obtain

$$i\omega R_1 + \overline{q^2}R_1' + \overline{q^2}U_1' - s_M^2\overline{\rho^2}\overline{q^2}V_1 - \sigma_N^2 W_1 = 0 . \tag{74}$$

The separation of the variables is now complete.

8. REDUCTION OF THE SYSTEM: AXISYMMETRIC NOZZLE

We first observe that the case $\nu = 0$, $h = 1$, resulting in $s_{\nu h} = 0$ corresponds to purely axial oscillations, since Θ and Ψ become constants. In this case (48) and (49) are automatically satisfied, and Equations (46), (47), (50), and (51) contain only the variables R , U , P , and S . The solution for this case has been discussed in References 2 and 3, but it will again be included as a particular case in the following treatment.

Similarly, the cases in which either $\nu = 0$ or $h = 1$ (purely radial or purely tangential oscillations) can be included in the following treatment as particular cases. We shall, therefore, discuss the case where ν and h are arbitrary integers.

In this case, in order to satisfy (48) and (49), the corresponding expressions in parentheses must vanish. Subtracting one from the other we eliminate P and obtain

$$i\omega(W - V) + \overline{q^2}\frac{d}{d\phi}(W - V) = 0 . \tag{75}$$

The general solution of this can be written in the form

$$W - V = C_0 f_0 , \tag{76}$$

where, for brevity, we have written

$$f_0(\phi) = e^{-i\omega} \int_0^{\phi} \frac{d\phi'}{\overline{q^2}(\phi')} , \tag{77}$$

ϕ' being an integration variable and the lower integration limit being arbitrarily set at the throat. Clearly the integration constant C_0 has to be taken zero in order to satisfy (55), with the aforementioned loss of one degree of freedom in the solution.

Equations (50) and (51) result in

$$S(\phi) = \frac{P(\phi) - \overline{c^2}R(\phi)}{\overline{c^2}} = \sigma f_0 , \tag{78}$$

$\sigma = S(\phi_{th})$ being an integration constant.

Introducing a function $\Phi(\phi)$ such that

$$U = \Phi' , \tag{79}$$

(47) can be integrated to

$$i\omega\Phi + \overline{q^2}\Phi' + P = \sigma f_1 , \tag{80}$$

with

$$f_1(\phi) = \frac{1}{2} \int_{\phi_{th}}^{\phi} f_0(\phi') \frac{d\overline{q^2}}{d\phi'} d\phi' . \tag{81}$$

The integration constant has been incorporated in Φ, which is defined anyway within an arbitrary constant.

Since V satisfies the equation

$$i\omega V + \overline{q^2}V' + P = 0 ,$$

we can eliminate P from this equation and (80), obtaining

$$i\omega(\Phi - V) + \overline{q^2} \frac{d}{d\phi} (\Phi - V) = \sigma f_1 ,$$

which, upon integration, provides

$$\Phi - V = \sigma f_2 + C_1 f_0 \tag{82}$$

with

$$f_2(\phi) = f_0(\phi)\int_{\phi_{th}}^{\phi} \frac{f_1(\phi')}{\overline{q^2}(\phi')f_0(\phi')}\,d\phi' . \tag{83}$$

Using (78), Equation (80) can be rewritten as

$$i\omega\Phi + \overline{q^2}\Phi' + \overline{c^2}R = \sigma(f_1 - \overline{c^2}f_0) .$$

Equation (60) can be written in the same dependent variables making use of (82), as

$$i\omega R + \overline{q^2}R' + \overline{q^2}\Phi'' - \frac{s_{\nu h}^2}{2\psi_w}\overline{\rho}\overline{q}\Phi = -\frac{s_{\nu h}^2}{2\psi_w}\overline{\rho}\overline{q}(\sigma f_2 + C_1 f_0) .$$

Elimination of R from these two equations and use of (13) produce the following equation

$$\overline{q^2}(\overline{c^2} - \overline{q^2})\Phi'' - \overline{q^2}\left(\frac{1}{\overline{c^2}}\frac{d\overline{q^2}}{d\phi} + 2i\omega\right)\Phi' + \left(\omega^2 - \frac{\gamma - 1}{2}\frac{i\omega\overline{q^2}}{\overline{c^2}}\frac{d\overline{q^2}}{d\phi} - \frac{s_{\nu h}^2}{2\psi_w}\overline{\rho}\overline{q}\overline{c^2}\right)\Phi$$

$$= -\sigma\overline{c^2}\left[i\omega\left(\frac{f_1}{\overline{c^2}} - f_0\right) + \overline{q^2}\frac{d}{d\phi}\left(\frac{f_1}{\overline{c^2}} - f_0\right) + \frac{s_{\nu h}^2}{2\psi_w}\overline{\rho}\overline{q}f_2\right] - C_1\frac{s_{\nu h}^2}{2\psi_w}\overline{\rho}\overline{q}\overline{c^2}f_0 . \tag{84}$$

This is the fundamental equation to which our system can be reduced; its solution can only be obtained by numerical integration*, using the appropriate boundary conditions.

The right-hand side of (84) can be written in a somewhat simpler form by introducing the function

$$f_3 = \frac{f_1}{\overline{c^2}f_0} \tag{85}$$

and observing that the first two terms within the brackets can be written as

$$i\omega f_0(f_3 - 1) + \overline{q^2}\frac{d}{d\phi}[f_0(f_3 - 1)] = \overline{q^2}\left\{\frac{d}{d\phi}[f_0(f_3 - 1)] - (f_3 - 1)\frac{df_0}{d\phi}\right\} = \overline{q^2}f_0\frac{df_3}{d\phi} .$$

Use has been made of the equation

$$\overline{q^2}\frac{df_0}{d\phi} + i\omega f_0 = 0 , \tag{86}$$

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* An exception is represented by the case of axial oscillations ($s_{\nu h} = 0$) and $\overline{q^2}$ linear with ϕ, in which case (84) reduces to the hypergeometric equation[2,3].

which f_0 satisfies. The last term within the brackets can also be expressed through f_3, observing that with the help of (86) we get from (83)

$$f_2 = \frac{f_0}{i\omega}\int_{\phi_{th}}^{\phi} f_1 d\left(\frac{1}{f_0}\right) = \frac{1}{i\omega}\left(f_1 - f_0\int_{\phi_{th}}^{\phi}\frac{df_1}{f_0}\right)$$

$$= \frac{1}{i\omega}\left(f_1 - f_0\,\frac{\overline{q^2} - \overline{q^2_{th}}}{2}\right) = \frac{f_0}{i\omega}\left(\overline{c^2}f_3 - \frac{\overline{q^2} - \overline{q^2_{th}}}{2}\right) . \tag{87}$$

Hence (84) can be written in the form

$$L(\Phi) = -\overline{c^2}f_0(C_1F^{(1)} + \sigma F^{(2)}) , \tag{88}$$

where $L(\Phi)$ represents the left-hand side of Equation (84) and the functions F are expressed in terms of f_3 alone by

$$F^{(1)} = \frac{s^2_{\nu h}}{2\psi_w}\bar{\rho}\bar{q} ; \qquad F^{(2)} = q^2\frac{df_3}{d\phi} + \frac{s^2_{\nu h}}{2\psi_w}\,\frac{\bar{\rho}\bar{q}}{2i\omega}\,(2\overline{c^2}f_3 + \overline{q^2_{th}} - \overline{q^2}) . \tag{89}$$

The flow rotation can also be calculated from (52). Because of (55) the ϕ-component vanishes, while the other two components are proportional to $U - V'$. In view of (79) and (82) this can be written as

$$\Phi' - V' = \sigma\frac{df_2}{d\phi} + C_1\frac{df_0}{d\phi} .$$

From the third expression (87) for f_2 and (81) one obtains

$$\frac{df_2}{d\phi} = -\frac{1}{2i\omega}\,\frac{df_0}{d\phi}\,(\overline{q^2} - \overline{q^2_{th}}) ,$$

where $\overline{q_{th}} = [2/(\gamma + 1)]^{\frac{1}{2}}$ is the value at the throat, obtained from (13) by setting $\bar{q} = \bar{c}$. Hence, making use of (86), one has

$$\Phi' - V' = \frac{f_0}{2q^2}\,[\sigma(\overline{q^2} - \overline{q^2_{th}}) - 2i\omega C_1] ,$$

and the vorticity* is given by

$$\nabla \times q = \frac{\sigma(\overline{q^2} - \overline{q^2_{th}}) - 2i\omega C_1}{2}\,\exp\left\{i\omega\left(t - \int_{\phi_{th}}^{\phi}\frac{d\phi'}{q^2}\right)\right\}\left(\frac{\Psi\Theta'}{r\bar{q}}\,e_{\psi} - r\bar{\rho}\Psi'\Theta e_{\theta}\right) .$$

* Taking into account that the variables are different we have

$$\Psi' = d\Psi/d\psi = (s_{\nu h}/2[\psi\psi_w]^{\frac{1}{2}})J'_{\nu}(s_{\nu h}[\psi/\psi_w]^{\frac{1}{2}}) .$$

Observe that both $\Psi\Theta'/r$ and $r\Psi'\Theta$ vanish at $r = 0$ for $\nu \neq 1$. Only for $\nu = 1$ are they finite at $r = 0$ and only for this ν can there exist a vorticity on the nozzle axis. Observe also that the subtractive term in the exponential represents the travel time of a gas particle to any station ϕ; therefore the exponential exhibits the transport of vorticity with the fluid elements. Clearly for isentropic oscillations ($\sigma = 0$) and $C_1 = 0$ the oscillations are also irrotational, but, if there is an entropy perturbation, vorticity must be present no matter how C_1 is chosen, except for axial oscillation, when $\Psi' = \Theta' = 0$.

9. REDUCTION OF THE SYSTEM: BIDIMENSIONAL NOZZLE

The developments of Section 8 can be repeated almost identically for the bidimensional nozzle. The case of purely axial oscillations (obtained for $M = N = 0$) provides equations which coincide with those of the axisymmetric nozzle. The difference between the two only appears when transverse oscillations are present. If M or N is zero the respective equation (64) or (65) is satisfied identically and the corresponding dependent variable V_1 or W_1 disappears from the equations. All these cases can be obtained as particular cases from the case when neither M nor N vanish; in which case we obtain, from (64) and (65),

$$W_1 - V_1 = C_0 f_0 . \tag{90}$$

C_0 being an integration constant which (contrary to the case of circular section) can be taken non-vanishing.

From (66) and (67) we get

$$S_1 = \frac{P_1 - \overline{c^2} R_1}{\overline{c^2}} = \sigma f_0 , \tag{91}$$

$\sigma = S(0)$ being again an arbitrary integration constant.

Taking

$$U_1 = \frac{d\Phi_1}{d\phi} \tag{92}$$

one obtains, from (63),

$$i\omega\Phi_1 + \overline{q^2}\Phi_1' + P = \sigma f_1 . \tag{93}$$

Combining this with (64) we obtain, as before,

$$\Phi_1 - V_1 = \sigma f_2 + C_1 f_0 . \tag{94}$$

Hence we obtain, from (91) and (93),

$$i\omega\Phi_1 + \overline{q^2}\Phi_1' + \overline{c^2}R_1 = \sigma(f_1 - \overline{c^2} f_0)$$

and, from (74), making use of (90), (92), and (94),

$$i\omega R_1 + \overline{q^2}R_1' + \overline{q^2}\Phi_1'' - (s_M^2\overline{\rho^2}\,\overline{q^2} + \sigma_N^2)\Phi = -(\sigma f_2 + C_1 f_0)(s_M^2\overline{\rho^2}\,\overline{q^2} + \sigma_N^2) + \sigma_N^2 C_0 f_0 .$$

Eliminating R from the last two equations we obtain

$$\overline{q^2}(\overline{c^2} - \overline{q^2})\Phi_1'' - \overline{q^2}\left(\frac{1}{\overline{c^2}}\frac{d\overline{q^2}}{d\phi} + 2i\omega\right)\Phi_1' + \left[\omega^2 - \frac{\gamma - 1}{2}\frac{i\omega\overline{q^2}}{\overline{c^2}}\frac{d\overline{q^2}}{d\phi} - (s_M^2\overline{\rho^2}\,\overline{q^2} + \sigma_N^2)\overline{c^2}\right]\Phi_1$$

$$= -\sigma\overline{c^2}\left[i\omega\left(\frac{f_1}{\overline{c^2}} - f_0\right) + \overline{q^2}\frac{d}{d\phi}\left(\frac{f_1}{\overline{c^2}} - f_0\right) + (s_M^2\overline{\rho^2}\,\overline{q^2} + \sigma_N^2)f_2\right] -$$

$$- C_1\overline{c^2}(s_M^2\overline{\rho^2}\,\overline{q^2} + \sigma_N^2)f_0 + C_0\overline{c^2}\sigma_N^2 f_0 , \quad (95)$$

which can also be written

$$L_1(\Phi_1) = -\overline{c^2}f_0(C_0F_1^{(0)} + C_1F_1^{(1)} + \sigma F_1^{(2)}) , \quad (96)$$

where $L_1(\Phi_1)$ represents the left-hand side of (95) and

$$\left.\begin{aligned} F_1^{(0)} &= -\sigma_N^2 ; \\ F_1^{(1)} &= s_M^2\overline{\rho^2}\,\overline{q^2} + \sigma_N^2 ; \\ F_1^{(2)} &= \overline{q^2}\frac{df_3}{d\phi} + (s_M^2\overline{\rho^2}\,\overline{q^2} + \sigma_N^2)\frac{2\overline{c^2}f_3 + \overline{q_{th}^2} - \overline{q^2}}{2i\omega} . \end{aligned}\right\} \quad (97)$$

From (68) we obtain the vorticity. Following a line similar to that of Section 8 we obtain

$$\nabla \times u = \left[C_0 b\overline{\rho}\,\overline{q}\psi_1' Y' e_\phi + \frac{\sigma(\overline{q^2} - \overline{q_{th}^2}) - 2i\omega(C_1 - C_0)}{2\overline{q}}\Psi_1 Y' e_\psi - \right.$$

$$\left. - b\overline{\rho}\frac{\sigma(\overline{q^2} - \overline{q_{th}^2}) - 2i\omega C_1}{2}\Psi_1' Y e_y\right] \exp\left\{i\omega\left(t - \int_{\phi_{th}}^{\phi}\frac{d\phi'}{\overline{q^2}}\right)\right\} .$$

Again we notice the exponential expressing the vorticity property of being transported with the fluid, and again we notice that the vorticity cannot vanish identically with noniseutropic oscillations, except for purely axial oscillations, when $\Psi_1' = Z' = 0$.

10. ADMITTANCE CONDITION AT THE ENTRANCE OF AN AXISYMMETRIC NOZZLE

The general solution of (88) is made of the general solution of the homogeneous equation and of a particular solution of (88). In other words, if we know the solutions of the equations

$$L(\Phi_h) = 0 \tag{98}$$

$$L(\Phi^{(j)}) = -\overline{c^2} f_0 F^{(j)} \qquad (j = 1, 2) , \tag{99}$$

the general solution is

$$\Phi = C_1 \Phi^{(1)} + \sigma \Phi^{(2)} + C_2 \Phi_h + C_3 \tilde{\Phi}_h , \tag{100}$$

where Φ_h , $\tilde{\Phi}_h$ are two independent solutions of (98) and C_2 , C_3 arbitrary constants. Now observe that (98) has the singular points $\bar{q} = 0$, $\bar{q} = \bar{c} = \overline{q_{th}} = \{2/(\gamma + 1)\}^{\frac{1}{2}}$ and $\bar{q} = \infty$. For a supercritical nozzle with a finite entrance section, only the sonic singularity at the throat is of concern. If the solution must be regular at the throat, only the solution, say Φ_h , which is regular there can appear in (100); and the particular solutions $\Phi^{(j)}$ must also be regular at the sonic point. Since all the singularity appears now in the independent solution $\tilde{\Phi}_h$, the condition of regularity at the throat is simply expressed by $C_3 = 0$.

Hence the proper solution of (88) is

$$\Phi = C_1 \Phi^{(1)} + \sigma \Phi^{(2)} + C_2 \Phi_h , \tag{101}$$

with $\Phi^{(1)}$, $\Phi^{(2)}$, and Φ_h regular at the sonic point.

From (101) one obtains, making use of (79), (82), (55), (80), and (78), the expressions

$$\left.\begin{aligned} U &= C_1 \frac{d\Phi^{(1)}}{d\phi} + \sigma \frac{d\Phi^{(2)}}{d\phi} + C_2 \frac{d\Phi_h}{d\phi} \\ V = W &= C_1(\Phi^{(1)} - f_0) + \sigma(\Phi^{(2)} - f_2) + C_2\Phi_h \\ P + \overline{q^2}U &= - C_1 i\omega\Phi^{(1)} - \sigma(i\omega\Phi^{(2)} - f_1) - C_2 i\omega\Phi_h \\ S &= \sigma f_0 . \end{aligned}\right\} \tag{102}$$

If this is considered to be a system of linear equations for the determination of C_1 , σ , and C_2 with given values of U , $V = W$, P , and S for a given ϕ , we see that only three of these values can be arbitrarily prescribed. In other words, a relation must exist between any such set of four values, a relation that can be obtained from the relation of compatibility of the four linear equations (102), that is from the vanishing of the determinant

$$\begin{vmatrix} U & \dfrac{d\Phi^{(1)}}{d\phi} & \dfrac{d\Phi^{(2)}}{d\phi} & \dfrac{d\Phi_h}{d\phi} \\ V & \Phi^{(1)} - f_0 & \Phi^{(2)} - f_2 & \Phi_h \\ P + \overline{q^2}U & -i\omega\Phi^{(1)} & f_1 - i\omega\Phi^{(2)} & -i\omega\Phi_h \\ S & 0 & f_0 & 0 \end{vmatrix} = 0 \ .$$

Developing the determinant, dividing by $f_0^2\Phi_h$, writing for simplicity

$$\zeta(\phi) = \frac{1}{\Phi_h}\frac{d\Phi_h}{d\phi}; \qquad \xi^{(j)}(\phi) = \frac{1}{\overline{c^2}f_0\Phi_h}\left[\Phi^{(j)}\frac{d\Phi_h}{d\phi} - \Phi_h\frac{d\Phi^{(j)}}{d\phi}\right] , \tag{103}$$

and applying the last relation (87) we obtain

$$U[\overline{q^2}(\overline{c^2}\xi^{(1)} - \zeta) - i\omega] + V i\omega\overline{c^2}\xi^{(1)} + P(\overline{c^2}\xi^{(1)} - \zeta) -$$
$$- S\overline{c^2}\left(\frac{\overline{q^2} - \overline{q_{th}^2}}{2}\xi^{(1)} + i\omega\xi^{(2)} - f_3\zeta\right) = 0 . \tag{104}$$

This relation (104) holds for any value of ϕ; in particular, it holds for the entrance section of the nozzle, where it represents the admittance condition we have been looking for. The admittance condition simplifies in the isentropic case by taking $S = 0$ identically. If the perturbed flow is irrotational according to the discussion of Section 9 on the flow rotation, it must be isentropic as well, so that $C_1 = \sigma = 0$. The corresponding admittance condition can be found directly from (102) which reduce to

$$U = C_2\frac{d\Phi_h}{d\phi}, \qquad V = C_2\Phi_h , \qquad P + \overline{q^2}U = -C_2 i\omega\Phi_h .$$

Eliminating C_2 one obtains

$$U = \zeta V = -\frac{\zeta}{\overline{q^2}\zeta + i\omega}P = \frac{\alpha}{\gamma}P , \tag{105}$$

which represents two admittance conditions when applied to the entrance of the nozzle. Here α represents the *irrotational admittance coefficient*.

For purely axial oscillations, since (48) and (49) are identically satisfied, one has to disregard Equation (82), which makes use of (48). The corresponding equations can be obtained from (102) disregarding the second one, and all terms in C_1.

With the same procedure used above one finds from these equations the corresponding admittance condition

$$U(\overline{q^2}\zeta + i\omega) + P\zeta + S\overline{c^2}(i\omega\xi^{(2)} - f_3\zeta) = 0 . \tag{106}$$

For isentropic oscillations this condition can be used in the form

$$U = -\frac{\zeta}{\overline{q^2}\zeta + i\omega} P = \frac{\alpha}{\gamma} P = \alpha R . \tag{107}$$

In this case α corresponds to the *admittance ratio* of the acousticians. Observe that, contrary to the α of (105), this α has to be calculated for $s_{rh} = 0$.

We see that in every case the knowledge of three functions, ζ , $\xi^{(1)}$, and $\xi^{(2)}$ defined in (103) is sufficient to determine the admittance condition. The numerical procedure to calculate these functions is explained in Section 14. Here, however, it is useful to obtain a more explicit expression for the $\xi^{(j)}$. From (98) and (99), recalling that $L(\Phi)$ is given by the first member of (84), we obtain

$$\frac{1}{\overline{q^2}} \{\Phi^{(j)}L(\Phi_h) - \Phi_h L(\Phi^{(j)})\} = (\overline{c^2} - \overline{q^2})\frac{d}{d\phi}(\overline{c^2}f_0\Phi_h\xi^{(j)}) - \left(\frac{1}{\overline{c^2}}\frac{d\overline{q^2}}{d\phi} + 2i\omega\right)(\overline{c^2}f_0\Phi_h\xi^{(j)})$$

$$= \frac{\overline{c^2}f_0}{q^2}\Phi_h F^{(j)} . \tag{108}$$

This first order equation has the general solution

$$\overline{c^2}f_0\Phi_h\xi^{(j)} = e^{\beta(\phi)}\left[\text{constant} + \int_{\phi_{th}}^{\phi} \frac{\overline{c^2}f_0}{\overline{q^2}(\overline{c^2} - \overline{q^2})}\Phi_h F^{(j)} e^{-\beta(\phi')}\, d\phi'\right] , \tag{109}$$

where

$$\beta(\phi) = 2i\omega\int^{\phi}\frac{d\phi'}{\overline{c^2} - \overline{q^2}} - \log_e\left(1 - \frac{\overline{q^2}}{\overline{c^2}}\right) \tag{110}$$

and ϕ' is again an integration variable, of which all quantities in the integrands are functions. The lower limit in the integral of (110) is essential. The presence of the second term on the right-hand side of (110) causes $\exp(\beta(\phi))$ to become infinite at the sonic point. Clearly the only change for $\xi^{(j)}$ to remain finite there (which is necessary if the solution must be regular) is the vanishing of the integration constant in (109), since the lower limit of the corresponding integral has been taken at the throat. Hence the proper form for the solution of (108) is

$$\xi^{(j)} = \frac{\exp\left\{\int^{\phi}\left(\frac{i\omega}{\overline{q^2}} + \frac{2i\omega}{\overline{c^2} - \overline{q^2}} - \zeta\right)d\phi'\right\}}{\overline{c^2} - \overline{q^2}}\int_{\phi_{th}}^{\phi}\exp\left\{-\int^{\phi'}\left(\frac{i\omega}{\overline{q^2}} + \frac{2i\omega}{\overline{c^2} - \overline{q^2}} - \zeta\right)d\phi''\right\}\frac{F^{(j)}(\phi')}{\overline{q^2}(\phi')}\, d\phi' . \tag{111}$$

where f_0 has been replaced by its expression (77) and Φ_h by

$$\Phi_h = \exp\left\{\int^{\phi} \zeta(\phi')\, d\phi\right\} ,$$

immediately obtained from (103).

We see that $\xi^{(j)}$ can, in principle, be obtained from (111) once ζ is known, using only integration processes, and that the solution of the Equations (99) is not necessary if our scope is limited to the admittance condition. However, in practice (111) presents nasty problems of evaluation, since – as the form of the exponents shows – both the integrand and the factor in front of it tend to oscillate faster and faster when the sonic point is approached, or when $\bar{q}$ becomes small, toward the nozzle entrance. These oscillations are artificial and tend to cancel each other, but they make it difficult to carry on the last integration which would require absurdly small integration intervals. A better way of calculating the $\xi^{(j)}$ is to make use of the equation

$$\frac{d}{d\phi}\,[(\overline{c^2} - \overline{q^2}]\xi^{(j)}] + \left(\zeta - \frac{i\omega}{\overline{q^2}} - \frac{2i\omega}{\overline{c^2} - \overline{q^2}}\right)\,[(\overline{c^2} - \overline{q^2})\xi^{(j)}] = \frac{1}{\overline{q^2}}\,F^{(j)} , \qquad (112)$$

the validity of which can be directly checked from (111).

11. ADMITTANCE CONDITION AT THE ENTRANCE OF A BIDIMENSIONAL NOZZLE

For the bidimensional nozzle one has to consider one constant and one equation more than in the previous case. That is, if the solutions of the equations

$$L_1(\Phi_{1h}) = 0 \qquad (113)$$

$$L_1(\Phi_1^{(j)}) = -\overline{c^2} f_0 F_1^{(j)} \qquad (j = 0, 1, 2) \qquad (114)$$

are known, the general solution of (96) remaining regular at $\bar{q} = \bar{c} = \{2/(\gamma + 1)\}^{\frac{1}{2}} = \overline{q_{th}}$ is

$$\Phi_1 = C_0\Phi_1^{(0)} + C_1\Phi_1^{(1)} + \sigma\Phi_1^{(2)} + C_2\Phi_{1h} ,$$

where all partial solutions must be regular at $\overline{q_{th}}$.

Hence we obtain, from (92), (94), (90), (93), and (91),

$$\left.\begin{aligned} U_1 &= C_0\frac{d\Phi_1^{(0)}}{d\phi} + C_1\frac{d\Phi_1^{(1)}}{d\phi} + \sigma\frac{d\Phi_1^{(2)}}{d\phi} + C_2\frac{d\Phi_{1h}}{d\phi} \\ V_1 &= C_0\Phi_1^{(0)} + C_1(\Phi_1^{(1)} - f_0) + \sigma(\Phi_1^{(2)} - f_2) + C_2\Phi_{1h} \\ V_1 - W_1 &= C_0 f_0 \end{aligned}\right| \qquad (115)$$

$$\left.\begin{aligned} P_1 + \overline{q^2}U_1 &= -C_0 i\omega\Phi_1^{(0)} - C_1 i\omega\Phi_1^{(1)} + \sigma(f_1 - i\omega\Phi_1^{(2)}) - C_2 i\omega\Phi_{1h} \\ S_1 &= \sigma f_0 \quad . \end{aligned}\right\} \tag{115}$$

Again the compatibility condition of these five equations between the four constants, requires the vanishing of the corresponding determinant. After development, and division by $f_0^3\Phi_h$, the condition becomes

$$\begin{aligned} &U_1[\overline{q^2}(\overline{c^2}\xi_1^{(1)} - \zeta) - i\omega] + V_1 i\omega\overline{c^2}(\xi_1^{(1)} - \xi_1^{(0)}) + W_1 i\omega\overline{c^2}\xi_1^{(0)} + \\ &\quad + P_1(\overline{c^2}\xi_1^{(1)} - \zeta) - S_1\overline{c^2}\left(\frac{\overline{q^2} - \overline{q_{th}^2}}{2}\xi_1^{(1)} + i\omega\xi_1^{(2)} - f_3\zeta_1\right) , \end{aligned} \tag{116}$$

with

$$\zeta_1(\phi) = \frac{1}{\Phi_{1h}}\frac{d\Phi_{1h}}{d\phi} ; \qquad \xi_1^{(j)}(\phi) = \frac{1}{\overline{c^2} f_0 \Phi_{1h}}\left[\Phi_1^{(j)}\frac{d\Phi_{1h}}{d\phi} - \Phi_{1h}\frac{d\Phi_1^{(j)}}{d\phi}\right] . \tag{117}$$

When applied at the nozzle entrance, (116) represents the admittance condition.

We notice that if the axial component of the vorticity vanishes ($W_1 = V_1$) Equation (116) becomes identical to (104). However, the functions appearing in the coefficients are different, depending on the solution of different equations. The isentropic case again can be derived immediately from (116) taking $S_1 = 0$, and the isentropic, irrotational case leads to two admittance conditions identical to (105), the only difference being the subscript 1 on the various quantities. Finally, the case of axial oscillations leads to equations identical with (106) and (107). Evidently in this case there is no difference between the quantities with the subscript 1 and those without subscript; indeed Equations (84) and (95) coincide when $s_{\nu h} = s_M = \sigma_N = 0$. About the functions ζ_1 and $\xi_1^{(j)}$, which are the only ones to be determined in every case for the purpose of obtaining the admittance condition, the same observations developed at the end of the preceding section for the corresponding quantities ζ and $\xi^{(j)}$ apply with identical conclusions.

12. SIMILARITY OF NOZZLES: VELOCITY DISTRIBUTION FOR REFERENCE NOZZLE

The solution of Equation (88), or of (98) and (99), depends on the parameters ω and $s_{\nu h}$ (ψ_w has a fixed value once the reference length is chosen; see footnote, p. 13); it depends moreover on the nozzle geometry through the function $\overline{q}(\phi)$ (from which $\overline{c}(\phi)$ and $\overline{\rho}(\phi)$ are derived). The results obtained for a given nozzle that we shall call the reference nozzle can, however, be immediately applied to a whole family of nozzles obtained by linear deformation of the axial scale. If z is the axial length coordinate measured from the throat and positive in the flow direction, β is the proper scale factor and $\overline{q}_r(z)$ is the axial distribution of velocity for the reference nozzle, $\overline{q}(z) = \overline{q_{ref}}(\beta z)$ will be the general velocity distribution

for the derived family of nozzles. From (22), since, in view of the one-dimensional assumption, $\delta s = dx$, we obtain, indicating with ϕ_{ref} the potential for the reference nozzle,

$$\beta d\phi = \beta \bar{q}(z)dz = \overline{q_{ref}}(\beta z)d(\beta z) = d\phi_{ref} .$$

It appears that $\beta(\phi - \phi_{th}) = \phi_{ref}$ is also a function only of $\beta z = z_{ref}$, and that as a result $\bar{q}$ can be considered to be the same function of ϕ_{ref} for the whole family.

It is immediately seen that if $L(\phi)$, that is the first member of (84), is divided by β^2, its form remains unaltered if the independent variable is taken to be ϕ_{ref} and the parameters ω and $s_{\nu h}$ are replaced by $\omega_{ref} = \omega/\beta$ and $s_{\nu h\,ref} = s_{\nu h}/\beta$; so that

$$\Phi_h(\phi; \omega, s_{\nu h}) = \Phi_{h\,ref}(\phi_{ref}, \omega_{ref}, s_{\nu h\,ref}) , \tag{118}$$

where $\Phi_{h\,ref}$ is the solution of (98) obtained for the reference nozzle. It appears from (103) that

$$\frac{1}{\beta}\zeta(\phi; \omega, s_{\nu h}) = \frac{1}{\beta\Phi}\frac{d\Phi}{d\phi} = \frac{1}{\Phi_{ref}}\frac{d\Phi_{ref}}{d\phi_{ref}} = \zeta_{ref}(\phi_{ref}, \omega_{ref}, s_{\nu h\,ref}) . \tag{119}$$

We conclude from (118) or (119) that it is sufficient to solve (98) only for the reference nozzle. However, the solution should be computed not only for the discrete set of values $s_{\nu h}$ of Table I (p.84), but for $s_{\nu h}$ continuously variable in a certain range, in order to cover the possible range of values of the scale factor β. We see also that if the nozzle entrance velocity is the same for all the nozzles of the given family (that is, if the area contraction ratio of these nozzles is prescribed), the value of ϕ_{ref} corresponding to the entrance section is also the same so that ζ_{ref} is only a function of ω_{ref} and $s_{\nu h\,ref}$. For given scale factor β and mode $(s_{\nu h})$, ζ_{ref}, as well as ζ, is only a function of ω.

Similar observations can be made on the quantities $\xi^{(j)}$. Inspection of (89) shows that

$$F^{(1)}(\phi; s_{\nu h}) = \beta^2 F^{(1)}_{ref}(\phi_{ref}; s_{\nu h\,ref}) ;$$

$$F^{(2)}(\phi; \omega, s_{\nu h}) = \beta F^{(2)}_{ref}(\phi_{ref}; \omega_{ref}; s_{\nu h\,ref}) .$$

Hence we obtain at once from (111) that

$$\left.\begin{aligned} \xi^{(1)}(\phi; \omega, s_{\nu h}) &= \beta \xi^{(1)}_{ref}(\phi_{ref}; \omega_{ref}, s_{\nu h} \quad) ; \\ \xi^{(2)}(\phi; \omega, s_{\nu h}) &= \xi^{(2)}_{ref}(\phi_{ref}; \omega_{ref}, s_{\nu h\,ref}) . \end{aligned}\right\} \tag{120}$$

We are particularly interested in the repercussion of these relations on the admittance condition (104). We can rewrite this condition in the form

$$U + AP + BV + CS = 0 , \tag{121}$$

where the *admittance coefficients* are

$$A = \frac{\overline{c^2}\xi^{(1)} - \zeta}{\overline{q^2}(\overline{c^2}\xi^{(1)} - \zeta) - i\omega} \tag{122}$$

$$B = \frac{i\omega\,\overline{c^2}\xi^{(1)}}{\overline{q^2}(\overline{c^2}\xi^{(1)} - \zeta) - i\omega} \tag{123}$$

$$C = -\frac{\overline{c^2}\left(\dfrac{\overline{q^2} - \overline{q^2_{th}}}{2}\xi^{(1)} + i\omega\xi^{(2)} - f_3\zeta\right)}{\overline{q^2}(\overline{c^2}\xi^{(1)} - \zeta) - i\omega} . \tag{124}$$

Because of (119) and (120) we see that

$$\left.\begin{aligned} A(\phi;\omega, s_{\nu h}) &= A_{ref}(\phi_{ref};\omega_{ref}, s_{\nu h\,ref}) \\ B(\phi;\omega, s_{\nu h}) &= \beta B_{ref}(\phi_{ref};\omega_{ref}, s_{\nu h\,ref}) \\ C(\phi;\omega, s_{\nu h}) &= C_{ref}(\phi_{ref};\omega_{ref}, s_{\nu h\,ref}) . \end{aligned}\right\} \tag{125a}$$

Hence, from the admittance coefficients calculated for the reference nozzle in the appropriate range of the independent variable (or of $\overline{q}$) and of the parameters, one can obtain the admittance coefficients for any nozzle of the family.

The case of the two-dimensional nozzle presents the same property of nozzle similarity. In fact, it is easily seen from equations (113) and (117) that, defining $s_{M\,ref} = s_M/\beta$ and $\sigma_{N\,ref} = \sigma_N/\beta$, we obtain

$$\Phi_{1h}(\phi;\omega, s_M, \sigma_N) = \Phi_{1h\,ref}(\phi_{ref};\omega_{ref}, s_{M\,ref}, \sigma_{N\,ref})$$

$$\zeta_1(\phi;\omega, s_M, \sigma_N) = \beta\zeta_{1\,ref}(\phi_{ref};\omega_{ref}, s_{M\,ref}, \sigma_{N\,ref})$$

and, from (97) and (111),

$$\xi_1^{(0)}(\phi;\omega, s_M, \sigma_N) = \beta\xi^{(0)}_{1\,ref}(\phi_{ref};\omega_{ref}, s_{M\,ref}, \sigma_{N\,ref})$$

$$\xi_1^{(1)}(\phi;\omega, s_M, \sigma_N) = \beta\xi^{(1)}_{1\,ref}(\phi_{ref};\omega_{ref}, s_{M\,ref}, \sigma_{N\,ref})$$

$$\xi_1^{(2)}(\phi;\omega, s_M, \sigma_N) = \xi^{(2)}_{1\,ref}(\phi_{ref};\omega_{ref}, s_{M\,ref}, \sigma_{N\,ref}) .$$

Here again the subscript ref denotes solutions relative to the reference nozzle. As a consequence if the admittance condition (116) is written in the form

$$U_1 + A_1P_1 + B_1V_1 + C_1S_1 + D_1W_1 = 0 ,$$

where the expression for admittance coefficients A_1, B_1, C_1, D_1 is immediately obtained by comparison with (116). Then it follows that

$$\left.\begin{aligned} A_1(\phi;\omega, s_M, \sigma_N) &= A_{1\,ref}(\phi_{ref};\omega_{ref}, s_{M\,ref}, \sigma_{N\,ref}) \\ B_1(\phi;\omega, s_M, \sigma_N) &= \beta B_{1\,ref}(\phi_{ref};\omega_{ref}, s_{M\,ref}, \sigma_{N\,ref}) \\ C_1(\phi;\omega, s_M, \sigma_N) &= C_{1\,ref}(\phi_{ref};\omega_{ref}, s_{M\,ref}, \sigma_{N\,ref}) \\ D_1(\phi;\omega, s_M, \sigma_N) &= \beta D_{1\,ref}(\phi_{ref};\omega_{ref}, s_{M\,ref}, \sigma_{N\,ref}) \,. \end{aligned}\right\} \quad (125b)$$

Hence from the admittance coefficients for the reference nozzle calculated in a sufficient range of variation of the independent variable and of the parameters one can obtain directly those for any nozzle of the family.

It is interesting to notice that, as (125a) and (125b) show clearly, the scale change from the reference nozzle to any nozzle of the family affects in a different fashion the relation between scalar or axial quantities (pressure, entropy, and axial velocity components) and those including the transverse components of velocity.

13. NONLINEAR ANALYSIS

The linearized analysis which has been performed applies to small-amplitude oscillations and is most useful in the treatment of spontaneous instabilities. It can be used in the prediction of the stability of the steady-state nozzle operation and, if the regime oscillation in the unstable situation has a small amplitude, it can be used to predict some characteristics of the oscillation. However, if the oscillation does not initiate spontaneously but instead requires a finite-size disturbance to the steady-state operation in order to excite an oscillation, the linearized analysis is not sufficient. Also, if the regime oscillation does not have a small amplitude (as is often the case), the linearized analysis does not accurately predict all of the characteristics of the oscillations. In these situations, a nonlinear analysis is better suited on the basis of accuracy.

The analysis of the axisymmetric nozzle was extended to include nonlinear effects by Zinn[8]. A perturbation series was employed where the perturbation quantity was an amplitude parameter. Of course, the first order solution is identical to the linearized solution discussed in previous sections. The second and higher order solutions represent the nonlinear effects. In his work, Zinn completed the analysis up to and including third order; however, for the sake of brevity only the second order results will be presented here and the reader is referred to the original work for third order results.

The same nondimensional scheme given in Section 2 has been employed so that the nonlinear system of equations describing the oscillatory phenomenon is given by Equations (5) through (9). It is convenient to transform the time variable by setting $y = \omega t$, where ω is the angular frequency, so that the solution is 2π-periodic in y. In accordance with the standard procedure in nonlinear mechanics, the following series are substituted into the nonlinear system of equations:

$$q = \bar{q} + \epsilon q^{(1)} + \epsilon^2 q^{(2)} + 0(\epsilon^3)$$

$$p = \bar{p} + \epsilon p^{(1)} + \epsilon^2 p^{(2)} + 0(\epsilon^3)$$

$$\rho = \bar{\rho} + \epsilon \rho^{(1)} + \epsilon^2 \rho^{(2)} + 0(\epsilon^3)$$

$$s = \bar{s} + \epsilon s^{(1)} + \epsilon^2 s^{(2)} + 0(\epsilon^3)$$

$$\omega = \omega^{(0)} + \epsilon\omega^{(1)} + \epsilon^2\omega^{(2)} + 0(\epsilon^3) ,$$

where ϵ is the amplitude parameter and barred quantities, as before, denote steady-state solutions. Separation of the equations according to powers of ϵ yields the same equations for the steady-state quantities as (11), (12), and (13). The equations for the first order coefficients $q^{(1)}$, $p^{(1)}$, $\rho^{(1)}$, and $s^{(1)}$ are the same as Equations (17) through (21) for q', p', ρ', and s', except that the partial-differential operator $\partial(\ )/\partial t$ is replaced by $\omega^{(0)}\partial(\ )/\partial y$.

The second order system of equations becomes

Continuity:

$$\omega^{(0)} \frac{\partial \rho^{(2)}}{\partial y} + \nabla.(\bar{q}\rho^{(2)} + q^{(2)}\bar{\rho}) = -\nabla.(\rho^{(1)}q^{(1)}) - \omega^{(1)}\frac{\partial \rho^{(1)}}{\partial y} . \tag{126}$$

Momentum:

$$\omega^{(0)} \frac{\partial q^{(2)}}{\partial y} + \nabla(\bar{q}.q^{(2)}) + (\nabla \times q^{(2)}) \times \bar{q} + \frac{1}{2}\frac{\rho^{(2)}}{\rho}\nabla(\bar{q}.\bar{q}) + \frac{1}{\gamma\bar{\rho}}\nabla p^{(2)}$$

$$= -\left[\omega^{(1)}\frac{\partial q^{(1)}}{\partial y} + \omega^{(0)}\frac{\rho^{(1)}}{\bar{\rho}}\frac{\partial q^{(1)}}{\partial y} + \frac{\rho^{(1)}}{\bar{\rho}}\nabla(\bar{q}.q^{(1)}) + \right.$$

$$\left. + \frac{\rho^{(1)}}{\bar{\rho}}(\nabla \times q^{(1)}) \times \bar{q} + \frac{1}{2}\nabla(q^{(1)}.q^{(1)}) + (\nabla \times q^{(1)}) \times q^{(1)}\right] . \tag{127}$$

Entropy Equation:

$$\omega^{(0)}\frac{\partial s^{(2)}}{\partial y} + \bar{q}.\nabla s^{(2)} = -q^{(1)}.\nabla s^{(1)} - \omega^{(1)}\frac{\partial s^{(1)}}{\partial y} . \tag{128}$$

Equation of State:

$$s^{(2)} - \frac{p^{(2)}}{\gamma\bar{p}} + \frac{\rho^{(2)}}{\bar{\rho}} = -\frac{1}{2}\left[\frac{1}{\gamma}\left(\frac{p^{(1)}}{\bar{p}}\right)^2 - \left(\frac{\rho^{(1)}}{\bar{\rho}}\right)^2\right] . \tag{129}$$

As done in the linear analysis, a stream function and a potential are defined for the axisymmetric case according to Equations (14) and (16). Furthermore, they are still used as independent variables, as discussed in Section 4. Then we have (consistent with (27)) u , v , and w such that $q = ue_\phi + ve_\psi + we_\theta$. Now the equations for $p^{(1)}$, $\rho^{(1)}$, $s^{(1)}$, $u^{(1)}$, $v^{(1)}$, and $w^{(1)}$ become identical to Equations (28) through (33) for the primed quantities except that again the time derivative has been transformed. The equations for the second order quantities become the following:

Continuity:

$$\omega^{(0)}\frac{\partial}{\partial y}\left(\frac{\rho^{(2)}}{\bar\rho}\right) + \overline{q}^2\frac{\partial}{\partial\phi}\left(\frac{\rho^{(2)}}{\bar\rho} + \frac{u^{(2)}}{\bar q}\right) + \overline{q}^2\frac{\partial}{\partial\psi}\left(r^2\overline{\rho}^2\frac{v^{(2)}}{r\bar\rho\bar q}\right) +$$

$$+\frac{1}{r^2}\frac{\partial}{\partial\theta}(rw^{(2)}) = -\omega^{(1)}\frac{\partial}{\partial y}\left(\frac{\rho^{(1)}}{\bar\rho}\right) - \overline{q}^2\frac{\partial}{\partial\phi}\left(\frac{\rho^{(1)}}{\bar\rho}\frac{u^{(1)}}{\bar q}\right) -$$

$$-\overline{q}^2\frac{\partial}{\partial\psi}\left(r^2\overline{\rho}^2\frac{v^{(1)}}{r\bar\rho\bar q}\frac{\rho^{(1)}}{\bar\rho}\right) - \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(rw^{(1)}\frac{\rho^{(1)}}{\bar\rho}\right) \equiv A\ . \tag{130}$$

ϕ-Component of Momentum:

$$\omega^{(0)}\frac{\partial}{\partial y}\left(\frac{u^{(2)}}{\bar q}\right) + \frac{\partial}{\partial\phi}\left(\overline{q}^2\frac{u^{(2)}}{\bar q}\right) + \frac{\partial}{\partial\phi}\left(\frac{p^{(2)}}{\gamma\bar\rho}\right) - \frac{1}{2}\frac{d\overline{q}^2}{d\phi}s^{(2)}$$

$$= -\omega^{(1)}\frac{\partial}{\partial y}\left(\frac{u^{(1)}}{\bar q}\right) + \frac{1}{4}\frac{d\overline{q}^2}{d\phi}\frac{\gamma}{c^4}\left(\frac{p^{(1)}}{\gamma\bar\rho}\right)^2 - \left(\frac{\rho^{(1)}}{\bar\rho}\right)^2 -$$

$$-\frac{1}{2}\frac{\partial}{\partial\phi}\left[\overline{q}^2\left(\frac{u^{(1)}}{\bar q}\right)^2\right] - \frac{\bar\rho\bar q r^2}{2}\frac{\partial}{\partial\phi}\left[\bar\rho\bar q\left(\frac{v^{(1)}}{\bar\rho\bar q r}\right)^2\right] -$$

$$-\frac{1}{2\bar\rho\bar q r^2}\frac{\partial}{\partial\phi}\left[\bar\rho\bar q(rw^{(1)})^2\right] - \frac{1}{r^2}rw^{(1)}\frac{\partial}{\partial\theta}\left(\frac{u^{(1)}}{\bar q}\right) -$$

$$-\frac{\partial}{\partial\phi}(rw^{(1)}) + \overline{\rho}^2\overline{q}^2r^2\frac{v^{(1)}}{\bar\rho\bar q r}\frac{\partial}{\partial\phi}\left(\frac{v^{(1)}}{\bar\rho\bar q r}\right) - \frac{\partial}{\partial\psi}\left(\frac{u^{(1)}}{\bar q}\right) +$$

$$+\frac{\rho^{(1)}}{\bar\rho}\frac{\partial}{\partial\phi}\left(\frac{p^{(1)}}{\gamma\bar\rho}\right) - \frac{1}{2}s^{(1)}\frac{d\overline{q}^2}{d\phi} \equiv \frac{\partial B}{\partial\phi}\ . \tag{131}$$

ψ-Component of Momentum:

$$\omega^{(0)}\frac{\partial}{\partial y}\left(\frac{v^{(2)}}{\bar{\rho}\,\bar{q}r}\right)+\bar{q}^2\frac{\partial}{\partial\phi}\left(\frac{v^{(2)}}{\bar{\rho}\bar{q}r}\right)+\frac{\partial}{\partial\psi}\left(\frac{p^{(2)}}{\gamma\bar{\rho}}\right) = -\,\omega^{(1)}\frac{\partial}{\partial y}\left(\frac{v^{(1)}}{\bar{\rho}\bar{q}r}\right)-\frac{\bar{\rho}^2\bar{q}^2}{2}\frac{\partial}{\partial\psi}\left[r^2\left(\frac{v^{(1)}}{\bar{\rho}\bar{q}r}\right)^2\right]-$$

$$-\frac{1}{2\bar{\rho}\bar{q}}\frac{\partial}{\partial\psi}\left[\frac{(rw^{(1)})^2}{r}\right]-\bar{q}^2\,\frac{u^{(1)}}{\bar{q}}\frac{\partial}{\partial\phi}\left(\frac{v^{(1)}}{\bar{\rho}\bar{q}r}\right)+\frac{\rho^{(1)}}{\bar{\rho}}\frac{\partial}{\partial\psi}\left(\frac{p^{(1)}}{\gamma\bar{\rho}}\right) = \frac{\partial C}{\partial\psi}. \tag{132}$$

θ-Component of Momentum:

$$\omega^{(0)}\frac{\partial}{\partial y}(rw^{(2)})+\bar{q}^2\frac{\partial}{\partial\phi}(rw^{(2)})+\frac{\partial}{\partial\theta}\left(\frac{p^{(2)}}{\gamma\bar{\rho}}\right) = -\,\omega^{(1)}\frac{\partial}{\partial y}(rw^{(1)})-\bar{\rho}\bar{q}\psi\frac{\partial}{\partial\theta}\left(\frac{v^{(1)}}{\bar{\rho}\bar{q}r}\right)^2-$$

$$-\frac{\bar{\rho}\bar{q}}{4\psi}\frac{\partial}{\partial\theta}(rw^{(1)})^2-\bar{q}^2\,\frac{u^{(1)}}{\bar{q}}\frac{\partial}{\partial\phi}(rw^{(1)})+\frac{\rho^{(1)}}{\bar{\rho}}\frac{\partial}{\partial\theta}\left(\frac{p^{(1)}}{\gamma\bar{\rho}}\right) \equiv \frac{\partial D}{\partial\theta}. \tag{133}$$

Entropy:

$$\omega^{(0)}\frac{\partial s^{(2)}}{\partial y}+\bar{q}^2\frac{\partial s^{(2)}}{\partial\phi} = -\,\omega^{(1)}\frac{\partial s^{(1)}}{\partial y}-\bar{q}^2\,\frac{u^{(1)}}{\bar{q}}\frac{\partial s^{(1)}}{\partial\phi}-$$

$$-\,2\bar{\rho}\,\bar{q}\psi\frac{v^{(1)}}{\bar{\rho}\bar{q}r}\frac{\partial s^{(1)}}{\partial\psi}-\frac{\bar{\rho}\bar{q}}{2\psi}rw^{(1)}\frac{\partial s^{(1)}}{\partial\theta} \equiv E. \tag{134}$$

Equation of State:

$$\bar{c}^2 s^{(2)}+\bar{c}^2\frac{\rho^{(2)}}{\bar{\rho}}-\frac{p^{(2)}}{\gamma\bar{\rho}} = \frac{1}{2}\,\bar{c}^2\left[\left(\frac{\rho^{(1)}}{\bar{\rho}}\right)^2-\frac{\gamma}{\bar{c}^4}\left(\frac{p^{(1)}}{\gamma\bar{\rho}}\right)^2\right] \equiv F. \tag{135}$$

The first order condition given by (52) and (55) (zero axial vorticity) has been used in deriving the above equations.

The time dependence of the first order solutions as shown by (44) and the form of the inhomogeneous parts of (130) through (135) indicate that the y dependence of those inhomogeneous terms is readily expressed as a Fourier series. This indicates that second order solutions are of the form

$$\left.\begin{aligned}\frac{u^{(2)}}{\bar{q}} &= \sum_n e^{iny}\xi_n(\phi,\psi,\theta)\\ \frac{v^{(2)}}{\bar{\rho}\bar{q}r} &= \sum_n e^{iny}\eta_n(\phi,\psi,\theta)\end{aligned}\right\} \tag{136}$$

$$
\left.\begin{aligned}
rw^{(2)} &= \sum_n e^{iny}\zeta_n(\phi, \psi, \theta) \\
\frac{\rho^{(2)}}{\bar{\rho}} &= \sum_n e^{iny}\kappa_n(\phi, \psi, \theta) \\
\frac{p^{(2)}}{\gamma\bar{\rho}} &= \sum_n e^{iny}\pi_n(\phi, \psi, \theta) \\
s^{(2)} &= \sum_n e^{iny}\sigma_n(\phi, \psi, \theta) \, .
\end{aligned}\right\} \qquad (136)
$$

As shown later, the solutions contain only terms corresponding to $n = 0$ and $n = 2$.

Substitution of (136) into (130) through (135) and separation of the Fourier components yields

$$
in\omega^{(0)}\kappa_n + \overline{q^2}\frac{\partial\kappa_n}{\partial\phi} + \overline{q^2}\frac{\partial\xi_n}{\partial\phi} + 2\overline{\rho}\overline{q}\frac{\partial}{\partial\psi}(\psi\eta_n) + \frac{\overline{\rho}\,\overline{q}}{2\psi}\frac{\partial\zeta_n}{2\theta} = A_n \qquad (130a)
$$

$$
in\omega^{(0)}\xi_n + \frac{\partial}{\partial\phi}(\overline{q^2}\xi_n) + \frac{\partial\pi_n}{\partial\phi} - \frac{1}{2}\sigma_n\frac{d\overline{q^2}}{d\phi} = \frac{\partial B_n}{\partial\phi} \qquad (131a)
$$

$$
in\omega^{(0)}\eta_n + \overline{q^2}\frac{\partial\eta_n}{\partial\phi} + \frac{\partial\pi_n}{\partial\psi} = \frac{\partial C_n}{\partial\psi} \qquad (132a)
$$

$$
in\omega^{(0)}\zeta_n + \overline{q^2}\frac{\partial\zeta_n}{\partial\phi} + \frac{\partial\pi_n}{\partial\theta} = \frac{\partial D_n}{\partial\theta} \qquad (133a)
$$

$$
in\omega^{(0)}\sigma_n + \overline{q^2}\frac{\partial\sigma_n}{\partial\phi} = E_n \qquad (134a)
$$

$$
\overline{c^2}(\sigma_n + \kappa_n) - \pi_n = F_n \, . \qquad (135a)
$$

where A_n , $\partial B_n/\partial\phi$, etc., are the coefficients in the Fourier expansion of A , $\partial B/\partial\phi$, etc.

In analogy with (79) we define

$$
\xi_n = \frac{\partial\Phi_n}{\partial\phi} \, . \qquad (137)
$$

Then (131a), (132a), (133a), and (137) yield, upon combination and integration,

$$
\eta_n = f_{0n}\frac{\partial}{\partial\psi}\left[\int_{\phi_{th}}^{\phi}\frac{C_n - G_n}{\overline{q^2}f_{0n}}d\phi' + \frac{\Phi_n}{f_{0n}} - \Phi_n(\phi_{th}, \psi, \theta)\right] + f_{0n}\eta_n(\phi_{th}, \psi, \theta) \qquad (138)
$$

$$\zeta_n = f_{on}\frac{\partial}{\partial\theta}\left[\int_{\phi_{th}}^{\phi}\frac{D_n - G_n}{\overline{q^2}f_{on}}\,d\phi' + \frac{\Phi_n}{f_{on}} - \Phi_n(\phi_{th}\,,\psi\,,\theta)\right] + f_{on}\zeta_n(\phi_{th}\,,\psi\,,\theta)\ , \tag{139}$$

where the definitions are made that

$$G_n = \frac{1}{2}\int_{\phi_{th}}^{\phi}\sigma_n\frac{d\overline{q^2}}{d\phi'}\,d\phi' + B_n$$

$$f_{on} = e^{-in\omega^{(0)}}\int_{\phi_{th}}^{\phi}\frac{d\phi'}{\overline{q^2}}\ .$$

In addition, (134a) is readily integrated to obtain

$$\sigma_n = f_{on}\left[\int_{\phi_{th}}^{\phi}\frac{E_n}{\overline{q^2}f_{on}}\,d\phi' + \sigma_n(\phi_{th}\,,\psi\,,\theta)\right]\ . \tag{140}$$

Using (131a), (135a), and (137) through (140) to substitute into (130a), we find that

$$\overline{q^2}(\overline{c^2} - \overline{q^2})\frac{\partial^2\Phi_n}{\partial\phi^2} - \overline{q^2}\left(2in\omega^{(0)} + \frac{1}{\overline{c^2}}\frac{d\overline{q^2}}{d\phi}\right)\frac{\partial\Phi_n}{\partial\phi} + \left[\left(n\omega^{(0)}\right)^2 - \frac{\overline{q^2}}{\overline{c^2}}\frac{\gamma - 1}{2}\,in\omega^{(0)}\frac{d\overline{q^2}}{d\phi}\right]\Phi_n +$$

$$+ 2\bar{\rho}\bar{q}\overline{c^2}\frac{\partial}{\partial\psi}\left(\psi\frac{\partial\Phi_n}{\partial\psi}\right) + \frac{\bar{\rho}\bar{q}\overline{c^2}}{2\psi}\frac{\partial^2\Phi_n}{\partial\theta^2} = \overline{c^2}\left[A_n + E_n - \overline{q^2}\frac{\partial}{\partial\phi}\left(\frac{F_n + G_n}{\overline{c^2}}\right)\right] -$$

$$- in\omega^{(0)}(F_n + G_n) - 2\bar{\rho}\bar{q}f_{on}\left[\frac{\partial}{\partial\psi}\left(\psi\frac{\partial H_n}{\partial\psi}\right) + \frac{1}{4\psi}\frac{\partial^2 H_n}{\partial\theta^2}\right] \equiv I_n\ . \tag{141}$$

where H_n is defined as

$$H_n = \int_{\phi_{th}}^{\phi}\frac{C_n - G_n}{\overline{q^2}f_{on}}\,d\phi' + \tilde{\eta}_n(\phi_{th}\,,\psi\,,\theta) - \Phi_n(\phi_{th}\,,\psi\,,\theta)$$

and $\tilde{\eta}_n$ is a potential function defined by the relation

$$\eta_n = \frac{\partial\tilde{\eta}_n}{\partial\psi}\ .$$

The inhomogeneous part I_n in (141) has been simplified by assuming zero axial vorticity to second order.

The form of Equations (137) through (141) and the form of the first order solution (as given by (44)), together with the boundary condition that the radial velocity vanishes at the nozzle wall, indicate how the radial and tangential dependencies of the second order solution may be obtained; the solution may conveniently be found in the form of an expansion in eigenfunctions. Following standard procedures, Zinn obtains the following for a wave traveling* in the negative θ direction

$$I_n^{(t)} = \left[\sum_{q=n}^{\infty} I_{n,n\nu,q}^{(t)}(\phi) J_{n\nu}\left(s_{n\nu,q}\left(\frac{\psi}{\psi_w}\right)^{\frac{1}{2}}\right)\right] e^{in\nu\theta} \qquad (142a)$$

while for a standing wave he obtains

$$I_n^{(s)} = \sum_{m=0,2}\left\{\left[\sum_{q=0}^{\infty} I_{n,m\nu,q}^{(s)}(\phi) J_{m\nu}\left(s_{m\nu,q}\left(\frac{\psi}{\psi_w}\right)^{\frac{1}{2}}\right)\right] \cos m\nu\theta\right\}. \qquad (142b)$$

If $n \neq 0,2$, $I_n^{(s)} = I_n^{(t)} = 0$. The symbol $\sum_{m=0,2}$ means that $m = 0$ and $m = 2$ produce the only nonzero values of $I_{n,m\nu,q}$. Note that $\omega^{(1)}$ was properly set equal to zero in the above relations for the combustion instability problem considered in Reference 8. Otherwise, the first harmonic would appear above. Note that $s_{m\nu,q}$ is the q^{th} root of $J'_{m\nu}(x) = 0$. Solutions are found in the following form for traveling waves

$$\left.\begin{aligned}
\Phi_n^{(t)} &= \left[\sum_{q=0}^{\infty} \Phi_{n,n\nu,q}^{(t)}(\phi) J_{n\nu}\left(s_{n\nu,q}\left(\frac{\psi}{\psi_w}\right)^{\frac{1}{2}}\right)\right] e^{in\nu\theta} \\
\xi_n^{(t)} &= \left[\sum_{q=0}^{\infty} U_{n,n\nu,q}^{(t)}(\phi) J_{n\nu}\left(s_{s\nu,q}\left(\frac{\psi}{\psi_w}\right)^{\frac{1}{2}}\right)\right] e^{in\nu\theta} \\
\eta_n^{(t)} &= \left[\sum_{q=0}^{\infty} V_{n,n\nu,q}^{(t)}(\phi) \frac{d}{d\psi} J_{n\nu}\left(s_{n\nu,q}\left(\frac{\psi}{\psi_w}\right)^{\frac{1}{2}}\right)\right] e^{in\nu\theta} \\
\zeta_n^{(t)} &= ni\nu\left[\sum_{q=0}^{\infty} W_{n,n\nu,q}^{(t)}(\phi) J_{n\nu}\left(s_{n\nu,q}\left(\frac{\psi}{\psi_w}\right)^{\frac{1}{2}}\right)\right] e^{in\nu\theta}
\end{aligned}\right\} \qquad (143a)$$

* Since transverse standing waves are the sum of two transverse traveling waves in the linear case, no distinction was necessary there. However, here in the nonlinear case, it is necessary since no simple relation applies between the two types of waves.

$$
\left.
\begin{aligned}
\pi_n^{(t)} &= \left[\sum_{q=0}^{\infty} P_{n,n\nu,q}^{(t)}(\phi) J_{n\nu}\left(s_{n\nu,q}\left(\frac{\psi}{\psi_w}\right)^{\frac{1}{2}}\right)\right] e^{in\nu\theta} \\
\kappa_n^{(t)} &= \left[\sum_{q=0}^{\infty} R_{n,n\nu,q}^{(t)}(\phi) J_{n\nu}\left(s_{n\nu,q}\left(\frac{\psi}{\psi_w}\right)^{\frac{1}{2}}\right)\right] e^{in\nu\theta} \\
\sigma_n^{(t)} &= \left[\sum_{q=0}^{\infty} S_{n,n\nu,q}^{(t)}(\phi) J_{n\nu}\left(s_{n\nu,q}\left(\frac{\psi}{\psi_w}\right)^{\frac{1}{2}}\right)\right] e^{in\nu\theta} \, .
\end{aligned}
\right\}
\tag{143a}
$$

In the case of standing waves, solutions are found in the form

$$
\left.
\begin{aligned}
\Phi_n^{(s)} &= \sum_{m=0,2}\left\{\left[\sum_{q=0}^{\infty} \Phi_{n,m\nu,q}^{(s)}(\phi) J_{m\nu}\left(s_{m\nu,q}\left(\frac{\psi}{\psi_w}\right)^{\frac{1}{2}}\right)\right] \cos m\nu\theta\right\} \\
\xi_n^{(s)} &= \sum_{m=0,2}\left\{\left[\sum_{q=0}^{\infty} U_{n,m\nu,q}^{(s)}(\phi) J_{m\nu}\left(s_{m\nu,q}\left(\frac{\psi}{\psi_w}\right)^{\frac{1}{2}}\right)\right] \cos m\nu\theta\right\} \\
\eta_n^{(s)} &= \sum_{m=0,2}\left\{\left[\sum_{q=0}^{\infty} V_{n,m\nu,q}^{(s)}(\phi) \frac{\psi}{d\psi} J_{m\nu}\left(s_{m\nu,q}\left(\frac{\psi}{\psi_w}\right)^{\frac{1}{2}}\right)\right] \cos m\nu\theta\right\} \\
\zeta_n^{(s)} &= -2\nu\left[\sum_{q=1}^{\infty} W_{n,2\nu,q}^{(s)}(\phi) J_{2\nu}\left(s_{2\nu,q}\left(\frac{\psi}{\psi_w}\right)^{\frac{1}{2}}\right)\right] \sin 2\nu\theta \\
\pi_n^{(s)} &= \sum_{m=0,2}\left\{\left[\sum_{q=0}^{\infty} P_{n,m\nu,q}^{(s)}(\phi) J_{m\nu}\left(s_{m\nu,q}\left(\frac{\psi}{\psi_w}\right)^{\frac{1}{2}}\right)\right] \cos m\nu\theta\right\} \\
\kappa_n^{(s)} &= \sum_{m=0,2}\left\{\left[\sum_{q=0}^{\infty} R_{n,m\nu,q}^{(s)}(\phi) J_{m\nu}\left(s_{m\nu,q}\left(\frac{\psi}{\psi_w}\right)^{\frac{1}{2}}\right)\right] \cos m\nu\theta\right\} \\
\sigma_n^{(s)} &= \sum_{m=0,2}\left\{\left[\sum_{q=0}^{\infty} S_{n,m\nu,q}^{(s)}(\phi) J_{m\nu}\left(s_{m\nu,q}\left(\frac{\psi}{\psi_w}\right)^{\frac{1}{2}}\right)\right] \cos m\nu\theta\right\} .
\end{aligned}
\right\}
\tag{143b}
$$

Again, the only allowable values for m are 0 and 2 .

Substitution of the proper forms of (142) and (143) into (141) and separation of the eigenfunctions leads to the following ordinary differential equation for both traveling and standing waves

$$\overline{q^2}(\overline{c^2} - \overline{q^2}) \frac{d^2}{d\phi^2} \Phi_{n,m\nu,q} - \overline{q^2}\left(\frac{1}{\overline{c^2}} \frac{d\overline{q^2}}{d\phi} + 2in\omega^{(0)}\right) \frac{d}{d\phi} \Phi_{n,m\nu,q} +$$

$$+ \left[(n\omega^{(0)})^2 - \frac{\gamma - 1}{2} \frac{in\omega^{(0)}\overline{q^2}}{\overline{c^2}} \frac{d\overline{q^2}}{d\phi} - \frac{s^2_{m\nu,q}}{2\psi_m} \overline{\rho q c^2}\right] \Phi_{n,m\nu,q} = I_{n,m\nu,q} \quad . \qquad (144)$$

Reference 8 shows that the inhomogeneous part may be organized into a more convenient form. For both traveling and standing waves, (141) and (142) yield

$$I_{n,m\nu,q}(\phi) = -\overline{c^2} f_{cn}(C_{1n,m\nu,q} F^{(1)}_{n,m\nu,q} + \sigma_{n,m\nu,q} F^{(2)}_{n,m\nu,q} + F^{(3)}_{n,m\nu,q}) \quad . \qquad (145)$$

where

$$F^{(1)}_{n,m\nu,q} = \frac{s^2_{m\nu,q}}{2\psi_m} \overline{\rho q}$$

$$F^{(2)}_{n,m\nu,q} = \overline{q^2} \frac{df_{3n}}{d\phi} + \frac{s^2_{m\nu,q}}{2\psi_m} \frac{\overline{\rho q}}{2in\omega^{(0)}} (2\overline{c^2} f_{3n} + \overline{q^2_{th}} - \overline{q^2})$$

$$F^{(3)}_{n,m\nu,q} = -\frac{1}{f_{cn}} (A_{n,m\nu,q} + E_{n,m\nu,q}) +$$

$$+ \frac{\overline{q^2}}{f_{cn}} \frac{d}{d\phi} \left[\frac{P_{n,m\nu,q} + B_{n,m\nu,q} + N_{n,m\nu,q}}{\overline{c^2}}\right] +$$

$$+ \frac{in\omega^{(0)}}{\overline{c^2} f_{cn}} [P_{n,m\nu,q} + B_{n,m\nu,q} + N_{n,m\nu,q}] -$$

$$- \overline{\rho q} \frac{s^2_{m\nu,q}}{2\psi_m} \int_{\phi_{th}}^{\phi} \frac{C_{n,m\nu,q} - B_{n,m\nu,q} - N_{n,m\nu,q}}{\overline{q^2} f_{cn}} d\phi'$$

$$f_{3n} = \frac{1}{2\overline{c^2} f_{cn}} \int_{\phi_{th}}^{\phi} \frac{d\overline{q^2}}{d\phi'} f_{cn} \, d\phi'$$

$$C_{1n,m\nu,q} = \Phi_{n,m\nu,q}(\phi_{th}) \quad V_{n,m\nu,q}(\phi_{th})$$

$$\sigma_{n,m\nu,q} = S_{n,m\nu,q}(\phi_{th})$$

$$N_{n,m\nu,q} = \frac{1}{2}\int_{\phi_{th}}^{\phi} f_{0n}\frac{d\overline{q^2}}{d\phi'}\left(\int_{\phi_{th}}^{\phi'} \frac{E_{n,m\nu,q}}{\overline{q^2} f_{0n}}\, d\phi''\right) d\phi' \ .$$

$A_{n,m\nu,q}$, $B_{n,m\nu,q}$, etc., are coefficients in the eigenfunction expansions of A_n, B_n, etc. The details of their evaluation are presented in Reference 8. Of course, the coefficients are different for traveling waves and standing waves.

The similarity between (145) and (75) is instructive; it indicates a similar method of solution should be pursued for the nonlinear problem as has been pursued for the linear problem. The solution of (144) is of the form

$$\Phi_{n,m\nu,q} = C_{1n,m\nu,q}\Phi^{(1)}_{n,m\nu,q} + \sigma_{n,m\nu,q}\Phi^{(2)}_{n,m\nu,q} + \\ + \Phi^{(3)}_{n,m\nu,q} + C_{2n,m\nu,q}\Phi_{hn,m\nu,q} + C_{3n,m\nu,q}\tilde{\Phi}_{hn,m\nu,q} \ . \tag{146}$$

Each of the terms above except one corresponds to a term of (100). The first two terms $\Phi^{(1)}_{n,m\nu,q}$ and $\Phi^{(2)}_{n,m\nu,q}$ are particular solutions due to the vorticity and entropy, respectively. The third term $\Phi^{(3)}_{n,m\nu,q}$ is a particular solution due to nonlinear irrotational effects and does not correspond to anything in (100). The last two terms are homogeneous solutions. One of these $\Phi_{hn,m\nu,q}$ is regular at the throat; in order to eliminate the singular solution $\tilde{\Phi}_{hn,m\nu,q}$, $C_{3n,m\nu,q} = 0$ is imposed.

Now (143) may be substituted into (137) through (140) and those equations may be separated to yield

$$U_{n,m\nu,q} = \frac{d\Phi_{n,m\nu,q}}{d\phi} \tag{147}$$

$$V_{n,m\nu,q} = f_{0n}\int_{\phi_{th}}^{\phi} \frac{C_{n,m\nu,q} - B_{n,m\nu,q} - N_{n,m\nu,q}}{\overline{q^2} f_{0n}}\, d\phi' + \\ + \Phi_{n,m\nu,q} - C_{1n,m\nu,q}f_{0n} - \sigma_{n,m\nu,q}f_{2n} \tag{148}$$

$$W_{n,m\nu,q} = V_{n,m\nu,q} \tag{149}$$

$$S_{n,m\nu,q} = f_{0n}\left[\int_{\phi_{th}}^{\phi} \frac{E_{n,m\nu,q}}{\overline{q^2} f_{0n}}\, d\phi' + \sigma_{u,m\nu,q}\right] . \tag{150}$$

where

$$f_{2n} = f_{on} \int_{\phi_{th}}^{\phi} \frac{f_{1n}}{\overline{q^2} f_{on}} d\phi'$$

$$f_{1n} = \frac{1}{2} \int_{\phi_{th}}^{\phi} f_{on} \frac{d\overline{q^2}}{d\phi'} d\phi' \quad .$$

Integration of (131a) and combination with (137), (143a), and (150) determines $P_{n,m\nu,q}$. Then this result is combined with (135), (143), and (150) to determine $R_{n,m\nu,q}$. The results are

$$P_{n,m\nu,q} = \frac{1}{2} \int_{\phi_{th}}^{\phi} \frac{d\overline{q^2}}{d\phi'} S_{n,m\nu,q} d\phi' + B_{n,m\nu,q} - in\omega^{(0)} \Phi_{n,m\nu,q} - \overline{q^2} \frac{d}{d\phi} \Phi_{n,m\nu,q} \tag{151}$$

$$R_{n,m\nu,q} = \frac{1}{\overline{c^2}} (F_{n,m\nu,q} + P_{n,m\nu,q}) - S_{n,m\nu,q} \quad . \tag{152}$$

Obviously, since the coefficients $A_{n,m\nu,q}$, $B_{n,m\nu,q}$, etc., are different for traveling and standing, so will the solutions given by (146) through (152) be different. Note further that the ω in the linear solutions given by (102) actually represents the $\omega^{(0)}$ of (144) through (152). In the linearized problem, it is not necessary to distinguish between the two, since the difference is of higher order. Similarity is noticed between (104) and (147) through (151).

Substitution of (146) with $C_{3n,m\nu,q} = 0$ into (147), (148), (150), and (151) leads to four equations which are linear in the three constants $\sigma_{n,m\nu,q}$, $C_{1n,m\nu,q}$, and $C_{2n,m\nu,q}$. In similar fashion to the development in Section 10, the requirement of compatibility leads to the following relationship between $U_{n,m\nu,q}$, $P_{n,m\nu,q}$, $V_{n,m\nu,q}$, and $S_{n,m\nu,q}$:

$$\begin{aligned}
& U_{n,m\nu,q}\left[\overline{q^2}(\overline{c^2}\xi^{(1)}_{n,m\nu,q} - \zeta_{n,m\nu,q}) - in\omega^{(0)}\right] + \\
& \quad + V_{n,m\nu,q} in\omega^{(0)} \overline{c^2} \xi^{(1)}_{n,m\nu,q} + P_{n,m\nu,q}(\overline{c^2}\xi^{(1)}_{n,m\nu,q} - \zeta_{n,m\nu,q}) - \\
& \quad\quad - S_{n,m\nu,q}\overline{c^2}\left[\frac{\overline{q^2} - \overline{q^2_{th}}}{2}\xi^{(1)}_{n,m\nu,q} + in\omega^{(0)}\xi^{(2)}_{n,m\nu,q} - f_{3n}\zeta_{n,m\nu,q}\right] \\
& \quad\quad\quad = in\omega^{(0)}\overline{c^2} f_{on}\xi^{(3)}_{n,m\nu,q} + K^{(1)}_{n,m\nu,q} in\omega^{(0)}\xi^{(1)}_{n,m\nu,q} + \\
& \quad\quad\quad\quad + K^{(2)}_{n,m\nu,q}(\overline{c^2}\xi^{(1)}_{n,m\nu,q} - \zeta_{n,m\nu,q}) - K^{(3)}_{n,m\nu,q}\overline{c^2}\left[\frac{\overline{q^2} - \overline{q^2_{th}}}{2}\xi^{(1)}_{n,m\nu,q} + \right. \\
& \quad\quad\quad\quad\quad \left. + in\omega^{(0)}\xi^{(2)}_{n,m\nu,q} - f_{3n}\zeta_{n,m\nu,q}\right] \quad .
\end{aligned} \tag{153}$$

The following definitions have been made in order to obtain the simplified version (153)

$$\zeta_{n,m\nu,q} = \frac{1}{\Phi_{hn,m\nu,q}} \frac{d}{d\phi} \Phi_{hn,m\nu,q} \tag{154}$$

$$\xi^{(j)}_{n,m\nu,q} = \frac{1}{\overline{c^2} f_{on} \Phi_{hn,m\nu,q}} \left[\Phi^{(j)}_{n,m\nu,q} \frac{d}{d\phi} \Phi_{hn,m\nu,q} - \Phi_{hn,m\nu,q} \frac{d}{d\phi} \Phi^{(j)}_{n,m\nu,q} \right] , \tag{155}$$

where $j = 1, 2, 3$,

$$K^{(1)}_{n,m\nu,q} = f_{on} \left\{ \int_{\phi_{th}}^{\phi} \frac{1}{\overline{q^2} f_{on}} \left[C_{n,m\nu,q} - B_{n,m\nu,q} - \right. \right.$$

$$\left. \left. - \frac{1}{2} \int_{\phi_{th}}^{\phi'} \frac{d\overline{q^2}}{d\phi''} f_{on} \left(\int_{\phi_{th}}^{\phi''} \frac{E_{n,m\nu,q}}{\overline{q^2} f_{on}} d\phi''' \right) d\phi'' \right] d\phi' \right\}$$

$$K^{(2)}_{n,m\nu,q} = \frac{1}{2} \int_{\phi_{th}}^{\phi} \frac{d\overline{q^2}}{d\phi'} f_{on} \left(\int_{\phi_{th}}^{\phi'} \frac{E_{n,m\nu,q}}{\overline{q^2} f_{on}} d\phi'' \right) d\phi' + B_{n,m\nu,q}$$

$$K^{(3)}_{n,m\nu,q} = f_{on} \int_{\phi_{th}}^{\phi} \frac{E_{n,m\nu,q}}{\overline{q^2} f_{on}} d\phi' .$$

Equation (153) applies everywhere along the convergent portion of the nozzle and, when applied at the nozzle entrance, is the nozzle admittance expression. The similarity with (104) is immediately seen.

In the irrotational case where $\sigma_{n,m\nu,q}$ and $C_{1n,m\nu,q}$ are zero the nozzle admittance expression is simplified. In that case (147), (148), and (151) yield, after application of the definitions (154) and (155),

$$\begin{aligned} U_{n,m\nu,q} - \zeta_{n,m\nu,q} V_{n,m\nu,q} &= -\overline{c^2} f_{on} \xi^{(3)}_{n,m\nu,q} + in\omega^{(0)} U_{n,m\nu,q} + \\ &\quad + \zeta_{n,m\nu,q} (P_{n,m\nu,q} + \overline{q^2} U_{n,m\nu,q}) \\ &= \zeta_{n,m\nu,q} K^{(2)}_{n,m\nu,q} - in\omega^{(0)} \overline{c^2} f_{on} \xi^{(3)}_{n,m\nu,q} . \end{aligned} \tag{156}$$

The problem remains to determine the functions $\zeta_{n,m\nu,q}$ and $\xi^{(j)}_{n,m\nu,q}$ to substitute into the relations (153) and (156). Reference 8 shows that $\zeta_{n,m\nu,q}$ is governed by a complex Riccati equation in a similar manner to the function ζ in the linear case

of Section 10. Also, if ω were replaced by $n\omega^{(0)}$, ζ by $\zeta_{n,m\nu,q}$, $\xi^{(j)}$ by $\xi^{(j)}_{n,m\nu,q}$, and $F^{(j)}$ by $F^{(j)}_{n,m\nu,q}$ in (112), the governing equation for $\xi^{(j)}_{n,m\nu,q}$ is obtained. So the same numerical techniques may be used to determine $\zeta_{n,m\nu,q}$ and $\xi^{(j)}_{n,m\nu,q}$ as are used to determine ζ and $\xi^{(j)}$. These methods are discussed with respect to the linear theory in the next section.

There is one important difference between the linear case and the nonlinear case. This difference appears on account of that inhomogeneous part of (144) proportional to $F^{(3)}_{n,m\nu,q}$ and, consequently, on account of the solution of the function $\xi^{(3)}_{n,m\nu,q}$. $F^{(3)}_{n,m\nu,q}$ is a sum of terms each of which is proportional to the squares or double products of the first order solutions for the flow properties as given in (102). Therefore, in order to calculate the second admittance coefficients, it is necessary to calculate the first order flow properties. Also, σ , C_1 , and C_2 in (102) should be known to completely determine the second order admittance coefficients. That would require a coupled solution of the oscillatory phenomena in the nozzle and in the chamber preceding the nozzle.

There is a preferred method of solution which uncouples the numerical integration of the nozzle equations from the analysis of the chamber flow, although it was not employed in Reference 8. Due to the nature of $F^{(3)}_{n,m\nu,q}$ and of (102), $F^{(3)}_{n,m\nu,q}$ and $\xi^{(3)}_{n,m\nu,q}$ may be written in the convenient forms

$$\begin{aligned} F^{(3)}_{n,m\nu,q} &= \delta_1(C_1)^2 + \delta_2(C_2)^2 + \delta_3(\sigma)^2 + \delta_4(C_1C_2) + \delta_5(C_1\sigma) + \delta_6(C_2\sigma) \\ \xi^{(3)}_{n,m\nu,q} &= \alpha_1(C_1)^2 + \alpha_2(C_2)^2 + \alpha_3(\sigma)^2 + \alpha_4(C_1C_2) + \alpha_5(C_1\sigma) + \alpha_6(C_2\sigma) \ , \end{aligned} \tag{157}$$

where δ_i and α_i are independent of C_1 , C_2 , and σ . Since the coefficients of δ_i are constant throughout the nozzle, δ_i may replace $F^{(3)}_{n,m\nu,q}$ and α_i may replace $\xi^{(3)}_{n,m\nu,q}$ in the modified version of (112). Of course, the α_i's may be determined by numerical integration without a knowledge of the constants C_1 , C_2 , and σ . Note that $K^{(1)}_{n,m\nu,q}$, $K^{(2)}_{n,m\nu,q}$, and $K^{(3)}_{n,m\nu,q}$ may also be written in identical form to (157) so that the inhomogeneous parts of (153) and (156) may be presented in that same form, where C_1 , C_2 , and σ remain to be determined.

Note that there are actually a double infinity (two n values and an infinity of q values) of relations (153) and (156) for traveling waves. In the standing wave case, there are a quadruple infinity (two n values, two m values, and an infinity of q values) of those relations. Each of these infinite number of relations is to be applied as the admittance relation for a particular eigenfunction. In a practical sense, convergence will occur with a finite number of terms.

Quite similar relations to (153) and (157) are presented in Reference 8 for the third order (in ϵ) quantities. Of course, the higher the order, the more cumbersome are the algebraic developments and the numerical integrations to be performed.

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PART II. APPLICATIONS

14. CALCULATIONS FOR "CONICAL" NOZZLES

The vast majority of nozzles in practical devices are axisymmetric, with the generatrix of the convergent portion (Fig. 17) consisting of a straight segment, generating a convergent cone, and circular arcs, generating the throat portion and possibly the entrance portion (which connects the conical convergent to the cylindrical combustion chamber).

The calculations were performed for a reference nozzle of this type, using the equations developed in Part I for the axisymmetric case. In order to include in a single calculation nozzles of various contraction ratios, the entrance portion of the nozzle was assumed to be part of the conical convergent, so that the slope is discontinuous at the entrance (Fig. 17). Within the one-dimensional approximation, this allows choosing each station in the calculations to be the nozzle entrance station. The effect of the presence of an entrance portion other than conical was studied separately (Section 15) and found indeed to be generally small.

It must be observed that, according to Section 12, a whole family of nozzles can be obtained by axially stretching or shrinking the reference nozzle. However, the generatrix of the nozzles thus obtained is made of straight segments and elliptical (rather than circular) arcs. This means that the correspondence between a practically designed nozzle (different from the reference nozzle) and a nozzle of the family is only approximate in the throat (and maybe the entrance) portions.

In Part I the physical quantities have been nondimensionalized with respect to the steady-state stagnation properties. Indeed, this is the most appropriate choice, because the same stagnation properties can be used as reference quantities in the study of the combustion chamber behavior, which makes use of the admittance condition at the nozzle entrance. For the same reason it is logical to choose the combustion chamber radius (or, in the two-dimensional case, its height) to be the appropriate reference length L^* introduced in Section 2.

On the other hand when this study was undertaken several years ago the reference quantities were taken to be the static properties at the throat and the throat radius (so that the nondimensional steady-state pressure, density, velocity and radius at the throat took the value of unity), and the initial programming for the computer was developed on this basis. In subsequent calculations, rather than changing the program, it was decided to stick with this choice so far as the calculations were concerned, only switching to the stagnation reference properties when the final admittance coefficients were calculated.

Although this may be a little confusing, the decision was made to keep in this Part II the alternate choice of the reference system so as to avoid the laborious conversion of numerical results or analytical developments based on the throat reference system to the stagnation reference system. However, we shall use for the variables the same notations used in Part I, explicitly stating when the stagnation reference system is used instead of the throat reference system.

In view of the relation

$$\bar{c}^*_{th} = \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{2}} \overline{c^{o*}} \; ,$$

which immediately follows from (13) and from the definition (9), Equations (12) and (13) become

$$\frac{\bar{p}}{\bar{\rho}} = \bar{c}^2 = \bar{\rho}^{\gamma-1} = \frac{\gamma + 1}{2} - \frac{\gamma - 1}{2}\bar{q}^2 \; .$$

Of course, since at the throat we have $\bar{p} = \bar{\rho} = \bar{q} = \bar{c} = \bar{r}_w = 1$ we obtain, from (43) $\psi_w = \frac{1}{2}$. Equation (88) becomes

$$\begin{aligned} L(\Phi) &= \bar{q}^2(\bar{q}^2 - \bar{c}^2)\Phi'' - \bar{q}^2\left(2i\omega + \frac{\gamma + 1}{2\bar{c}^2}\frac{d\bar{q}^2}{d\phi}\right)\Phi' + \\ &\quad + \left(\omega^2 - \frac{\gamma - 1}{2}\, i\omega \frac{\bar{q}^2}{\bar{c}^2}\frac{d\bar{q}^2}{d\phi} - \bar{q}\bar{c}^{2\gamma/(\gamma-1)} s^2_{\nu h}\right)\Phi \qquad (158) \\ &= -\,(C_1 F^{(1)} + \sigma F^{(2)})\bar{c}^2 f_0 \; . \end{aligned}$$

where, instead of (89), (77) and (85), we have

$$F^{(1)} = s^2_{\nu h}\bar{q}\bar{c}^{\,2/(\gamma-1)} \; ;$$

$$F^{(2)} = \bar{q}^2 \frac{df_3}{d\phi} + \frac{s^2_{\nu h}\bar{q}\bar{c}^{\,2/(\gamma-1)}}{2i\omega}\,(2\bar{c}^2 f_3 + 1 - \bar{q}^2) \; ;$$

$$f_0 = \exp\left\{-i\omega\int_0^{\phi}\frac{d\phi'}{\overline{q^2}}\right\} ;$$

$$f_3 = \frac{1}{2\overline{c}^2 f_0}\int_0^{\phi} f_0\frac{d\,\overline{q^2}}{d\phi'}d\phi' .$$

The appropriate solution of the equation for Φ is given by the equation corresponding to (101),

$$\Phi = C_1\Phi^{(1)} + \sigma\Phi^{(2)} + C_2\Phi_h ,$$

where all the Φ's must be regular at the throat. The admittance relation can be written in the form corresponding to (121),

$$U + AP + BV + CS = 0 , \tag{159}$$

with U , P , V and S still defined by (44) and A,B,C by (122), (123) and (124) with $\overline{q_{th}} = 1$.

This, however, is not the most convenient form for use as a boundary condition on the oscillatory flow in the combustion chamber. Indeed, the equations in the cylindrical chamber are usually separated by setting

$$\left.\begin{aligned} u' &= \mathcal{U}(z)J_\nu(s_{\nu h}r)\Theta(\theta)e^{i\omega t} \\ v' &= \mathcal{V}(z)\frac{dJ_\nu}{dr}(s_{\nu h}r)\Theta(\theta)e^{i\omega t} \\ p' &= \mathcal{P}(z)J_\nu(s_{\nu h}r)\Theta(\theta)e^{i\omega t} \\ s' &= \mathcal{S}(z)J_\nu(s_{\nu h}r)\Phi(\theta)e^{i\omega t} , \end{aligned}\right\} \tag{160}$$

where r is nondimensionalized by the chamber radius, u' , v' by the stagnation sound velocity $\overline{c^{o*}}$ and p' by the stagnation pressure.

Comparing (160) with (44), and taking into account the difference of the reference systems, we obtain

$$U = \left(\frac{\gamma+1}{2}\right)^{\frac{1}{2}}\frac{\mathcal{U}}{\overline{q}} ; \qquad V = \left(\frac{\gamma+1}{2}\right)^{\frac{1}{2}}\frac{\mathcal{V}}{\overline{q}^{\frac{1}{2}}\overline{c}^{1/(\gamma-1)}} ;$$

$$P = \left(\frac{\gamma+1}{2}\right)^{\gamma/(\gamma-1)}\frac{\mathcal{P}}{\overline{c}^{2/(\gamma-1)}} ; \qquad S = \mathcal{S} .$$

After substitution the admittance condition becomes

$$\mathcal{U} + \mathcal{A}\mathcal{P} + \mathcal{B}\mathcal{V} + \mathcal{C}\mathcal{S} = 0 , \tag{161}$$

where

$$\mathcal{A} = \frac{1}{\gamma}\left(\frac{\gamma+1}{2}\right)^{\frac{1}{2}(\gamma+1)/(\gamma-1)} \frac{\bar{q}}{\bar{c}^{2/(\gamma-1)}} \frac{\overline{c^2}\xi^{(1)} - \zeta}{\overline{q^2}(\overline{c^2}\xi^{(1)} - \zeta) - i\omega}$$

$$\mathcal{B} = i\,\frac{\bar{q}^{\frac{1}{2}}\omega}{\bar{c}^{1/(\gamma-1)}} \frac{\overline{c^2}\xi^{(1)}}{\overline{q^2}(\overline{c^2}\xi^{(1)} - \zeta) - i\omega}$$

$$\mathcal{C} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}} \frac{1}{\overline{qc^2}} \frac{\frac{1}{2}(1-\overline{q^2})\xi^{(1)} - i\omega\xi^{(2)} + f_3\zeta}{\overline{q^2}(\overline{c^2}\xi^{(1)} - \zeta) - i\omega} .$$

Certain (otherwise redundant) additional definitions are made necessary by the way the original computer program was written. Instead of ϕ we introduce a new independent variable

$$\hat{\phi} = 2\kappa(\phi - \phi_{th}) .$$

The value κ derives from the following simple considerations. Since, by definition, $d\phi = \bar{q}dz$, we find $d\hat{\phi} = 2\kappa\bar{q}dz$, with z representing again the physical axial coordinate. Hence

$$\frac{d\overline{q^2}}{d\hat{\phi}} = \frac{1}{\kappa}\frac{d\bar{q}}{dz} .$$

The particular choice

$$\kappa = \left(\frac{d\bar{q}}{dz}\right)_{th}$$

results in $d\overline{q^2}/d\hat{\phi} = 1$ at $\hat{\phi} = 0$. This is the value of κ that was adopted in the following calculations. For the throat geometry of Figure 17, κ can be expressed explicitly (using, for instance, the procedure outlined in Reference 9) as

$$\kappa = \left(\frac{2}{\gamma+1}\frac{r^*_{th}}{R^*}\right)^{\frac{1}{2}} = \left(\frac{2}{\gamma+1}\frac{1}{R}\right)^{\frac{1}{2}} .$$

In conjunction with the independent variable $\hat{\phi}$ the additional definitions were made

$$\zeta = 2\kappa\hat{\zeta} , \qquad \xi^{(1)} = 2\kappa\hat{\xi}^{(1)} , \qquad \omega = \kappa\hat{\omega} , \qquad s_{\nu h} = \kappa\hat{s}_{\nu h} .$$

In terms of the variables the admittance coefficients may be written as follows:

$$\left.\begin{aligned} \mathcal{A} &= \frac{1}{\gamma}\left(\frac{\gamma+1}{2}\right)^{\frac{1}{2}(\gamma+1)/(\gamma-1)} \frac{\bar{q}}{\bar{c}^{2/(\gamma-1)}} \frac{\overline{c^2}\hat{\xi}^{(1)} - \hat{\zeta}}{\overline{q^2}(\overline{c^2}\hat{\xi}^{(1)} - \hat{\zeta}) - \frac{1}{2}i\hat{\omega}} \\ \mathcal{B} &= i\kappa\,\frac{\hat{\omega}\bar{q}^{\frac{1}{2}}}{\bar{c}^{1/(\gamma-1)}} \frac{\overline{c^2}\hat{\xi}^{(1)}}{\overline{q^2}(\overline{c^2}\hat{\xi}^{(1)} - \hat{\zeta}) - \frac{1}{2}i\hat{\omega}} \end{aligned}\right\} \qquad (162)$$

$$C = \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{2}} \overline{qc^2} \left. \frac{\frac{1}{2}(1 - \overline{q^2})\hat{\xi}^{(1)} - \frac{1}{2}i\hat{\omega}\xi^{(2)} + f_3\hat{\zeta}}{\overline{q^2}(\overline{c^2}\hat{\xi}^{(1)} - \hat{\zeta}) - \frac{1}{2}i\hat{\omega}} \right\} . \tag{162}$$

The admittance coefficients are complex numbers since $\hat{\zeta}$, $\hat{\xi}^{(1)}$. $\xi^{(2)}$, f_3 and $i\hat{\omega}$ are complex.

An interesting observation is that the second order differential equation (158) need not be solved for Φ_h , $\Phi^{(1)}$ and $\Phi^{(2)}$. It is sufficient indeed to solve directly the equations which are satisfied by $\hat{\zeta}$, $\hat{\xi}^{(1)}$ and $\xi^{(2)}$, that is*

$$\left(\frac{d\hat{\zeta}}{d\hat{\phi}} + \hat{\zeta}^2\right) = (g + ih)\hat{\zeta} - (j - ik) \tag{163}$$

$$\frac{d}{d\hat{\phi}}[(1 - \overline{q^2})\hat{\xi}^{(1)}] = -\left\{\hat{\zeta} - i\hat{\omega}\left(\frac{1}{2\overline{q^2}} + \frac{2}{(\gamma + 1)(1 - \overline{q^2})}\right)\right\}[(1 - \overline{q^2})\hat{\xi}^{(1)}] + \\ + \frac{2}{\gamma + 1}\frac{\hat{s}^2_{\nu h}}{4}\frac{\overline{c}^{2/(\gamma-1)}}{\overline{q}} \tag{164}$$

$$\frac{d}{d\hat{\phi}}[(1 - \overline{q^2})\xi^{(2)}] = -\left\{\hat{\zeta} - i\hat{\omega}\left(\frac{1}{2\overline{q^2}} + \frac{2}{(\gamma + 1)(1 - \overline{q^2})}\right)\right\}[(1 - \overline{q^2})\xi^{(2)}] + \\ + \frac{2}{\gamma + 1}\left\{\frac{df_3}{d\hat{\phi}} + \frac{\hat{s}^2_{\nu h}\overline{c}^{2/(\gamma-1)}\overline{q}}{2i\hat{\omega}}\left(\frac{1 - \overline{q^2}}{2\overline{q^2}} + \frac{\overline{c^2}}{\overline{q^2}}f_3\right)\right\} \tag{165}$$

where the following definitions apply:

$$b = \overline{q^2}(\overline{c^2} - \overline{q^2})$$

$$g = \frac{\gamma + 1}{2}\frac{\overline{q^2}}{\overline{c^2}}\frac{d\overline{q^2}}{d\hat{\phi}}$$

$$h = \hat{\omega}\overline{q^2}$$

$$j = \frac{\hat{\omega}^2}{4}\frac{\overline{qc}^{2\gamma/(\gamma-1)}}{4}\hat{s}^2_{\nu h}$$

$$k = \frac{\gamma - 1}{4}\frac{\overline{q^2}}{\overline{c^2}}\frac{d\overline{q^2}}{d\hat{\phi}}\hat{\omega} .$$

* Equation (163) is the Riccati equation obtained directly from (158) upon substitution of $\hat{\zeta}$ for $(1/\Phi)(d\Phi/d\hat{\phi})$. Equations (164) and (165) are obtained, after the proper adjustment, from Equations (112).

The nonlinear, first order equation for $\hat{\zeta}$ is a complex Riccati equation and may only be solved by numerical integration. Once this is done, the linear, first order equations for $\hat{\xi}^{(1)}$ and $\xi^{(2)}$ may be solved, obtaining the standard integral forms which are essentially identical to the form given by (111). However, rather than numerically evaluating the integral solutions, it was more convenient to solve all three complex (or six real) equations simultaneously by numerical integration.

The numerical integration requires statements on the initial conditions. These are provided by the condition of regularity at the throat ($\hat{\phi} = 0$). After assuming a Taylor series expansion about the throat and evaluating the coefficients in the series, we find

$$\hat{\zeta}(0) = \alpha_0 + i\beta_0$$

$$\hat{\xi}^{(1)}(0) = 0$$

$$\xi^{(2)}(0) = 0 \;,$$

where

$$\alpha_0 = \frac{\dfrac{\gamma + 1}{2}\,\dfrac{\hat{\omega}^2 - \hat{s}_{\nu h}^2}{4} - \dfrac{\gamma - 1}{4}\,\hat{\omega}^2}{\hat{\omega}^2 + \left(\dfrac{\gamma + 1}{2}\right)^2}$$

$$\beta_0 = -\frac{\dfrac{\gamma^2 - 1}{8} + \dfrac{\hat{\omega}^2 - \hat{s}_{\nu h}^2}{4}}{\hat{\omega}^2 + \left(\dfrac{\gamma + 1}{2}\right)^2} \;.$$

Since the Riccati equation is singular at the throat ($\overline{c^2} - \overline{q^2} = 0$), the numerical integration cannot begin exactly there. So, in addition to the above conditions the first derivative of $\hat{\zeta}$ at the initial point must be obtained from the regular expansion and supplied to the numerical integration scheme. This initial condition is

$$\left(\frac{d\hat{\zeta}}{d\hat{\phi}}\right)_{th} = \alpha_1 + i\beta_1 \;,$$

where

$$\alpha_1 = \frac{1}{\hat{\omega}^2 + (\gamma + 1)^2}\left\{\frac{(\gamma + 1)^2}{2}(\alpha_0^2 - \beta_0^2) + (\gamma + 1)\hat{\omega}\alpha_0\beta_0 + \right.$$

$$+ \left[\hat{\omega}^2 + \frac{(\gamma + 1)^3}{4} + (\gamma + 1)^2\bar{b}\right]\alpha_0 + \frac{(\gamma + 1)(\gamma - 3)}{4}\hat{\omega}\beta_0 +$$

$$\left. + \frac{\gamma^2 - 1}{8}(\hat{\omega}^2 - \hat{s}_{\nu h}^2) + \frac{\gamma - 1}{2}\hat{\omega}\bar{b}\right\}$$

$$\beta_1 = \frac{1}{\hat{\omega}^2 + (\gamma + 1)^2} \left\{ (\gamma + 1)^2 \alpha_0 \beta_0 - \frac{\gamma + 1}{2} \hat{\omega}(\alpha_0^2 - \beta_0^2) + \right.$$

$$+ (\gamma + 1) \frac{1 - \gamma}{2} \hat{\omega}\alpha_0 - (\gamma + 1)\hat{\omega}\alpha_0 \tilde{b} + \left[\hat{\omega}^2 + \frac{(\gamma + 1)^3}{4} \right] \beta_0$$

$$\left. + \frac{\gamma - 1}{8} \hat{\omega} \, [\hat{s}_{\nu h}^2 + (\gamma + 1)^2] + \frac{\gamma^2 - 1}{2} \hat{\omega}\tilde{b} \right\} .$$

Note that $\tilde{b}$ is a coefficient in the power series expansion for $\overline{q^2}$. That is $\overline{q^2} = 1 + \hat{\phi} + \tilde{b}\hat{\phi}^2 + \ldots$, the value of $\tilde{b}$ being determined by the nozzle geometry in the throat vicinity. The above-mentioned power series shows the convenience of the transformation of the independent variable from ϕ to $\hat{\phi}$; the coefficient of the first order term in the power series is unity.

Rather than numerically evaluating the integral for f_3 , it is convenient to solve a first order differential equation for f_3 simultaneously with the equations for $\hat{\zeta}$, $\hat{\xi}^{(1)}$, and $\xi^{(2)}$. As directly checked, the proper equation is as follows:

$$\frac{d}{d\hat{\phi}} (\overline{c^2} f_3) - \frac{i\hat{\omega}}{2\overline{q^2}} (\overline{c^2} f_3) = \frac{1}{2} \frac{d\overline{q^2}}{d\hat{\phi}} , \tag{166}$$

subject to the initial condition that $f_3(0) = 0$.

The steady-state velocity profile $\overline{q}(\hat{\phi})$ must be determined for the given geometry of the convergent portion of the nozzle. This cannot be found in closed form but must be determined either approximately or indirectly. It is difficult to find an approximate form of $\overline{q}(\hat{\phi})$ which is very accurate over a wide class of nozzle geometries. By indirect means the exact form of $\overline{q}$ versus $\hat{\phi}$ may be determined and a table may be constructed. This would proceed in the following manner:

$$\hat{\phi} = 2\kappa(\phi - \phi_{th}) = 2\kappa \int_0^z \overline{q} \, dz' ,$$

where z is the axial coordinate. Furthermore, for circular cross-sections, the continuity, isentropic, and isoenergetic relations may be used to relate the chamber radius r to the velocity $\overline{q}$. The result is

$$r = r(\overline{q}) = \frac{1}{\overline{q}^{\frac{1}{2}} \left(\frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} \overline{q}^2 \right)^{1/\{2(\gamma - 1)\}}} .$$

This equation could be used directly for given nozzle geometry, to calculate $\overline{q}(\hat{\phi})$. This procedure, however, is rather clumsy from the point of view of numerical calculations, for several reasons that we shall not discuss.

Instead, a much simpler procedure was employed which results in a straightforward computational procedure. By differentiation of the above relations, a first order differential equation is found to describe $\bar{q}(\hat{\phi})$. The equation can merely be solved simultaneously with the equations for $\hat{\zeta}$, $\hat{\xi}^{(1)}$, $\xi^{(2)}$, and f_3. The equation is of the form

$$\frac{d\bar{q}}{d\hat{\phi}} = \frac{d\bar{q}}{dr}\frac{dr}{dz}\frac{dz}{d\phi}\frac{d\phi}{d\hat{\phi}} = \frac{1}{2\kappa\bar{q}\, dr/d\bar{q}}\frac{dr}{dz},$$

so that it only remains to find an analytical form for dr/dz. For typical geometries, this is a simple matter.

The major portion of the calculations has been performed for the nozzle which we have already considered, with the generatrix shaped as a circular arc near the throat and with a smooth transition to a conical nozzle in the remainder of the convergent portion, as shown in Figure 1. The differential equation for this case is found to be:

$$\frac{d\bar{q}}{d\hat{\phi}} = \left\{ \begin{array}{ll} -\dfrac{(2R(r-1)-(r-1)^2)^{\frac{1}{2}}}{2\kappa\bar{q}(R+1-r)\, dr/d\bar{q}} & \text{in circular region} \\ \\ -\dfrac{\tan\theta_1}{2\kappa\bar{q}\, dr/d\bar{q}} & \text{in conical region ,} \end{array} \right\} \tag{167}$$

where all lengths are nondimensionalized with respect to the throat radius. According to (167), $\bar{q}$ and $d\bar{q}/d\hat{\phi}$ (but not $d^2\bar{q}/d\hat{\phi}^2$) are continuous at the matching point ($z = -R\sin\theta_1$) between the circular and conical portions. This is necessary and sufficient to perform the integration.

The initial condition is $\bar{q}(0) = 1$. Note that $\hat{\phi} = 0$ is a singular point since $r = 1$ and $dr/d\bar{q} = 0$ there. One can, however, find a regular expansion which gives $d\bar{q}/d\hat{\phi}(0) = \frac{1}{2}$ (in agreement with the value $(d\bar{q}^2/d\hat{\phi})_{th} = 1$) thereby allowing the numerical integration to be performed. From this same series expansion, the term $\tilde{b}$ appearing in the initial conditions for $\hat{\zeta}$ may be found. For the present geomentry, it is

$$\tilde{b} = \frac{2-\gamma^2}{6(\gamma+1)} .$$

The relationship between the nondimensional distance from the throat z and the mean velocity $\bar{q}$ is

$$z = \left\{ \begin{array}{ll} -(2R(r-1)-(r-1)^2)^{\frac{1}{2}} & \text{if} \quad 0 \geqslant z \geqslant -R\sin\theta_1 \\ \dfrac{1}{\tan\theta_1}\left(1 - r + R\dfrac{\cos\theta_1 - 1}{\cos\theta_1}\right) & \text{if} \quad z < -R\sin\theta_1 . \end{array} \right\} \tag{168}$$

The system of differential equations (163) through (167) have been solved using a Milne variable-step-size technique by means of a Fortran IV program on an IBM 7094 computer. The calculations are made with double precision and a predictor-corrector

scheme is employed which gives an accuracy of seven (or, in other words, the ratio of the error in the integration to the correct value is of the order of 10^{-7}). These are found to be the optimum precision and accuracy within the limits of the operating range.

The value $\gamma = 1.2$ was taken; ω values were in the range $0 \leqslant \omega \leqslant 10$ and $s_{\nu h}$ values in the range $0 \leqslant s_{\nu h} \leqslant 9$. For each combination of ω and $s_{\nu h}$, the system was integrated along the nozzle length beginning at the throat where the Mach number* is unity and extending to the point where the Mach number is 0.05. The solutions were determined at certain specified values of the Mach number by interpolation and then the admittance coefficients were calculated at these specified points by means of (162).

In using these results, one would take the values of the admittance coefficients at that Mach number which equals the entrance Mach number of the particular nozzle of interest. In this way, as already indicated at the beginning of this section, one integration presents information for an infinity of contraction ratios. Of course, we are considering here a conical nozzle joined directly to a cylindrical chamber. In practice, a smooth transition would occur between the conical and cylindrical portions. However, an exact calculation of this actual situation would require one integration for the contraction ratio. So the approximation employed in the calculations is a large-time-saving technique. The validity of this approximation was verified by calculations with the more realistic nozzle shape of Figure 17. These calculations are discussed in Section 15.

The radius of curvature of the circular portion of the nozzle wall was equal to the throat diameter (R = 2) while the semi-angle of the conical portion was 30° in these calculations (Figure 1 is actually drawn with these specifications). It is important to remember, however, that the admittance coefficients may be applied by a scaling procedure to a whole class of nozzle shapes. As shown in Section 13 and illustrated in Figure 2, any nozzle shape which can be obtained by "stretching" the wall of the reference nozzle uniformly in the axial direction by a factor $1/\beta$ is a member of this class. "Shrinking" the nozzle shape is simply considered by letting β be larger than unity. The scaling rules are given in the figure. Scaling in the radial direction is trivial, since all lengths are nondimensionalized with respect to the throat radius. Note that the radius of curvature at the throat R and the convergence angle θ_1 do not vary independently in the scaling procedure. So all "conical" nozzles cannot be obtained exactly by scaling from any one reference nozzle. However, for the range of shapes of practical interest, good approximations can usually be obtained by scaling from the reference nozzle. This is discussed in detail in Section 19.

Now that the possibility of scaling has been included, $s_{\nu h}$ has been replaced by $s_{\nu h}/\beta$ and this value will no longer necessarily correspond to an eigenvalue. Therefore, rather than taking the usual eigenvalues for $s_{\nu h}$, it is more convenient to use integral values. If results are desired for non-integral values of $s_{\nu h}$, interpolations may be made on certain convenient cross-plots.

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* Note that the mean flow Mach number may replace the axial coordinate as the independent variable for the purpose of presenting the results.

Figures 3 through 6 show results of the numerical integration for a sample case ($\omega = 0.5$, $s_{\nu h} = 1.0$) where the frequency is not so high. In Figure 3, $\hat{\zeta}_r$ is plotted versus axial distance showing a gradual change in $\hat{\zeta}_r$ due to the relatively large pressure wavelength. Figure 4 shows $\xi_r^{(2)}$ to be undulating* rapidly due to the relatively small entropy and vorticity wavelengths. Note, of course, that pressure waves propagate with the speed of sound, while entropy and vorticity waves propagate with the subsonic gas velocity. Figures 5 and 6 show the admittance coefficients $\mathcal{A}_r$ and $\mathcal{B}_i$ (which are the most pertinent from a stability point of view) plotted versus axial distance. Superimposed upon a gradual change due to the pressure waves, we see a rapid undulation due to the entropy and vorticity waves. It is apparent that the admittance coefficients, and therefore the stability of the motor operation, can be quite sensitive to contraction ratio variations.

At higher frequencies the oscillations become more severe, since undulations in the admittance coefficients occur due to pressure waves in that case. The undulations due to entropy and vorticity waves become still more rapid. It is worthwhile to discuss the nature of the phenomenon at higher frequencies and its effect upon the utility of the calculations.

An interesting phenomenon occurs in the solution of the second order equation for Φ and, therefore, in $\hat{\zeta}$, $\hat{\xi}^{(1)}$, $\xi^{(2)}$, and the admittance coefficients. The real part of the coefficient of Φ in the second order Equation (158) is

$$\omega^2 - \bar{q}\bar{c}^{2/(\gamma-1)} s_{\nu h}^2 = 4\kappa^2 j .$$

This may become negative for low values of the ratio $\omega/s_{\nu h}$, especially in the high Mach number region (where $\bar{q}$ is large). Note that this never happens for longitudinal oscillations where $s_{\nu h} = 0$. If, for a given oscillation (ω and $s_{\nu h}$ are fixed), this number changes sign as $\bar{q}$ varies with $\hat{\phi}$, the equation has a turning point. In the region of low $\bar{q}$, where this number is positive, the solution for $\Phi(\hat{\phi})$ is undulatory, while in the region of high $\bar{q}$ where the number is negative, $\Phi(\hat{\phi})$ is not undulatory. Undulations in $\Phi(\hat{\phi})$ imply that longitudinal undulations as well as transverse undulations occur in the nozzle.

This turning point occurs in the range of physical interest for either transverse mode undulations in configurations where the nozzle volume is large compared to the chamber† volume or for mixed mode undulations. The physical reason is that the same frequency may be a low frequency in the small diameter, high $\bar{q}$, region near the throat while it is at the same time a high frequency in the large diameter, low $\bar{q}$, region away from the throat. In the case of transverse mode undulations in configurations where the nozzle volume is large compared to the chamber volume, the frequency is determined by a characteristic dimension which is smaller than the chamber radius. Similarly, the characteristic dimension for mixed mode undulations is smaller than the chamber radius. So, in both cases, the nozzle may have two separate regions. One region, where the radius is large compared to the characteristic dimension, contains longitudinal undulations as well as transverse undulations. This region obviously will always occur for mixed mode undulations, but even in that case need

* "Undulation" pertains to space-wise variations while "oscillation" pertains to time-wise variations.

† As already indicated, the chamber is that portion which is of constant diameter while the nozzle is the convergent portion.

not extend all the way to the throat. The other region, where the radius is small compared to the characteristic dimension, contains only transverse undulations. This region always occurs for transverse mode undulations and, in that case, unless the nozzle volume is quite large compared to the chamber volume, it extends over the whole volume. It is seen, therefore, that in a given situation either or both regions may occur.

Any undulation in Φ_h means that Φ_{hr} and Φ_{hi} pass through zero, although, in general, not simultaneously. This means $\hat{\zeta}$ can become large locally with the result that $\hat{\xi}^{(1)}$, $\xi^{(2)}$, and the admittance coefficients also exhibit wild behavior locally. These regions of wild behavior do appear in the results of the calculations for certain ranges of high $\omega/s_{\nu h}$ and low Mach number. The simple physical interpretation is that nodal points occur in these regions and the reciprocal of the admittance coefficient (which is itself an admittance coefficient) is small. So, the wild behavior in the mathematics is seen not to be a physical wild behavior by merely observing the reciprocal of the results.

However, since wild behavior means large changes over a small range, it is not feasible to interpolate between two calculated values in this range to obtain a third value*. It is desirable therefore to find another means for determining the admittance values in this troublesome range. This has been accomplished by the development of asymptotic solutions for $\hat{\zeta}$, $\hat{\xi}^{(1)}$, $\xi^{(2)}$, and f_3 which apply to the low Mach number range of the conical nozzle. This asymptotic theory is discussed in Section 17.

In addition to the admittance coefficients $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$, two other complex admittance coefficients are useful and have been calculated. They are

$$\alpha = -\left(\frac{\gamma + 1}{2}\right)^{(\gamma+1)/\{2(\gamma-1)\}} \frac{\bar{q}}{\bar{c}^{2/(\gamma-1)}} \frac{\hat{\zeta}}{\bar{q}^2\hat{\zeta} + i\hat{\omega}/2}$$

$$\mathcal{E} = \gamma\mathcal{A} - \frac{\mathcal{B}}{i\omega_c} ,$$

where ω_c is the frequency nondimensionalized, as in Part I, by the ratio of the steady-state stagnation speed of sound to the nozzle entrance radius.

α is the admittance coefficient to be used in the relation $U = (\alpha/\gamma)P$ at the nozzle entrance in the absence of vorticity and entropy perturbations. This is essentially identical to (105) and (107), except that the separation of variables scheme and nondimensional scheme are slightly different. (See the discussion following (160)). Of course, when $s_{\nu h} = 0$, α is also the admittance coefficient corresponding to (107) for isentropic longitudinal oscillations.

$\mathcal{E}$ is a combination of $\mathcal{A}$ and $\mathcal{B}$ which becomes important in typical combustion instability applications, as will be shown in a later section treating the application of the results. It can be shown that, for low Mach numbers, α and $-\mathcal{E}$ are approximately equal. This means that at low mean-flow Mach numbers $\mathcal{E}$ becomes approximately independent of $\hat{\xi}^{(1)}$, even though $\mathcal{A}$ and $\mathcal{B}$ are dependent upon it.

* Due to time and expenses, neither is it feasible to perform the calculations with a sufficiently fine mesh in the parameter space to allow interpolation.

The results of the calculations for the real and imaginary parts of the five coefficients $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{E}$, and α are presented as functions of frequency ω with $s_{\nu h}$ and Mach number as parameters in Figures 7 to 11. Here the frequency is nondimensionalized by the ratio of the steady-stage stagnation speed of sound to the nozzle throat radius. In order to obtain ω_c, ω must be multiplied by the square root of the area contraction ratio.

One of the most interesting results is that the nozzle may have a destabilizing effect upon the transverse modes of oscillation. This is indicated by negative values of the real part of α and positive values of the real part of $\mathcal{E}$. (The importance of the signs of these quantities will be discussed in Section 19.) Negative α_r and positive $\mathcal{E}_r$ generally seem to occur in the range of "purely" transverse modes where ω_c is close to $s_{\nu h}$. So here the nozzle would have a destabilizing effect. For longitudinal modes and those mixed modes where the longitudinal dimensions are most significant in determining the frequency $(\omega_c >> s_{\nu h})$, α_r is positive and $\mathcal{E}_r$ is negative, so that the nozzle has a damping effect upon the oscillations.

It is seen from the figures that the wild behavior of the solutions at low Mach numbers makes interpolation of the results most difficult. For this reason, as already mentioned, the asymptotic solution has been employed at these low Mach numbers in order to determine the admittance coefficients in that range.

The figures indicate that the value of the admittance coefficient $\mathcal{C}$ are generally quite small compared to the coefficients $\mathcal{A}$ and $\mathcal{B}$. This and the fact that the amplitude of the entropy oscillation is small compared to the amplitude of the pressure and velocity oscillations in most situations of physical interest mean that usually (159) may be simplified to the following form

$$\mathcal{U} + \mathcal{A}\mathcal{P} + \mathcal{B}\mathcal{V} = 0 . \tag{159a}$$

As shown in Figure 8, $\mathcal{B} = 0$ whenever $s_{\nu h} = 0$. Furthermore $\hat{\xi}^{(1)}$ also equals zero and it follows that $(-\alpha/\gamma)$ and $\mathcal{A}$ are identical in that case.

It should be noted that the results of the calculations for the standard three-dimensional axisymmetric nozzle may be scaled for use with certain annular nozzles. The major restriction is that the inner wall of the annular nozzle must have the same shape as a stream tube contour in the three-dimensional nozzle. This implies that the two nozzle flows are identical in the steady state (that is, of course, only in the common region where both flows exist). Also, under the long-nozzle, one-dimensional steady-state flow assumption, this means that the ratio of the outer wall radius to the inner wall radius is constant along the convergent section of the nozzle.

The equations for the annular nozzle may be separated in the same manner and the same differential equations remain to be integrated as in the three-dimensional case. However, now $s_{\nu h}$ is no longer the h^{th} root of $J'_\nu(x) = 0$ but rather it is the h^{th} root of $J'_\nu(x)Y'_\nu(\beta x) - J'_\nu(\beta x)Y'_\nu(x) = 0$. Here J_ν and Y_ν are Bessel functions of the first and second kind, respectively, and β is the ratio of the inner to outer wall diameter. (ν is an integer, here.) So using the proper value of $s_{\nu h}$, the results of the three-dimensional nozzle calculations for both admittances and flow properties may be used for the annular nozzle. The values of $s_{\nu h}$ for various annuli may be found in Reference 10*.

* In that Reference, β is defined as the reciprocal of our β, so that their value of $s_{\nu h}$ must be multiplied by their β to obtain our value of $s_{\nu h}$.

A limited number of calculations have been performed wherein the throat wall curvature, the cone angle, and the ratio of specific heats have been changed. It was found that changing the last parameter from the standard values of $\gamma = 1.2$ to $\gamma = 1.4$ generally produced a change in the admittance coefficients of only a few percent. The other two parameters affected the results more significantly, as shown in Figures 12 through 16 where the results for different values of these parameters are compared.

Calculations were made with $R = 3.0$ (versus $R = 2.0$ in the standard cases) and $\theta_1 = 30^\circ$ and also with $R = 2.0$ and $\theta_1 = 15^\circ$ (versus $\theta_1 = 30^\circ$ in the standard cases). When R was changed and θ_1 left constant, the results changed most significantly in the high Mach number range near the throat. Further upstream in the low Mach number range, the difference between the $R = 2.0$ and $R = 3.0$ cases is smaller On the basis of this small amount of evidence, it seems that far away from the throat the results do not depend very strongly on the particulars of the nozzle shape near the throat. When θ_1 was changed and R left constant, the solution near the throat did not change, of course. Only in the conical portion of the nozzle was a change produced.

15. EFFECT OF TRANSITION REGION BETWEEN CYLINDRICAL CHAMBER AND CONICAL CONVERGENT NOZZLE

As mentioned in the previous section, it is expedient to disregard the actual shape of the nozzle entrance portion, by means of the approximate assumption that the conical portion and the chamber are directly connected. With this approximation the admittance coefficients for an infinity of contraction ratios may be obtained from only one numerical integration.

In order to ascertain whether this approximation does indeed produce the negligible error which would be expected, coefficients were calculated for a very limited number of cases with more realistic nozzle entrance portions. As Figure 17 indicates, these calculations have two phases. The first phase involves the determination of the admittance coefficients at the entrance to the actual nozzle with contour ABCD. For the sake of fair comparison with the results of the calculations for the approximate nozzle shape with contour ABE, we must include the effect of the cylindrical portion ED. So the second phase involves the calculation of the admittance coefficients at the E-end of the cylinder given the coefficients at the D-end (which were the results of the first-phase calculations). It is the results of these second-phase calculations (that is, the coefficients at the E-end of the cylinder) which must be compared to the results of the calculations for the approximate contour. Obviously the entrance Mach numbers for the two nozzles shown in the figure are identical.

In the calculation of the admittance coefficients for the actual nozzle, all the differential equations remain the same except the equation for the velocity $\bar{q}$ which has a different form in the transition region. In that region the equation becomes

$$\left.\begin{aligned} \frac{d\bar{q}}{dx} &= -\frac{1}{2\kappa\bar{q}\, dr/d\bar{q}}\,\frac{(2R_2(r_e - r) - (r - r_e)^2)^{\frac{1}{2}}}{r_e - r - R_2} \\ z &= z_e + (2R_2(r_e - r) - (r_e - r)^2)^{\frac{1}{2}} \ , \end{aligned}\right\} \qquad (169)$$

for the range $z_t > z > z_e$, where

$$z_e = z_t - R_2(2(1 - \cos\theta_1) - (1 - \cos\theta_1)^2)^{\frac{1}{2}}$$

$$z_t = \frac{R_2(1 - \cos\theta_1) + R\left(1 - \dfrac{1}{\cos\theta_1}\right) + 1 - r_e}{\tan\theta_1} .$$

Here the transition region wall contour is given by an arc of a circle with radius of curvature R_2 and the radius of the nozzle entrance cross-section is r_e . The velocity is still governed by (167) in the other regions of the convergent portion of the nozzle.

With the above simple modifications it is possible to find the admittance coefficients for this nozzle by integration of the equations. Now it remains to calculate what the proper value of the admittance coefficient at the end of the approximate nozzle would have to be. This value then must be compared to the results presented in the previous section in order to determine the accuracy of using the approximate nozzle shape.

This second calculation involves the determination of the admittance coefficients at one end of a cylinder given the coefficients at the other end. So, an analysis of three-dimensional oscillations with vorticity waves and entropy waves in a cylinder with uniform mean flow is involved. This analysis is essentially a specialization of that already performed for the variable cross-sectional area nozzle and is outlined in the following discussion.

The linearized equations of motion (17) through (20) may be separated in a cylindrical coordinate system by the scheme indicated by (160) with the addition that

$$w' = \mathbb{W}(z)\,\frac{1}{r}\,J_{\nu h}(s_{\nu h}r)\Theta'(\theta)e^{i\omega t}$$

$$\rho' = \mathbb{R}(z)J_{\nu h}(s_{\nu h}r)\Theta(\theta)e^{i\omega t} .$$

The reference length in the nondimensional scheme is now taken as the radius of the cylinder walls. $z = 0$ is one end of the cylinder and $z = \ell$ is the other end. A positive uniform mean flow $\bar{q}$ in the axial direction exists in the cylinder.

The resulting system of ordinary differential equations may be solved to obtain the following:

$$\left.\begin{aligned}
\mathbb{P}(z) &= \alpha e^{i\lambda_1 z} + \beta e^{i\lambda_2 z} \\
\mathbb{U}(z) &= -\frac{\alpha}{\gamma}\frac{1}{\omega + \bar{q}\lambda_1}\,e^{i\lambda_1 z} - \frac{\beta}{\gamma}\frac{\lambda_2}{\omega + \bar{q}\lambda_2}\,e^{i\lambda_2 z} + \frac{\left(\bar{q}\,\dfrac{s_{\nu h}}{\omega}\right)^2}{1 + \left(\bar{q}\,\dfrac{s_{\nu h}}{\omega}\right)^2}\,e^{-i\frac{\omega}{\bar{q}}z}
\end{aligned}\right\} \quad (170)$$

$$
\left.\begin{aligned}
\mathcal{V}(z) &= \mathcal{U}(z) = i\frac{\alpha}{\gamma}\frac{e^{i\lambda_1 z}}{\omega + \bar{q}\lambda_1} + \frac{\beta}{\gamma}\frac{e^{i\lambda_2 z}}{\omega + \bar{q}\lambda_2} - i\delta\frac{\omega\bar{q}}{\omega^2 + \bar{q}^2 s_{\nu h}^2}\, e^{-i\frac{\omega}{\bar{q}} z} \\
\mathcal{S}(z) &= \gamma\epsilon\, e^{-i\frac{\omega}{\bar{q}} z} \\
\mathcal{R}(z) &= \frac{1}{\gamma}\left[\mathcal{P}(z) - \mathcal{S}(z)\right] ,
\end{aligned}\right\} \tag{170}
$$

where α , β , δ , and ϵ are undetermined constants and

$$
\lambda_{1,2} = \left[\omega\bar{q} \pm (\omega^2 - s_{\nu h}^2 + \bar{q}^2 s_{\nu h}^2)^{\frac{1}{2}}\right]/(1 - \bar{q}^2) .
$$

In the case of interest the admittance coefficients at $z = 0$ are known; that is, the complex numbers $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ are known in the relation

$$
\mathcal{U}(0) + \mathcal{A}\mathcal{P}(0) + \mathcal{B}\mathcal{V}(0) + \mathcal{C}\mathcal{S}(0) = 0 \tag{171}
$$

by means of the first-phase calculation for the nozzle.

Now, first by applying (170) to determine $\mathcal{U}(0)$, $\mathcal{P}(0)$, $\mathcal{V}(0)$, and $\mathcal{S}(0)$ in (171) and then using (170) to obtain four relations, one identity for each of $\mathcal{U}(\ell)$, $\mathcal{P}(\ell)$, $\mathcal{V}(\ell)$ and $\mathcal{S}(\ell)$, we have a system of five linear equations for the four unknown constants α , β , δ , and ϵ . This means a certain relation must exist between the flow properties at $z = \ell$. The relation is readily obtained by setting the determinant of the system of five equations equal to zero. It is of the form

$$
\mathcal{U}(\ell) + \bar{\mathcal{A}}\mathcal{P}(\ell) + \bar{\mathcal{B}}\mathcal{V}(\ell) + \bar{\mathcal{C}}\mathcal{S}(\ell) = 0 .
$$

Since the formulas for $\bar{\mathcal{A}}$, $\bar{\mathcal{B}}$, and $\bar{\mathcal{C}}$ are rather cumbersome, they are not presented here. The reader may easily produce them solely by means of (170) and (171). The formulas allow the calculation of $\bar{\mathcal{A}}$, $\bar{\mathcal{B}}$ and $\bar{\mathcal{C}}$ given γ , ω , $s_{\nu h}$, ℓ , $\bar{q}$, $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$. (Note that α , β , δ , and ϵ have been eliminated.) Due to the fact that different reference lengths have been used in the two phases of the calculations, the above value of ω is greater than the ω used in the calculations of $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ by a factor equal to the chamber radius divided by the throat radius. Also since ℓ must correspond to the length ED in Figure 17, we see that $\ell = (R_2/r_e)\tan(\theta_1/2)$.

The calculations were performed for various values of ω , $s_{\nu h}$, R_2 , and $\bar{q} = \bar{q}_e$ evaluated at the nozzle entrance (which determines the contraction ratio). $\gamma = 1.2$, $\theta_1 = 30^\circ$, and $R = 2.0$ were taken in the calculations. Table II presents sample results which show the error produced in using the approximate nozzle shape.

The first complex number given for the coefficients is the value calculated at the entrance of the actual nozzle while the second number is the proper effective value to be used at the entrance of the approximate nozzle. (This number resulted from the second phase of the calculations.) The third number is the result of the calculations for the approximate nozzle discussed in Section 14. So the error is seen by comparing the second and third numbers. Note that, instead of comparing the real and imaginary parts separately, one should more properly compare magnitudes and phases.

In the first four examples the frequency ω is large compared to the eigenvalue $s_{\nu h}$. In those cases the frequency is primarily determined by the longitudinal dimension. The solutions are varying slowly through the transition region as indicated by the small difference between the first two numbers. Most importantly, the small difference between the second and third numbers shows that the error introduced by the approximate nozzle is small even when the radius of curvature R_2 is not-so-small.

In the next four cases ω and $s_{\nu h}$ are equal. Since the frequency is nondimensionalized with respect to the throat radius, the chamber frequency ω_c is actually larger than the eigenvalue. This is the region of mixed mode oscillations where, for the flow velocities indicated, the radial and longitudinal dimensions are equally important in determining the frequency. Here the flow properties may vary more rapidly through the transition region, resulting in the possibility of significant errors by use of the approximate nozzle. Still, as shown in Table II, the error generally is satisfactorily small for small radii of curvature at the nozzle entrance. This is expected, since the actual nozzle approaches the approximate nozzle as R_2 tends to zero.

The last example, when compared with the one immediately preceding it, indicates the improvement in the accuracy of the approximation as the frequency decreases below the eigenvalue. There, the range of "pure" transverse oscillations is approached.

In conclusion, the calculations show that the approximation usually introduces little error. However, in those ranges where the flow properties vary rapidly in the axial direction, significant errors may be obtained for large radii of curvature at the nozzle entrance.

16. FLOW PROPERTIES

It is interesting to examine the actual velocities, pressure, and entropy of the oscillatory nozzle flow, even though the knowledge of these quantities is not required for the determination of the admittance coefficients. The determination of the flow properties should lead to a better physical understanding of the nature of the oscillation in the nozzle.

In order to determine these quantities one must solve (158) for Φ and substitute the solution into (102). In addition f_0, f_1, and f_2 must be determined and σ, C_1, and C_2 must be specified. In order to solve (158) it was convenient to first change the independent variable from ϕ to $\hat{\phi} = 2\kappa(\phi - \phi_{th})$. The differential equation becomes

$$b\frac{d^2\Phi}{d\hat{\phi}^2} - (g + ih)\frac{d\Phi}{d\hat{\phi}} + (j - ik)\Phi = -(G + H) , \tag{172}$$

where the coefficients b, g, h, j, and k are those defined immediately following (165) and G and H are given by

$$G = C_1\bar{c}^{2\gamma/(\gamma-1)}f_0\bar{q}\hat{s}_{\nu h}^2/4$$

$$H = (\sigma/2\kappa)\overline{c^2}f_0\left[\overline{q^2}\frac{df_3}{d\hat{\phi}} + \frac{\hat{s}_{\nu h}^2\bar{c}^{2/(\gamma-1)}\bar{q}}{2i\hat{\omega}}\left(\overline{c^2}f_3 + \frac{1-\overline{q^2}}{2}\right)\right] .$$

It is convenient for the purpose of numerical integration to reduce this second-order complex differential equation to four first order equations by means of the following definitions: $\Phi = y_1 + iy_2$, $d\Phi/d\hat{\phi} = y_3 + iy_4$. $G = G_r + iG_i$. and $H = H_r + iH_i$. Then the system of equations becomes

$$\left.\begin{aligned} y_1' &= y_3 \\ y_2' &= y_4 \\ y_3' &= \left[gy_3 - jy_1 - hy_4 - ky_2 - G_r - H_r\right]/b \\ y_4' &= \left[gy_4 - jy_2 + hy_3 + ky_1 - G_i - H_i\right]/b . \end{aligned}\right\} \tag{173}$$

Since $b = 0$ at the initial point of the integration (as before), one must insure that the regular solution is obtained by expanding the solution in a Taylor series about the singular point at the throat in order to determine the value of the solution and its first derivative at that point. These values are then used in the first step of the numerical integration. Again, we use the steady-state velocity profile $\overline{q^2} = 1 + \hat{\phi} + \bar{b}\hat{\phi}^2 + \ldots$ So, by expanding the coefficients and inhomogeneous parts of (173) in a series in $\hat{\phi}$ and determining the coefficients in the series solution for (173) in the vicinity of the throat, one finds the following initial conditions*:

$$\left.\begin{aligned} y_1(0) &= 1 , & y_1'(0) &= B_r \\ y_2(0) &= 0 , & y_2'(0) &= B_i \\ y_3(0) &= B_r , & y_3'(0) &= 2C_r \\ y_4(0) &= B_i , & y_4'(0) &= 2C_i . \end{aligned}\right\} \tag{174}$$

where the following definitions apply

$$B_r = \frac{\hat{\omega} I_2 - \dfrac{\gamma+1}{2} I_1}{\hat{\omega}^2 + \left(\dfrac{\gamma+1}{2}\right)^2}$$

$$B_i = \frac{\dfrac{\gamma+1}{2} I_2 + \hat{\omega} I_1}{\hat{\omega}^2 + \left(\dfrac{\gamma+1}{2}\right)^2}$$

---

* In linearized problems dealing with periodic phenomenon the amplitude and time-phase are arbitrary in the final solution. So with no loss of generality $\Phi(0) = 1$ was chosen.

$$C_r = \frac{2\hat{\omega}I_4 - 2(\gamma + 1)I_3}{4[\hat{\omega}^2 + (\gamma + 1)^2]}$$

$$C_i = \frac{2\hat{\omega}I_3 + 2(\gamma + 1)I_4}{4[\hat{\omega}^2 + (\gamma + 1)^2]} .$$

Defining further, we have

$$I_1 = \frac{\hat{s}_{\nu h}^2 - \hat{\omega}^2}{4} - \frac{\hat{s}_{\nu h}^2}{4} C_{1r} - \frac{\sigma_r}{2\kappa(\gamma + 1)}$$

$$I_2 = -\frac{\gamma - 1}{4}\hat{\omega} + \frac{\hat{s}_{\nu h}^2}{4} C_{1i} - \frac{\sigma_i}{2\kappa(\gamma + 1)}$$

$$I_3 = \left[\left(\frac{\gamma + 1}{2}\right)^2 + (\gamma + 1)\tilde{b}\right] B_r + \frac{\hat{s}_{\nu h}^2 - \hat{\omega}^2}{4} B_r - (\gamma - 1)\frac{\hat{s}_{\nu h}^2}{8} -$$

$$- \frac{3 + \gamma}{4}\hat{\omega}B_i - \frac{\hat{s}_{\nu h}^2}{8}\left[\hat{\omega}C_{1i} - (\gamma - 1)C_{1r}\right] +$$

$$+ \frac{\gamma^2 - 6\gamma - 3}{2(\gamma + 1)^2}\frac{\sigma_r}{2\kappa} + \frac{\hat{\omega}\gamma}{\gamma + 1}\frac{\sigma_i}{4\kappa}$$

$$I_4 = -\left[\left(\frac{\gamma + 1}{2}\right)^2 + (\gamma + 1)\tilde{b}\right] B_i + \frac{\hat{\omega}^2 - \hat{s}_{\nu h}^2}{4} B_i -$$

$$- \frac{3 + \gamma}{4}\hat{\omega}B_r - \frac{\gamma^2 - 1}{8}\hat{\omega} - \frac{\gamma - 1}{2}\hat{\omega}\tilde{b} -$$

$$- \frac{\hat{s}_{\nu h}^2}{8}(\gamma - 1)C_{1i} - \hat{\omega}C_{1r} - \frac{\gamma^2 - 6\gamma - 3}{(\gamma + 1)^2}\frac{\sigma_i}{4} + \frac{\hat{\omega}\gamma}{\gamma + 1}\frac{\sigma_r}{4\kappa} .$$

When solving the system (173) and (174), f_0, f_1, f_2, and f_3 are determined simultaneously. They are governed by the following system of equations which may be obtained from (77), (81), (83), and (85):

$$\left.\begin{aligned} f_{0r}'(\hat{\phi}) &= \frac{\hat{\omega}}{2}\frac{f_{0i}(\hat{\phi})}{q^2} ; \qquad & f_{0r}(0) &= 1 \\ f_{0i}'(\hat{\phi}) &= -\frac{\hat{\omega}}{2}\frac{f_{0r}(\hat{\phi})}{q^2} ; \qquad & f_{0i}(0) &= 0 \end{aligned}\right\} \tag{175}$$

$$
\left.
\begin{aligned}
f'_{1r}(\hat{\phi}) &= \frac{1}{2} f_{or} \frac{d\overline{q^2}}{d\hat{\phi}} ; & f_{1r}(0) &= 0 \\
f'_{1i}(\hat{\phi}) &= \frac{1}{2} f_{oi} \frac{d\overline{q^2}}{d\hat{\phi}} ; & f_{oi}(0) &= 0 \\
f'_{2r}(\hat{\phi}) &= \frac{\hat{\omega} f_{2i}(\hat{\phi})}{2\overline{q^2}} + \frac{f_{1r}}{2\kappa \overline{q^2}} ; & f_{2r}(0) &= 0 \\
f'_{2i}(\hat{\phi}) &= \frac{\hat{\omega} f_{2r}(\hat{\phi})}{2\overline{q^2}} + \frac{f_{1i}}{2\kappa \overline{q^2}} ; & f_{2i}(0) &= 0 \\
(\overline{c^2} f_3)'_r &= -\frac{\hat{\omega}\overline{c^2} f_{3i}}{2\overline{q^2}} + \frac{1}{2}\frac{d\overline{q^2}}{d\hat{\phi}} ; & \overline{c^2} f_{3r}(0) &= 0 \\
(\overline{c^2} f_3)'_i &= \frac{\hat{\omega}\overline{c^2} f_{3r}}{2\overline{q^2}} ; & \overline{c^2} f_{3i}(0) &= 0 .
\end{aligned}
\right\} \qquad (175)
$$

The numerical integration of Equations (175) is simpler than the numerical evaluation of the integrals appearing in their exact analytical solution. The flow properties were determined for the 30° conical nozzle described in Section 14. So (167) and (168) were solved simultaneously with Equations (175). The integrations were performed with an IBM 7094 computer in similar fashion to those previously mentioned.

Substituting for Φ and $d\Phi/d\phi = 2\kappa d\Phi/d\hat{\phi}$ in (102), we obtain, after certain manipulations,

$$
\left.
\begin{aligned}
U_{mod} &= 2\kappa (y_3^2 + y_4^2)^{\frac{1}{2}} \\
U_{arg} &= \text{arc tan } (y_4/y_3) \\
P_{mod} &= \Big[(\kappa\hat{\omega} y_2 + \sigma_r f_{1r} - \sigma_i f_{1i} - 2\kappa \overline{q^2} y_3)^2 + \\
&\qquad + (-2\kappa \overline{q^2} y_4 - \kappa\hat{\omega} y_1 + \sigma_r f_{1i} + \sigma_i f_{1r})^2\Big]^{\frac{1}{2}} \\
P_{arg} &= \text{arc tan} \frac{\sigma_r f_{1i} + \sigma_i f_{1r} - \kappa\hat{\omega} y_1 - 2\kappa \overline{q^2} y_4}{\sigma_r f_{1r} - \sigma_i f_{1i} + \kappa\hat{\omega} y_2 - 2\kappa \overline{q^2} y_3} \\
V_{mod} = W_{mod} &= \Big[(y_1 - C_{1r} f_{or} + C_{1i} f_{oi} - \sigma_r f_{2r} + \sigma_i f_{2i})^2 + \\
&\qquad + (y_2 - C_{1r} f_{oi} - C_{1i} f_{or} - \sigma_r f_{2i} - \sigma_i f_{2r})^2\Big]^{\frac{1}{2}} \\
V_{arg} = W_{arg} &= \text{arc tan} \frac{y_2 - C_{1r} f_{oi} - C_{1i} f_{or} - \sigma_r f_{2i} - \sigma_i f_{2r}}{y_1 - C_{1r} f_{or} + C_{1i} f_{oi} - \sigma_r f_{2r} + \sigma_i f_{2i}}
\end{aligned}
\right\} \qquad (176)
$$

$$
\left.\begin{aligned}
S_{mod} &= (\sigma_r^2 + \sigma_i^2)^{\frac{1}{2}} \\
S_{arg} &= \text{arc tan}\,\frac{\sigma_r f_{oi} + \sigma_i f_{or}}{\sigma_r f_{or} - \sigma_i f_{oi}} \,,
\end{aligned}\right\} \qquad (176)
$$

where, for example, $P = P_{mod} \exp(i P_{arg})$.

The results given in (176) are nondimensionalized with respect to static throat conditions and are consistent with the variable separation scheme given by (44). It was desired to have the final results nondimensionalized with respect to stagnation conditions and to be consistent with the variable separation scheme given by (160). This was readily achieved by multiplying, in (176),

$$P_{mod} \;\text{ by }\; \frac{2\gamma}{\gamma+1}\left[1 - \frac{\gamma-1}{\gamma+1}\,\bar{q}^2\right]^{1/(\gamma-1)} \,,$$

$$U_{mod} \;\text{ by }\; \left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}} \bar{q}$$

and

$$V_{mod} \;\text{ and }\; W_{mod} \;\text{ by }\; \left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}} .$$

As already specified, r in (160) (as well as in the equation for w' and ρ' given in Section 15) must be interpreted as the radius divided by the local wall radius.

In order to readily compare the results for different cases, a phase was added to the arguments of (176) such that $P_{arg}(0)$ always was equal to zero. Therefore the phase angle Δ to be added was given by

$$\Delta = -\,\text{arc tan}\,\frac{-(2B_i + \hat{\omega})}{-2B_r} \;.$$

Sample results of the flow properties calculations are given in Figures 18, 19, and 20 for the case of an oscillations with $s_{\nu h} = 1.84129$ and $\omega = 1.00$. Here the transverse variation corresponds to that of the first tangential mode. The same nozzle shape as discussed in Section 14 was considered. Both σ and C_1 are real and equal to 0.5 for this case.

In Figure 18, it is seen that there is a gradual axial variation of the pressure due to the acoustic wave phenomenon in the nozzle but there is essentially no variation due to the presence of entropy and vorticity waves. The axial component of velocity has a gradual variation due to the acoustic wave plus a more rapid undulation of smaller amplitude due to the entropy and vorticity waves, as shown in Figure 19. The transverse components of velocity have rather severe undulations, due to the entropy and vorticity waves, as indicated by the variation of the radial velocity component shown in Figure 20.

These entropy and vorticity waves travel at the particle velocity, which is subsonic, so that their wavelength is shorter than the acoustic wavelength but increases as the wave moves towards the throat. The values for σ and C_1 in the figures were taken to be considerably higher than those normally found in practice for the purpose of demonstration. The results are qualitatively similar for a wide range of values of these complex numbers. Even though the pressure does not exhibit the presence of entropy waves, the density and temperature would.

Note that the admittance coefficients are independent of the values of σ and C_1, and therefore independent of the intensity of the entropy and vorticity waves. Of course, as σ and C_1 go to zero, the general admittance relation can be shown to reduce to the irrotational admittance relation.

17. ASYMPTOTIC BEHAVIOR OF THE ADMITTANCE COEFFICIENTS

Due to the wild behavior observed in the numerical solutions of Equations (163), (164), (165), and (166) in the low Mach number range of the nozzle, it was desirable to obtain some analytical prediction of the solutions to those differential equations. This was achieved by developing solutions which were asymptotic in the sense that they apply in the limit as Mach number goes to zero. These solutions are satisfactory for the present purpose since it is in the low Mach number region (away from the throat location) that the behavior must be examined.

In the low Mach number region of the nozzle, we can say (using the continuity relation and neglecting terms of order $\overline{q}^2$) that

$$(2/(\gamma + 1))^{1/\{2(\gamma-1)\}} = r(\overline{q})^{\frac{1}{2}} ,$$

where the same nondimensional scheme as discussed in Section 14 is employed here. Noting that for the conical portion of the nozzle $dr = - dz \tan\theta = - d\hat{\phi}(\tan\theta)/2\kappa\overline{q}$, we find, after differentiation of the above relation, that

$$\left(\frac{\gamma + 1}{2}\right)^{1/\{2(\gamma-1)\}} \frac{\tan\theta_1}{2\kappa} d\hat{\phi} = \frac{1}{2\overline{q}^{\frac{1}{2}}} d\overline{q} .$$

This may readily be integrated to obtain

$$\overline{q} = (a\hat{\phi} + c)^2 , \tag{177}$$

where c is determined by specifying* $\overline{q}$ at some value of $\hat{\phi}$ and

$$a = \left(\frac{\gamma + 1}{2}\right)^{1/\{2(\gamma-1)\}} \frac{\tan\theta_1}{2\kappa} .$$

It is convenient to begin the development of the asymptotic solutions by a transformation of the independent variable which is suggested by the form (177).

---

* For the purposes here, specification of the value of c is unnecessary.

Setting $y = a\hat{\phi} + c$, it readily follows that

$$\left.\begin{aligned} \bar{q} &= y^2 \\ \bar{c} &= \frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} y^4 \\ \frac{d}{d\hat{\phi}} &= a \frac{d}{dy} . \end{aligned}\right\} \tag{178}$$

$\hat{\zeta}$ will be determined by finding the homogeneous solution to (158) and then taking the derivative of the logarithm of that solution. Using (178) to substitute into (172), we have the equation with complex coefficients

$$\Phi_h'' + (A - iC)\Phi_h' + (B - iD)\Phi_h = 0 , \tag{179}$$

where

$$A = -8y^3/(\gamma + 1) + O(y^7)$$

$$B = \frac{\hat{\omega}^2}{2(\gamma + 1)a^2} \frac{1}{y^4} + \frac{\hat{\omega}^2}{2(\gamma + 1)a^2} - \frac{\hat{s}_{\nu h}^2}{4a^2} \left(\frac{\gamma + 1}{2}\right)^{1/(\gamma - 1)} \left(\frac{1}{y^2} - \frac{\gamma}{\gamma + 1} y^2\right) + O(y^6)$$

$$C = \frac{2\hat{\omega}}{(\gamma + 1)a} + \frac{2\hat{\omega}y^4}{(\gamma + 1)a} + O(y^8)$$

$$D = \frac{4(\gamma - 1)}{a^2(\gamma + 1)^2} y^3 + O(y^7) .$$

The linear equation (179) with variable coefficients cannot be solved exactly; however, there is the well-known WKBJ method for solving a second order linear equation. The important criterion is that the solutions vary much more rapidly than the coefficients in the differential equation. This is satisfied if $\hat{\omega}/a$ is large, which implies that the axial gradients in the unsteady state are larger than those in the steady state, since a is proportional to $\tan \theta_1$.

The WKBJ method involves the determination of the two solutions in the form

$$\Phi_h = e^{\psi(y)} . \tag{180}$$

For equations with constant coefficients the derivative ψ' is a constant. In the present situation, where the solution varies much more rapidly than the coefficients, this quantity is "nearly" constant, so that ψ'' is of higher order. Therefore, in the first approximation we may neglect ψ'' so that, substituting (180) into (179), we have

$$\psi' = -\frac{A}{2} + \frac{iC}{2} \pm i \left(B + \frac{C^2}{4}\right)^{\frac{1}{2}} + O(y^5) .$$

This first approximation is differentiated to obtain

$$\psi'' = \mp \frac{2\hat{\omega}}{a\{2(\gamma+1)\}^{\frac{1}{2}}}\frac{1}{y^3} \pm i\frac{\hat{\omega}}{a\{2(\gamma+1)\}^{\frac{1}{2}}}\left[\frac{\gamma+3}{\gamma+1} - \frac{1}{4}\left(\frac{\hat{s}_{\nu h}}{\hat{\omega}}\right)^4\left(\frac{\gamma+1}{2}\right)^{2\gamma/(\gamma-1)}\right] + O(y^2) \ . \qquad (181)$$

Now, in the second approximation, the ψ'' is kept when (180) is substituted into (179), with the following result:

$$\psi' \;=\; -\frac{A}{2} + \frac{iC}{2} \pm i(B + \psi'' + C^2/4)^{\frac{1}{2}} \ , \qquad (182)$$

where ψ'' is given by (181).

For the third approximation, the result of the second approximation is differentiated and substituted into (182), which then is solved for ψ'. Of course, this iteration may be continued indefinitely. The result of the fifth approximation is conveniently expressed as follows (setting $\psi = \psi_r + i\psi_i$):

$$\left.\begin{aligned} \psi_r' &= \frac{1}{y} + K_1 y + K_3 y^3 + O(y^5) \\ \psi_i' &= \pm\left\{\frac{\hat{\omega}}{a\{2(\gamma+1)\}^{\frac{1}{2}}}\frac{1}{y^2} - \frac{1}{2}\frac{\hat{s}^2_{\nu h}}{\hat{\omega} a}\left(\frac{\gamma+1}{2}\right)^{(\gamma+1)/\{2(\gamma-1)\}} \pm \right. \\ &\qquad \left. \pm \frac{\hat{\omega}}{a(\gamma+1)} + K_2 y^2 + O(y^4) \ ,\right\} \end{aligned}\right\} \qquad (183)$$

where the following definitions apply

$$K_1 = \frac{1}{2}\left(\frac{\hat{s}_{\nu h}}{\hat{\omega}}\right)^2\left(\frac{\gamma+1}{2}\right)^{\gamma/(\gamma-1)}$$

$$\begin{aligned} K_2 = {} & \frac{1}{2}\frac{\gamma+3}{\gamma+1}\frac{\hat{\omega}}{a\{2(\gamma+1)\}^{\frac{1}{2}}} + \frac{3}{4}\frac{a\hat{s}^2_{\nu h}}{\hat{\omega}^3}\{2(\gamma+1)\}^{\frac{1}{2}}\left(\frac{\gamma+1}{2}\right)^{\gamma/(\gamma-1)} + \\ & + \frac{9}{8}(\gamma+1)\{2(\gamma+1)\}^{\frac{1}{2}}\left(\frac{a}{\hat{\omega}}\right)^3 - \frac{1}{8}\frac{\hat{s}^4_{\nu h}}{a\hat{\omega}^3\{2(\gamma+1)\}^{\frac{1}{2}}}\left(\frac{\gamma+1}{2}\right)^{2\gamma/(\gamma-1)} \end{aligned}$$

$$\begin{aligned} K_3 = {} & \frac{3}{4}\frac{a\hat{s}^2_{\nu h}\{2(\gamma+1)\}^{\frac{1}{2}}}{\hat{\omega}^3}\left(\frac{\gamma+1}{2}\right)^{\gamma/(\gamma-1)} - \frac{3}{2}\frac{\{2(\gamma+1)\}^{\frac{1}{2}}(\gamma+1)a^3}{\hat{\omega}^3} - \\ & - \frac{\gamma+3}{\gamma+1} - 11\frac{a^2\hat{s}^2_{\nu h}}{\hat{\omega}^4}\left(\frac{\gamma+1}{2}\right)^{(2\gamma-1)/(\gamma-1)} - 10(\gamma+1)^2\frac{a^2}{\hat{\omega}^4} - \\ & - \frac{1}{8}\frac{\hat{s}^4_{\nu h}}{\hat{\omega}^4}\left(\frac{\gamma+1}{2}\right)^{2\gamma/(\gamma-1)} \ . \end{aligned}$$

(183) shows that two solutions exist for ψ_i' , which means that ψ , and therefore Φ_h , also have two solutions.

Integrating (183), we find that

$$\left.\begin{aligned} \psi_r &= b + \log y + \frac{K_1 y^2}{2} + \frac{K_3 y^4}{4} + O(y^6) \\ \psi_i &= \psi_i^{(+)} - c \qquad \text{or} \qquad -\psi_i^{(-)} + d \, , \end{aligned}\right\} \tag{184}$$

where b , c , and d are constants of integration and

$$\left.\begin{aligned} \psi_i^{(+)} &= -\frac{\hat{\omega}}{a\{2(\gamma+1)\}^{\frac{1}{2}}}\frac{1}{y} - \frac{1}{2}\frac{\hat{s}_{\nu h}^2}{\hat{\omega} a}\left(\frac{\gamma+1}{2}\right)^{(\gamma+1)/\{2(\gamma-1)\}} y + \\ &\qquad + \frac{\hat{\omega} y}{a(\gamma+1)} + \frac{K_2 y^3}{3} + O(y^5) \\ \psi_i^{(-)} &= -\frac{\hat{\omega}}{a\{2(\gamma+1)\}^{\frac{1}{2}}}\frac{1}{y} - \frac{1}{2}\frac{\hat{s}_{\nu h}^2}{\hat{\omega} a}\left(\frac{\gamma+1}{2}\right)^{(\gamma+1)/\{2(\gamma-1)\}} y - \\ &\qquad - \frac{\hat{\omega} y}{a(\gamma+1)} + \frac{K_2 y^3}{3} + O(y^5) \, . \end{aligned}\right\} \tag{185}$$

It can be shown that $\psi_i^{(+)}$ and $\psi_i^{(-)}$ correspond to waves travelling in opposite directions; $\psi_i^{(+)}$ corresponds to a wave moving towards the nozzle entrance and $\psi_i^{(-)}$ corresponds to a wave moving towards the nozzle throat.

Substitution of (184) into (180) yields the following results:

$$\Phi_h = \bar{A} e^{\psi_r} \exp\{i(\psi_i^{(+)} - c)\} + \bar{B} e^{\psi_r} \exp\{-i(\psi_i^{(-)} - d)\} \, , \tag{186}$$

where $\bar{A}$, $\bar{B}$, c and d remain unknown. Since exp (b) may be included in $\bar{A}$ and $\bar{B}$ with no loss of generality we will consider $b = 0$ in (184).

Noting that

$$\hat{\zeta} = \left(\frac{1}{\Phi_h}\right)\left(\frac{d\Phi_h}{d\phi}\right) = \left(\frac{a}{\Phi_h}\right)\left(\frac{d\Phi_h}{dy}\right),$$

(186) may be used to deduce that

$$\left.\begin{aligned} \hat{\zeta}_r &= a\psi_r' - \frac{a(\psi_i^{(+)\prime} + \psi_i^{(-)\prime})(\bar{A}/\bar{B}) \sin(\psi_i^{(+)} + \psi_i^{(-)} - (c+d))}{(\bar{A}/\bar{B})^2 + 1 + 2(\bar{A}/\bar{B}) \cos(\psi_i^{(+)} + \psi_i^{(-)} - (c+d))} \\ \hat{\zeta}_i &= \frac{a(\psi_i^{(+)\prime} - \psi_i^{(-)\prime})}{2} + \frac{\dfrac{a(\psi_i^{(+)\prime} + \psi_i^{(-)\prime})}{2}\{(\bar{A}/\bar{B})^2 - 1\}}{(\bar{A}/\bar{B})^2 + 1 + 2(\bar{A}/\bar{B}) \cos(\psi_i^{(+)} + \psi_i^{(-)} - (c+d))} \, . \end{aligned}\right\} \tag{187}$$

It is seen that the unknown constants appear in two combinations, $(\bar{A}/\bar{B})$ and $(c + d)$. If $\hat{\zeta}_r$ and $\hat{\zeta}_i$ were given at some initial value of y, the constants $(\bar{A}/\bar{B})$ and $(c + d)$ could be calculated, since ψ_r, $\psi_i^{(+)}$ and $\psi_i^{(-)}$ and their derivatives are known as functions of y. Once these constants are known, (187) could be used to determine $\hat{\zeta}_r$ and $\hat{\zeta}_i$ at other values of y.

If the relations given by (177) and (178) are used to transform (166), the result is that

$$\frac{d}{dy}(\overline{c^2} f_3) = \frac{i\hat{\omega}}{2ay^4}(\overline{c^2} f_3) + 2y^3 .$$

Upon integration we have the exact solution as follows:

$$\overline{c^2} f_3 = k \exp(-i\hat{\omega}/6ay^3) + 2 \exp(-i\hat{\omega}/6ay^3) \int_0^y \overline{y^3} \exp(-i\hat{\omega}/6a\overline{y^3})\, d\bar{y} ,$$

where $\bar{y}$ is a dummy variable.

The definition is made that

$$I_n(y) = \exp(-i\hat{\omega}/6ay^3) \int_0^y \overline{y^n} \exp(-i\hat{\omega}/6a\overline{y^3})\, d\bar{y} . \qquad (188)$$

Then we have the following form for the solution:

$$\overline{c^2} f_3 = k \exp(-i\hat{\omega}/6ay^3) + 2I_3 , \qquad (189)$$

where k is a constant which is determined when f_3 is known at a certain value of y.

It remains to determine $I_n(y)$; an asymptotic series may be developed by successive integration by parts. First, the following transformations are made

$$y = \left(\frac{\hat{\omega}}{6at}\right)^{1/3} , \qquad \eta = \frac{n+4}{3} .$$

This means that (188) becomes

$$I_n = \left(\frac{\hat{\omega}}{6a}\right)^{(n+1)/3} \frac{e^{-it}}{3} \int_t^\infty e^{i\tau} \frac{d\tau}{\tau^\eta} . \qquad (188a)$$

Integration by parts generates the asymptotic series approximation

$$I_n \simeq \left(\frac{\hat{\omega}}{2a}\right)^{(n+1)/3} \left(\frac{1}{3t}\right)^\eta \left[\sum_{m=1}^{M} a_m \left(\frac{1}{t^{2m-1}}\right) + i\left(1 + \sum_{m=1}^{M} b_m \frac{1}{t^{2m}}\right)\right] . \qquad (190)$$

where

$$a_1 = \eta , \qquad a_{m+1} = -a_m(\eta + 2m)(\eta + 2m - 1) ,$$

$$b_1 = -\eta(\eta + 1) , \qquad b_{m+1} = -b_m(\eta + 2m + 1)(\eta + 2m) .$$

The integral which would remain after $2M + 1$ integrations represents the error in using the series approximation in place of the exact function I_n . This error can be shown to be bounded above by the quantity

$$E = \frac{(\eta + 2M - 1)(\eta + 2M - 2) - - - - - - - - - (\eta + 1)}{(3)^{\eta} t^{\eta + 2M}} .$$

For a given value of the argument t , therefore, an optimum value of the integer M exists which minimizes the upper bound E . This optimum situation occurs when M equals one plus the greatest integer in the quantity $(t - \eta)/2$. The smaller the Mach number is, the larger t is, and the larger the optimum value for M is. In the limit of zero Mach number, t is infinite and the optimum value of M becomes infinite.

I_n is not a rapidly undulating function of y for the values of $\hat{\omega}$ in the range of interest. In particular, I_3 is proportional to y^7 in its leading term, so that $c^2 f_3$ is essentially given by its homogeneous solution. That solution portrays the effect of properties which propagate with the gas velocity rather than the sound velocity since $\exp(-i\hat{\omega}/6ay^3)$ is an approximation to $\exp(-i\hat{\omega}\int(1/2\overline{q}^2)dx)$.

Asymptotic solutions may also be found for Equations (164) and (165). Equations (117) and (178) may be used to transform those differential equations to the following forms*:

$$a\frac{d}{dy}M^{(1)} + \left[\hat{\zeta} - i\hat{\omega}\left(\frac{1}{2y^4} + \frac{2}{(\gamma + 1)(1 - y^4)}\right)\right] M^{(1)} = F^{(1)}(y) , \tag{191}$$

where

$$M^{(1)} = (1 - y^4)\hat{\xi}^{(1)}$$

$$M^{(2)} = (1 - y^4)\xi^{(2)}$$

$$F^{(1)} = \left(\frac{\gamma + 1}{2}\right)^{(2-\gamma)/(\gamma-1)} \frac{\hat{s}^2_{\nu h}}{4y^2}\left(1 - \frac{\gamma - 1}{\gamma + 1}y^4\right)^{1/(\gamma-1)}$$

$$F^{(2)} = \frac{2}{\gamma + 1}\left[a\frac{df_3}{dy} + \frac{\hat{s}^2_{\nu h}}{2i\hat{\omega}}\left(\frac{\gamma + 1}{2} - \frac{\gamma - 1}{4}y^4\right)^{1/(\gamma-1)}\left(\frac{1 - y^4}{2y^2} + \frac{\overline{c^2}f_3}{y^2}\right)\right] .$$

Equations (189) and (190) may be employed to simplify these equations as follows:

---

* $F^{(1)}$ is not exactly the same quantity here as it is in (88).

$$F^{(1)} = \left(\frac{\gamma + 1}{2}\right)^{(2-\gamma)/(\gamma-1)} \frac{\hat{s}_{\nu h}^2}{4} \frac{1}{y^2} + O(y^2)$$

$$F^{(2)} = \frac{2i\hat{\omega}k}{(\gamma + 1)^2 y^4} \exp\,(-i\hat{\omega}/6ay^3) +$$

$$+ \left(\frac{\gamma + 1}{2}\right)^{(2-\gamma)/(\gamma-1)} \frac{\hat{s}_{\nu h}^2}{4i\hat{\omega}} \frac{1}{y^2} \left[1 + 2k \exp\,(-i\hat{\omega}/6ay^3)\right] + O(y^2) \,.$$

Equation (191) is readily integrated to obtain

$$M^{(1)} = \frac{\lambda_i \exp\left[(i\hat{\omega}/a) \int \left(\frac{1}{2y^4} + \frac{2}{(\gamma + 1)(1 - y^4)}\right) dy\right]}{\Phi_h} +$$

$$+ \frac{\exp\left[(i\hat{\omega}/a) \int \left(\frac{1}{2y^4} + \frac{2}{(\gamma + 1)(1 - y^4)}\right) dy\right]}{h} \times$$

$$\times \int_0^y F^{(1)}\Phi_h \exp\left[(-i\hat{\omega}/a) \int \left(\frac{1}{2\bar{y}^4} + \frac{2}{(\gamma + 1)(1 - \bar{y}^4)}\right) d\bar{y}\right] d\bar{y} \,, \qquad (192)$$

where λ_i is a complex constant of integration, $\bar{y}$ is a dummy variable and it has been noted that $\Phi_h = \exp\,[(1/a) \int \hat{\zeta}\, d\bar{y}]$.

The integrals appearing in (192) must now be approximated by an asymptotic solution. (185) and (186) will be used to evaluate Φ_h . One finds for the integrand

$$F^{(1)}\Phi_h \exp\left[(-i\hat{\omega}/a) \int \left(\frac{1}{2y} + \frac{2}{(\gamma + 1)(1 - y^4)}\right) dy\right]$$

$$= \left(\frac{\gamma + 1}{2}\right)^{(2-\gamma)/(\gamma-1)} \frac{\hat{s}_{\nu h}^2}{4} \left(\frac{1}{y} + \frac{K_1}{2} y + O(y^3)\right) \times$$

$$\times \left(\bar{A} e^{-ic} \exp\left[(i\hat{\omega}/a) \left(\frac{1}{6y^3} - \frac{1}{\{2(\gamma + 1)\}^{\frac{1}{2}}} \frac{1}{y}\right)\right] + \right.$$

$$\left. + \bar{B} e^{-id} \exp\left[(i\hat{\omega}/a) \left(\frac{1}{6y^3} + \frac{1}{\{2(\gamma + 1)\}^{\frac{1}{2}}} \frac{1}{y}\right)\right]\right) e^{O(iy)} \,.$$

For our purposes, it will be proper and consistent to take $\exp(O(iy))$ as sufficiently-slowly varying to be replaced by unity. We see then that integrals of a certain type sum to produce the integral in (192). This type of integral is

$$\int_0^y \bar{y}^n \exp\left[(i\hat{\omega}/a)\left(\frac{1}{6\bar{y}^3} \pm \frac{1}{\{2(\gamma+1)\}^{\frac{1}{2}}\bar{y}}\right)\right] d\bar{y} \ .$$

In particular, for our desired accuracy we are interested only in the cases where $n = 1$ and $n = -1$. Note that the $\exp(O(i/y))$ term represents wave propagation effects and varies much more slowly than the $\exp(O(i/y^3))$ term, which represents particle propagation effects. So there is a preferred way in which to integrate by parts. One finds that, to the desired accuracy,

$$\int_0^y \bar{y}^{-1} \exp\left[(i\hat{\omega}/a)\left(\frac{1}{6\bar{y}^3} \pm \frac{1}{\{2(\gamma+1)\}^{\frac{1}{2}}\bar{y}}\right)\right] d\bar{y}$$

$$= \exp\left[(i\hat{\omega}/a)\left(\frac{1}{6y^3} \pm \frac{1}{\{2(\gamma+1)\}^{\frac{1}{2}}y}\right)\right]\left(I_{-1}(y) \pm \left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}} I_1(y)\right)$$

$$\int_0^y \bar{y} \exp\left[(i\hat{\omega}/a)\left(\frac{1}{6\bar{y}^3} \pm \frac{1}{\{2(\gamma+1)\}^{\frac{1}{2}}\bar{y}}\right)\right] d\bar{y}$$

$$= \exp\left[(i\hat{\omega}/a)\left(\frac{1}{6y^3} \pm \frac{1}{\{2(\gamma+1)\}^{\frac{1}{2}}y}\right)\right] I_1(y) \ ,$$

where (190) gives $I_1(y)$ and $I_{-1}(y)$. Now (192) yields

$$M^{(1)} = (1-y^4)\hat{\xi}^{(1)} = \frac{\lambda_1 e^{-i\hat{\omega}/6ay^3}}{\left(y + \frac{K_1}{2}y^3 + \frac{2K_3 + K_1^2}{8}y^5\right)\left((\bar{A}/\bar{B})\, e^{-(c+d)}\, e^{-im/y} + e^{im/y}\right)} +$$

$$+ \left(\frac{\gamma+1}{2}\right)^{(2-\gamma)/(\gamma-1)} \cdot \frac{\hat{s}_{\nu h}^2}{4y}\left[\frac{I_{-1}}{1 + \frac{1}{2}K_1 y^2} + \frac{1}{2}K_1 I_1 + \right.$$

$$\left. + \left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}} I_1 \left(\frac{(\bar{A}/\bar{B})\, e^{-i(c+d)}\, e^{-2im/y} - 1}{(\bar{A}/\bar{B})\, e^{-i(c+d)}\, e^{-2im/y} + 1}\right)\right] \ , \qquad (193)$$

where the quantity m has been defined as

$$m = \frac{\hat{\omega}}{a\{2(\gamma+1)\}^{\frac{1}{2}}} \ .$$

In developing the asymptotic form of (192) for $\xi^{(2)}$, integrals appear of the same form as appeared in the relation for $\hat{\xi}^{(1)}$. In addition integrals of the form

$$\mathcal{G}_n(y) = \int_0^y \bar{y}^n \exp\left[\pm \frac{i\hat{\omega}}{a\{2(\gamma+1)\}^{\frac{1}{2}}} \frac{1}{\bar{y}}\right] d\bar{y}$$

appear. It can be shown that $\mathcal{G}_n \exp\,[\mp i\hat{\omega}/a\{2(\gamma+1)\}^{\frac{1}{2}}y]$ can be put into a form similar to (188a) with $t = \mp \hat{\omega}/a\{2(\gamma+1)\}^{\frac{1}{2}}y$ and $\eta = n+2$. Then it follows that

$$\mathcal{G}_n(y) = 3\left[\pm 3\left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}}\right]^{n+1} \exp\left[\pm \frac{i\hat{\omega}}{a\{2(\gamma+1)\}^{\frac{1}{2}}y}\right] \tilde{I}_{3n+2}(y) .$$

where $\tilde{I}_{3n+2}(y)$ is evaluated from (190) by using the above-mentioned relations for t and η. The "wiggly" sign over I has been used to denote the change in the argument as a function of n and y from the previous usage even though it remains as the same function of t and η. Note that $\mathcal{G}_{-3}$ is special in that it is improper; its amplitude goes to infinity as $1/y$, as y goes to zero. This really occurs only since we chose to set the lower limit of the integral at $y = 0$. If we set the lower limit at some finite number, say ϵ, the indefinite integral evaluated at the lower limit is merely a constant which may be incorporated with the constant which appears before the homogeneous solution for M_2. This is done with no loss of generality so that we may take

$$\mathcal{G}_{-3} = -\frac{2a^2(\gamma+1)}{\hat{\omega}^2}\left(1 \mp \frac{i\hat{\omega}}{a\{2(\gamma+1)\}^{\frac{1}{2}}} \frac{1}{y}\right) \exp\left[\pm \frac{i\hat{\omega}}{a\{2(\gamma+1)\}^{\frac{1}{2}}} \frac{1}{y}\right] .$$

Note that the above asymptotic solution which contains only two terms in the series is in fact an exact solution for the integral.

It now follows that

$$\begin{aligned} M^{(2)} &= (1 - y^4)\xi^{(2)} \\ &= \frac{\dfrac{2\hat{\omega}k}{(\gamma+1)^2 m} \exp(-i\hat{\omega}/6ay^3)}{\left(y^2 + \dfrac{K_1}{2}y^4 + \dfrac{K_3 + K_1^2}{8}y^6\right)} \frac{(\bar{A}/\bar{B})e^{-i(c+d)}e^{-2im/y} - 1}{(\bar{A}/\bar{B})e^{-(c+d)}e^{-2im/y} + 1} - \frac{\dfrac{2i\hat{\omega}k}{(\gamma+1)^2 m^2} \exp(-i\hat{\omega}/6ay^3)}{y + \frac{1}{2}K_1 y^3} + \\ &\quad + \frac{\lambda_2 \exp(-i\hat{\omega}/6ay^3)}{(y + \frac{1}{2}K_1 y^3)((\bar{A}/\bar{B})e^{-i(c+d)}e^{-im/y} + e^{im/y})} + \\ &\quad + \frac{3i\left[\dfrac{\hat{\omega}kK_1}{(\gamma+1)^2} - \left(\dfrac{\gamma+1}{2}\right)^{(2-\gamma)/(\gamma-1)} \dfrac{\hat{s}_{\nu b}^2 k}{2\hat{\omega}}\right] \exp(-i\hat{\omega}/6ay^3)}{y + \frac{1}{2}K_1 y^3} \times \end{aligned} \tag{194}$$

$$\times \frac{(\bar{A}/\bar{B})\, e^{-i(c+d)}\, e^{-2im/y}\, I_{-1}^{(-)} + I_{-1}^{(+)}}{(\bar{A}/\bar{B})\, e^{i(c+d)}\, e^{-2im/y} + 1} + \frac{54\, i \exp(-i\hat{\omega}/6ay^3)}{(\gamma+1)y} \times$$

$$\times \left[\frac{\hat{\omega}k}{(\gamma+1)^2}\, \frac{2K_3 + K_1^2}{4} - \left(\frac{\gamma+1}{2}\right)^{(2-\gamma)/(\gamma-1)} \frac{\hat{s}_{\nu h}^2\, \kappa K_1}{4\hat{\omega}}\right] \times$$

$$\times \frac{(\bar{A}/\bar{B})\, e^{-i(c+d)}\, e^{-2im/y}\, I_5^{(-)} + I_5^{(+)}}{(\bar{A}/\bar{B})\, e^{-(c+d)}\, e^{-2im/y} + 1} - i\left(\frac{\gamma+1}{2}\right)^{(2-\gamma)/(\gamma-1)} \frac{\hat{s}_{\nu h}^2}{4\hat{\omega}y} \times$$

$$\times \left(\frac{I_{-1}}{1 + \frac{1}{2}K_1\, y^2} + \tfrac{1}{2}K_1\, I_1\right) . \tag{194}$$

The plus or minus in the superscript of the I function indicates which sign is taken for t in (190).

The solutions given by the asymptotic theory provide a great amount of insight into the oscillatory flow in nozzles. (186) shows that $\hat{\Phi}$ is the sum of two solutions, one corresponding to upstream wave propagation and the other to downstream wave propagation. The equations following (185) show that ψ_1 goes as $1/y$ to leading order. Noting that $d\hat{\phi} = 2\kappa\, d\phi$, $dy = a d\hat{\phi}$, and $d\phi/dz = \bar{q} = y^2$ we find $1/y$ is linear in z , the axial coordinate, so that the solutions are undulatory with constant wavelength but their amplitudes vary*. The radical behavior in $\hat{\zeta}$ is now easy to explain. It is seen that $\hat{\Phi}$ behaves as a nearly-sinusoidal function, so that $\hat{\zeta} = (1/\hat{\Phi})(d\hat{\Phi}/d\hat{\phi})$ behaves nearly as a cotangent which becomes quite large in certain ranges. The appearance of the cosine function in the denominators of (187) shows that periodically $\hat{\zeta}$ may become large.

It is especially interesting that to leading order $\hat{\zeta}$ does not depend upon $\hat{s}_{\nu h}$ but only upon $\hat{\omega}$. The wavelength can be shown to be approximately that of a planar wave generated by a source of frequency $\hat{\omega}$. In a cylindrical chamber, one would have found a larger wavelength since $(\hat{\omega}^2 - \hat{s}_{\nu h}^2)^{\frac{1}{2}}$ is the effective factor rather than $\hat{\omega}$.

Equations (193) and (194) show that $\hat{\xi}^{(1)}$ and $\xi^{(2)}$ are affected not only by acoustic oscillations but by oscillations of entropy and vorticity as well. The acoustic behavior appears through the $\exp(\pm im/y)$ terms, while the entropy and vorticity behavior appears through the $\exp(-i\hat{\omega}/6ay^3)$ terms. Both $\hat{\xi}^{(1)}$ and $\xi^{(2)}$ have sinusoidal and cosinusoidal terms appearing in denominators of their expressions (193) and (194) so they become large periodically for the same reason that $\hat{\zeta}$ becomes large.

18. ASYMPTOTIC DEVELOPMENT OF THE FLOW PROPERTIES

In addition to the development of the asymptotic theory for the admittance coefficients, it is convenient to develop an asymptotic theory to determine the pressure,

* The variation is, of course, due to convergence of the nozzle.

density, and velocity undulations. This theory describes the effects of the acoustic waves and the entropy and vorticity waves upon the oscillatory flow properties, at least for the low Mach number region of the nozzle. Of course, it provides information that could be found qualitatively in the results of Section 16 only by means of hindsight. Quantitatively it gives the orders of magnitude of various interesting effects.

The asymptotic solution of (172) for low Mach number is required. The relations (177) and (178) are employed again and the coefficients* of (172) may be represented by their asymptotes as follows:

$$\left.\begin{aligned} b &= \frac{\gamma + 1}{2.} y^4(1 - y^4) \\ g &= 4ay^7 + O(y^{11}) \\ h &= \hat{\omega}y^4 \\ j &= \frac{\hat{\omega}^2}{4} \frac{\hat{s}_{\nu h}^2}{4} \left(\frac{\gamma + 1}{2}\right)^{\gamma/(\gamma-1)} \left(y^2 - \frac{\gamma}{\gamma + 1} y^6\right) + O(y^{10}) \\ k &= 2\left(\frac{\gamma - 1}{\gamma + 1}\right) y^7 + O(y^{11}) \;. \end{aligned}\right\} \tag{195}$$

The transformations discussed in Section 14 are used in conjunction with (77), (177) and (178) to yield

$$f_0 = \lambda_3 \exp (i\hat{\omega}/6ay^3)$$

where

$$\lambda_3 = \exp (-i\hat{\omega}/6ay^3) \exp\left(-\frac{i\hat{\omega}}{2}\int_0^{\hat{\phi}_1} \frac{d\hat{\phi}}{q^2}\right) . \tag{196}$$

y_1 and $\hat{\phi}_1$ correspond to the same axial position somewhere in the low Mach number region of the nozzle. Now, using the definitions following (172), together with (177), (178), and (196), we find that the inhomogeneous part of (172) has the following asymptote:

$$\begin{aligned} G + H = {} & C_1\lambda_3 \frac{\hat{s}_{\nu h}^2}{4}\left(\frac{\gamma + 1}{2}\right)^{\gamma/(\gamma-1)} \left(y^2 - \frac{\gamma}{\gamma + 1} y^6\right) e^{i\hat{\omega}/6ay^3} + \\ & + \frac{\sigma}{2\kappa} \lambda_3 \frac{\hat{s}_{\nu h}^2}{4i\hat{\omega}}\left(\frac{\gamma + 1}{2}\right)^{\gamma/(\gamma-1)} \left(y^2 - \frac{2\gamma + 1}{\gamma + 1} y^6\right) e^{i\hat{\omega}/6ay^3} + \\ & + \frac{\sigma}{2\kappa} ik\lambda_3 \left[\frac{\hat{\omega}}{2} - \frac{\hat{s}_{\nu h}^2}{2\hat{\omega}}\left(\frac{\gamma + 1}{2}\right)^{\gamma/(\gamma-1)} \left(y^2 - \frac{\gamma}{\gamma + 1} y^6\right)\right] + \\ & + O(y^7) + O(y^{10} e^{i\hat{\omega}/6ay^3}) \;. \end{aligned} \tag{197}$$

* Actually, these coefficients are defined immediately after (165).

Inspecting the coefficients (195) and the inhomogeneous part (197), we expect to find a particular solution* Φ_p of the form

$$\Phi_p = e^{i\hat{\omega}/6ay^3} \sum_{n=-\infty}^{\infty} A_n y^n \sum_{n=-\infty}^{\infty} B_n y^n . \tag{198}$$

Substitution of (195), (197), and (198) into (172) allows the determination of the coefficients A_n and B_n. The particular solution is found to be

$$\begin{aligned}\Phi_p = & -\frac{\lambda_3 i}{\kappa\hat{\omega}}\left(i\hat{\omega}\kappa C_1 + \frac{\sigma}{2}\right)\left(\frac{\hat{s}_{\nu h}}{\hat{\omega}}\right)^2\left(\frac{\gamma+1}{2}\right)^{1/(\gamma-1)} e^{i\hat{\omega}/6ay^3}\left(y^6 - \frac{20\,i\,a}{\hat{\omega}}y^9\right) + \\ & + \lambda_3\frac{i(\sigma\kappa)}{\kappa\hat{\omega}}\left[1 - \frac{2a^2\hat{s}_{\nu h}^2}{\hat{\omega}^2}\left(\frac{\gamma+1}{2}\right)^{(2\gamma-1)/(\gamma-1)}\left(1-\frac{4}{\hat{\omega}^2}\right)y^4 - \right. \\ & - \hat{s}_{\nu h}^2\left(1-\frac{4}{\hat{\omega}^2}\right)\left(\frac{\gamma+1}{2}\right)^{1/(\gamma-1)}\left\{\frac{\gamma}{8} + \frac{(\gamma+1)^2a^2}{2\hat{\omega}^2}\left[6a^2(\gamma+1) - \right.\right. \\ & \left.\left.\left. - \left(\frac{\hat{s}_{\nu h}}{\hat{\omega}}\right)^2\left(\frac{\gamma+1}{2}\right)^{\gamma/(\gamma-1)}\right]\right\}y^6\right] + O(y^7) + O(y^{10}\,e^{i\hat{\omega}/6ay^3}) .\end{aligned} \tag{199}$$

The solution is given to higher order in the undulating terms than the others, since their gradients are large and consistent accuracy in the gradients is required. The above solution when added to $C_2\Phi_h$, where Φ_h is given by (186), represents the asymptotic solution of (172), so we write

$$\Phi = C_2\Phi_h + \Phi_p .$$

Use of (81), (83), (177), (178), (196), and the transformations discussed in Section 14 yields

$$\left.\begin{aligned} f_1 &= \lambda_4 + 2\lambda_3\, e^{i\hat{\omega}/6ay^3}\, I_n(y) = \lambda_4 + O(y^7\, e^{i\hat{\omega}/6ay^3}) \\ f_2 &= \lambda_5\, e^{i\hat{\omega}/6ay^3} - \frac{3ik}{\hat{\omega}\kappa}\lambda_3 , \end{aligned}\right\} \tag{200}$$

where

$$\lambda_4 \equiv \frac{1}{2}\int_0^\infty f_0 \frac{d\overline{q^2}}{d\hat{\phi}}\, d\hat{\phi}$$

$$\lambda_5 \equiv \frac{\lambda_3}{2\kappa}\left[\int_0^{\hat{\phi}_1} \frac{\overline{c^2} f_3}{\overline{q^2}}\, d\hat{\phi} + \frac{6ik}{\hat{\omega}}\, e^{-i\hat{\omega}/6ay_1^3}\right] .$$

* Clearly, this particular solution must essentially equal the particular solution $C_1\Phi^{(1)} + \sigma\Phi^{(2)}$ of (88); the only difference is in the nondimensionalization scheme, as explained at the beginning of Section 14.

The relations (102) give the formulas for the axial dependence of the flow properties (of course, now the nondimensionalization scheme differs). Using the transformations given in Sections 14 and 17, we replace ω by $\kappa\hat{\omega}$ and $d\Phi/d\phi$ by $2a\kappa d\Phi/dy$ in those relations. Also, $C_1\Phi^{(1)} + \sigma\Phi^{(2)}$ is replaced by Φ_ν as given in (199) and Φ_h, f_0, f_1 and f_2 are given by (186), (196), and (200), respectively. Under this procedure, the relation (102) for the axial component of the velocity perturbation becomes, taking note of the form of (44), as follows:

$$
\begin{aligned}
\bar{q}U = {} & 2a\kappa(\bar{B}e^{id}C_2)iy\left[\frac{\bar{A}}{\bar{B}}e^{-i(c+d)}\,e^{i\psi_1^{(+)}} - e^{-i\psi_1^{(-)}}\right] \times \\
& \times\left[\frac{\hat{\omega}}{a\{2(\gamma+1)\}^{\frac{1}{2}}} + y^2\left\{\frac{K_1\hat{\omega}}{2a\{2(\gamma+1)\}^{\frac{1}{2}}} - \frac{1}{2}\frac{\hat{s}_{\nu h}^2}{\hat{\omega}a}\left(\frac{\gamma+1}{2}\right)^{(\gamma+1)/\{2(\gamma-1)\}}\right\} + \right. \\
& \left. + y^4\left\{\frac{\hat{\omega}}{a\{2(\gamma+1)\}^{\frac{1}{2}}}\,\frac{2K_3+K_1^2}{8} + K_2 - \frac{K_1}{4}\frac{\hat{s}_{\nu h}^2}{\hat{\omega}a}\left(\frac{\gamma+1}{2}\right)^{(\gamma+1)/\{2(\gamma-1)\}}\right\}\right] + \\
& + 2a\kappa(\bar{B}e^{id}C_2)y^2\left[\frac{\bar{A}}{\bar{B}}e^{-i(c+d)}\,e^{i\psi_1^{(+)}} + e^{-i\psi_1^{(-)}}\right] \times \\
& \times\left[1 + \frac{\hat{\omega}}{a(\gamma+1)}y + \frac{3}{2}K_1y^2 + \frac{K_1}{2a(\gamma+1)}y^3 + \frac{10K_3+5K_1^2}{8}y^4\right] - \\
& - \left(i\hat{\omega}\kappa C_1 + \frac{\sigma}{2}\right)\frac{\hat{s}_{\nu h}^2}{\hat{\omega}^2}\left(\frac{\gamma+1}{2}\right)^{1/(\gamma-1)} y^4\,e^{i\hat{\omega}/6ay^3} - \\
& - 16(\sigma k)i\,\frac{a^3\hat{s}_{\nu h}^2}{\hat{\omega}^3}\left(\frac{\gamma+1}{2}\right)^{(2\gamma-1)/(\gamma-1)}\left(1-\frac{4}{\hat{\omega}^2}\right)y^5 + O(y^7)\,.
\end{aligned}
\tag{201}
$$

Realizing that y is of the order of the square root of M (the Mach number along the nozzle) we see that the axial component of the perturbation velocity goes to zero as $(M)^{\frac{1}{2}}$. This component of velocity is primarily of an acoustic nature with vorticity and entropy effects of the order of σM^2 and C_1M^2. As discussed in Reference 12, whenever the entropy and vorticity waves are generated by the combustion process they are typically of $O(M)$. In that case, σ and C_1 would therefore by of $O(M)$, causing the effect of vorticity and entropy waves upon the axial velocity component to be of $O(M^3)$.

The relations (102) for the transverse components become

$$
\begin{aligned}
V = W = &- \left[\frac{i}{\kappa} \left(i\hat{\omega}\kappa C_1 + \frac{\sigma}{2} \right) \frac{\hat{s}_{\nu h}^2}{\hat{\omega}^3} \left(\frac{\gamma + 1}{2} \right)^{1/(\gamma-1)} - (\sigma\lambda_3 + C_1\lambda_5) \right] e^{i\hat{\omega}/6ay^3} - \\
&- \frac{2i(\sigma k)}{\kappa\hat{\omega}} \left[1 - \frac{a^2\hat{s}_{\nu h}^2}{2\hat{\omega}^2} \left(\frac{\gamma + 1}{2} \right)^{(2\gamma-1)/(\gamma-1)} \left(1 - \frac{4}{\hat{\omega}^2} \right) y^4 - \frac{\hat{s}_{\nu h}^2}{8} \left(1 - \frac{4}{\hat{\omega}^2} \right) \left(\frac{\gamma+1}{2} \right)^{1/(\gamma-1)} \times \right. \\
&\quad \left. \times \left\{ \frac{\gamma}{4} + \frac{(\gamma + 1)^2 a^2}{\hat{\omega}^2} \left[6a^2(\gamma + 1) - \frac{\hat{s}_{\nu h}^2}{\hat{\omega}^2} \left(\frac{\gamma + 1}{2} \right)^{\gamma/(\gamma-1)} \right] \right\} y^6 \right] + \\
&+ (\bar{B} e^{id} C_2) y \left(\frac{\bar{A}}{\bar{B}} e^{-i(c+d)} e^{i\psi_i^{(+)}} e^{i\psi_i^{(-)}} \right) \times \\
&\times \left(1 + \frac{K_1}{2} + \frac{2K_3 + K_1^2}{8} y^4 \right) + O(y^7) \quad . \qquad (202)
\end{aligned}
$$

(44) shows that the radial velocity perturbation v' behaves as $\bar{\rho}\bar{q}rV$ and the tangential velocity perturbation w' behaves as $(1/r)W = (1/r)V$. From the steady-state continuity equation, we have that $\bar{\rho}\bar{q}r_w^2$ is a constant along the nozzle. So, combining r_w/r with the radial dependence, we obtain that both v' and w' have axial dependencies which behave as V/r_w or, therefore, as $VM^{\frac{1}{2}}$ at low Mach numbers. So, each of the two transverse velocity components are primarily affected by the entropy and vorticity waves rather than the acoustic waves. The rotational effects have both an undulatory and a non-undulatory portion. These effects appear to be $O(M^{\frac{1}{2}}\sigma)$ while the acoustic effect is $O(M)$.

The thermodynamic properties now remain to be determined. Following the same procedure as above, (102) gives the following relation for the axial dependence of the pressure perturbation:

$$
\begin{aligned}
\gamma\bar{\rho}P = &- \gamma \left(\frac{\gamma + 1}{2} \right)^{1/(\gamma-1)} (C_2 \bar{B} e^{id}) i\hat{\omega}\kappa y \left(\frac{\bar{A}}{\bar{B}} e^{-i(c+d)} e^{i\psi_i^{(+)}} + e^{i\psi_i^{(-)}} \right) \times \\
&\times \left[1 + \frac{K_1}{2} y^2 - \frac{2ai}{\hat{\omega}} y^3 \left(\frac{2K_3 + K_1^2}{8} - \frac{2i}{\gamma + 1} \right) y^4 - \frac{3iaK_1}{\hat{\omega}} y^5 \right] - \\
&- 2a\kappa\gamma \left(\frac{\gamma + 1}{2} \right)^{1/(\gamma-1)} (C_2 \bar{B} e^{id}) y^3 \left[\frac{\bar{A}}{\bar{B}} e^{-(c+d)} e^{i\psi_i^{(+)}} - e^{-i\psi_i^{(-)}} \right] \times \\
&\times \left[\frac{\hat{\omega}}{a\{2(\gamma + 1)\}^{\frac{1}{2}}} + \left\{ \frac{\hat{\omega}K_1}{2a\{2(\gamma + 1)\}^{\frac{1}{2}}} - \frac{1}{2} \frac{\hat{s}_{\nu h}^2}{\hat{\omega} a} \left(\frac{\gamma + 1}{2} \right)^{(\gamma+1)/\{2(\gamma-1)\}} \right\} y^2 \right] + \qquad (203)
\end{aligned}
$$

$$+\gamma\left(\frac{\gamma+1}{2}\right)^{1/(\gamma-1)}(\sigma k)\left\{\frac{\lambda_4}{k}+1-\frac{2a^2\hat{s}_{\nu h}^2}{\hat{\omega}^2}\left(\frac{\gamma+1}{2}\right)^{(2\gamma-1)/(\gamma-1)}\left(1-\frac{4}{\hat{\omega}^2}\right)y^4+\right.$$

$$+\frac{\hat{s}_{\nu h}^2}{4}\left(1-\frac{4}{\hat{\omega}^2}\right)\left(\frac{\gamma+1}{2}\right)^{1/(\gamma-1)}\left[\frac{\gamma}{4}+\frac{(\gamma+1)^2a^2}{\hat{\omega}^2}\times\right.$$

$$\left.\left.\times\left\{6a^2(\gamma+1)-\frac{\hat{s}_{\nu h}^2}{\hat{\omega}^2}\left(\frac{\gamma+1}{2}\right)^{\gamma/(\gamma-1)}\right\}\right]y^6\right\}+O(y^7)\quad. \tag{203}$$

The pressure perturbation has an acoustic behavior which is of $O(M^{\frac{1}{2}})$. To $O(M^3)$ there is no undulatory effect due to the entropy and vorticity waves. There is, however, a non-undulatory effect of $O(\sigma)$ due to the entropy waves.

Finally, we find that the entropy and density are given as

$$\left.\begin{aligned} S &= \sigma f_0 = \sigma\lambda_3\, e^{i\hat{\omega}/6ay^3} \\ \bar{\rho}R &= -\sigma\lambda_3\left(\frac{\gamma+1}{2}\right)^{1/(\gamma-1)}\left(1-\frac{1}{\gamma+1}y^4\right)e^{i\hat{\omega}/6ay^3}+\frac{2(\gamma\bar{\rho}P)}{\gamma(\gamma+1)}\left(1+\frac{\gamma-1}{\gamma+1}y^4\right). \end{aligned}\right\} \tag{204}$$

It is seen that the entropy wave has a constant amplitude and the density has an undulating effect (due to entropy) which is of $O(\sigma)$.

The results of this section agree qualitatively with the results of Section 16. Figures 18, 19, and 20 show that pressure has the slightest effect due to entropy and vorticity, axial velocity has a small effect, and transverse velocities have a large effect.

It should be noted that there is a slight inconsistency throughout the last two sections. The steady-state density was considered as a constant of O(1) but, actually, compressibility causes variations of $O(y^4)$. This means that any terms in the previous results which are of $O(y^4)$ (or $O(M^2)$) higher than the leading terms cannot be considered as quantitatively accurate. The reason, however, that the analyses were continued to higher orders, was to establish the orders of magnitude of the acoustic, entropy, and vorticity effects. While the steady-state compressibility would modify the solution to higher orders, it would not change the orders of magnitudes of these effects. In principle, the steady-state density variation could have been considered but the simplicity of (177) would have been destroyed.

The solutions contain several groups of parameters: $(C_2\bar{B}e^{id})$, $(\bar{A}/\bar{B}\,e^{-(c+d)})$, $(i\hat{\omega}\kappa C_1+\sigma/2)$, (σk) , $(\sigma\lambda_3)$, $(\sigma\lambda_5+C_1\lambda_3)$, and (λ_4/k) . The first two parameters plus σ and C_1 are given by initial conditions on the problem. Then the other five parameters may be calculated. Actually, one of the parameters should be considered as unity and the other parameters should be referenced to it since the problem is linear.

Instead of calculating the asymptotes for $\hat{\xi}^{(1)}$ and $\xi^{(2)}$ as described in Section 17, one could take the results for the particular solutions of (172) and substitute them into the defining relations following (162).

19. RESULTS OF THE NOZZLE ADMITTANCE CALCULATIONS AND THEIR APPLICATIONS

The results of the nozzle admittance calculations are presented in Tables III through LXXX. The values of the real and imaginary parts of the coefficients $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, α, and $\mathcal{E}$ are given for various values of the nozzle entrance Mach number M, the angular frequency ω, and the eigenvalue $s_{\nu h}$. As previously noted, the frequency ω must be multiplied by the square root of the contraction ratio in order to obtain ω_c (the frequency nondimensionalized by the chamber radius). The coefficients were calculated at constant intervals of the variable $\hat{\phi}$ which do not correspond to constant intervals of the Mach number.

Of course, if one were interested in values of $s_{\nu h}$, ω, and/or M other than those presented it would be necessary to interpolate or extrapolate. This can readily be done, as will be shown in some of the examples given later in this section.

The most important admittance coefficients in the combustion instability application are α and $\mathcal{E}$. The coefficient α depends solely upon the function $\hat{\zeta}$ and therefore does not contain the effects of vorticity and entropy which are contained in $\hat{\xi}^{(1)}$, $\xi^{(2)}$, and f_3. $\mathcal{E}$ is a combination of $\mathcal{A}$ and $\mathcal{B}$ and therefore contains neither $\xi^{(2)}$ nor $\hat{f}_3$. Furthermore, $\mathcal{A}$ and $\mathcal{B}$ combine in such a way so as to suppress the effect of $\hat{\xi}^{(1)}$ at low Mach numbers. So $\mathcal{E}$ contains only slight effects of the vorticity and no effects of the entropy waves. For these reasons α and $\mathcal{E}$ do not behave as widly as the coefficients $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ and interpolations are more accurate for α and $\mathcal{E}$ than for $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$.

Certain relationships have been developed which provide necessary and sufficient conditions for the neutral stability of small perturbations in a rocket combustion chamber*. The derivations of these relationships may be found in References 1, 3 and 12. A typical relationship for purely transverse oscillations which is found in Reference 12 is as follows†:

$$\frac{1}{\bar{q}_e}\int_0^L Q_{\nu h}\,dz = \frac{\gamma+1}{\gamma} - \frac{\mathcal{E}}{\gamma\bar{q}_e} + \frac{2i\omega_1 L}{\gamma\bar{q}_e}, \tag{205}$$

where $Q_{\nu h}$ is the axial dependence of the energy release per unit volume, L is the combustion chamber length, $\bar{q}_e$ is the gas velocity to stagnation speed of sound ratio at the nozzle entrance, and ω_1 is the difference between the angular frequency ω_c and the eigenvalue $s_{\nu h}$. A similar relationship for longitudinal oscillations which employs α instead of $\mathcal{E}$ is also developed in Reference 12.

* Presently, the science of combustion instability has practical use only for the admittance coefficients α and $\mathcal{E}$; $\mathcal{A}$ and $\mathcal{B}$ have been used only in the combined form given as $\mathcal{E}$ and $\mathcal{C}$ have been neglected. Here a more general viewpoint was adopted, so that all of the admittance coefficients have been calculated at little extra expense.

† Actually, in the reference, another term due to droplet drag has been included. However, it is not important for the purpose of the example.

In (205) the term on the left-hand side represents the integral effect over the chamber of the perturbation of energy release divided by the pressure perturbation. In general, it will be a complex number and have a destabilizing effect. On the right-hand side, $(\gamma + 1)/\gamma$ has a stabilizing effect and is due to various terms in the differential equations of the gas motion. The second term on the right-hand side represents the effect of the nozzle upon the stability and upon the frequency. Whenever the real part of $\mathcal{E}$ is positive it has a destabilizing effect, while whenever the real part of $\mathcal{E}$ is negative it has a stabilizing effect. The last term represents the change in frequency from the acoustic frequency. It has no effect upon the stability since it is imaginary.

Now, suppose for example that one wishes to determine the stability characteristics of a combustion chamber with a nozzle of the shape employed in the calculations (30° half-angle in the conical section and wall radius of curvature of the throat equal to the throat diameter). The nozzle entrance Mach number in this example is 0.113 and then the contraction ratio is 5.28. The first tangential mode is examined so that $s_{\nu h} = 1.84$ and, taking the frequency as $\omega_c = 1.72$, one has $\omega = \omega_c/(5.28)^{\frac{1}{2}} = 0.750$.

In order to find the values of $\mathcal{E}_r$ and $\mathcal{E}_i$, interpolations of the results presented in the tables must be made. The following summarizes the information extracted from the tables of results for the purpose of this example:

ω	$s_{\nu h}$	M	$\mathcal{E}_r$	$\mathcal{E}_i$
0.5	1	0.099	0.074	0.114
1.0	1	0.099	-1.080	-1.538
0.5	2	0.099	0.223	1.132
1.0	2	0.099	0.097	0.063
0.5	1	0.124	0.100	0.216
1.0	1	0.124	-0.383	-1.008
0.5	2	0.124	0.390	1.319
1.0	2	0.124	0.130	0.230

By a linear interpolation over the frequency parameter we obtain, for $\omega = 0.75$,

$s_{\nu h}$	M	$\mathcal{E}_r$	$\mathcal{E}_i$
1	0.099	-0.503	-0.712
2	0.099	0.160	0.598
1	0.124	-0.142	-0.396
2	0.124	0.260	0.775

By another linear interpolation over the eigenvalue parameter, we obtain the following, for $\omega = 0.75$ and $s_{\nu h} = 1.84$,

M	$\mathcal{E}_r$	$\mathcal{E}_i$
0.099	0.054	0.388
0.124	0.195	0.588

Now, finally interpolating over the Mach number, we have for $\omega = 0.75$, $s_{\nu h} = 1.84$, and $M = 0.113$

$$\mathcal{E}_r = 0.133 ; \qquad \mathcal{E}_i = 0.500 .$$

For the purpose of comparison, the equations discussed in Section 14 were integrated and the admittance coefficients were calculated for the values $\omega = 0.75$ and $s_{\nu h} = 1.84$. With $M = 0.113$ that exact solution was

$$\mathcal{E}_r = 0.136 ; \qquad \mathcal{E}_i = 0.481 .$$

This compares favorably with the results obtained by linear interpolations; there is 2% error in $\mathcal{E}_r$ and 4% error in $\mathcal{E}_i$.

There are many cases where significant errors are introduced by the linear interpolation. Often the situation may be improved by employing a more sophisticated interpolation scheme. For example, if the value of the admittance coefficient at three values of a parameter were known, a parabola could be fitted to determine the admittance coefficient at neighboring values of the parameters. If f represents the particular admittance coefficient and x represents a particular parameter, the following formula results from such a parabolic fit to the points x_0 , x_1 , and x_2

$$\left.\begin{aligned} f(x) = {} & \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} f(x_0) \\ & + \frac{(x - x_2)(x - x_0)}{(x_1 - x_2)(x_1 - x_0)} f(x_1) \\ & + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} f(x_2) . \end{aligned}\right\} \qquad (206)$$

The use of (206) shall be demonstrated in an example where interpolations are performed only over the $s_{\nu h}$ parameter. Of course, it may be used successively to interpolate over as many parameters as desired. Consider $\omega_c = 2.36$, $s_{\nu h} = 1.84$, and $M = 0.250$. The contraction ratio is 2.45 so that $\omega = 1.50$. The following summarizes the pertinent results from the tables:

$s_{\nu h}$	$\mathcal{E}_r$	$\mathcal{E}_i$
1	-0.601	-0.625
2	0.116	-0.060
3	0.406	0.689

Using the results for $s_{\nu h} = 1$ and $s_{\nu h} = 2$, a linear interpolation gives, for $s_{\nu h} = 1.84$,

$$\mathcal{E}_r = 0.001 ; \qquad \mathcal{E}_i = -0.154 .$$

On the other hand, (206) may be used with x representing $s_{\nu h}$ and f representing $\mathcal{E}_r$ at first and secondly $\mathcal{E}_i$. Then $x_0 = 1$, $x_1 = 2$, and $x_2 = 3$. The result is that, for $s_{\nu h} = 1.84$,

$$\mathcal{E}_r = 0.030 \; ; \qquad \mathcal{E}_i = -0.163 \; .$$

An exact solution for $\omega = 1.50$, $s_{\nu h} = 1.84$, and $M = 0.250$ yields

$$\mathcal{E}_r = 0.036 \; ; \qquad \mathcal{E}_i = -0.198 \; .$$

So it is seen that the more sophisticated interpolation scheme is more accurate in this case, although about 20% error still exists*.

In both cases considered above $\mathcal{E}_r$ is positive so that the nozzle has a destabilizing effect. For low M, $\bar{q} \simeq M$, so that in the first case $\mathcal{E}_r/\gamma\bar{q}_e = 1.00$ while in the second case it equals 0.120. For $\gamma = 1.2$, $(\gamma + 1)/\gamma = 1.83$ in (205), so that the destabilizing effect is significant by comparison in the first case although small in the second case.

Suppose now that we did not have a 30° but instead a 15° nozzle. The scaling factor β introduced in Section 12 is chosen as $(\tan 15°)/(\tan 30°) = 0.465$. In general the scaled nozzle will not be identical to the actual nozzle in the throat portion, although it has been made identical in the conical portion. So, while there is some error here in replacing the actual nozzle by the scaled nozzle, this error is expected to be small whenever the conical portion of the nozzle is considerably larger than the throat portion.

In the present example consider the throat wall radius of curvature $R = 2$, $\omega_c = 2.97$, $s_{\nu h} = 1.0$, and the entrance Mach number $M = 0.152$. Then the contraction ratio is 3.94 and $\omega = 1.50$. ω/β equals 3.22 and $s_{\nu h}/\beta = 2.15$. Since

$$\mathcal{A}(M, \omega, s_{\nu h}) = \mathcal{A}_{ref}(M, \omega/\beta, s_{\nu h}/\beta)$$

and

$$\mathcal{B}(M, \omega, s_{\nu h}) = \beta \mathcal{B}_{ref}(M, \omega/\beta, s_{\nu h}/\beta) \; ,$$

it follows that

$$\mathcal{E}(M, \omega, s_{\nu h}) = \mathcal{E}_{ref}(M, \omega/\beta, s_{\nu h}/\beta) \; .$$

It remains now to determine $\mathcal{E}_{ref}$ for $M = 0.152$, $\omega = 3.22$, and $s_{\nu h} = 2.15$. From the tables of results, one obtains the following information for $M = 0.152$:

ω	$s_{\nu h}$	$\mathcal{E}_r$	$\mathcal{E}_i$
3.0	2	-0.894	-0.134
3.5	2	-0.982	-0.145
3.0	3	-0.669	-0.096
3.5	3	-0.883	-0.193

* The errors are considerably smaller when the more proper comparison is made with the differences between the values of $\mathcal{E}_r$ (and $\mathcal{E}_i$) at the points $s_{\nu h} = 1, 2,$ and 3.

By means of successive linear interpolations over the frequency and eigenvalue parameters, it is found that, for $M = 0.152$, $\omega = 3.22$, and $s_{\nu h} = 2.15$,

$$\mathcal{E}_r = -0.908 \; ; \qquad \mathcal{E}_i = -0.163 \; .$$

An exact calculation for these values of ω and $s_{\nu h}$ with $\theta_1 = 15^o$ and $R = 2$ yields

$$\mathcal{E}_r = -1.050 \; ; \qquad \mathcal{E}_i = -0.129 \; ,$$

which favorably agrees with the approximate results obtained by using the scaled nozzle. The difference results because the throat wall radius of curvature R for the scaled nozzle is greater than that for the reference nozzle by a factor $(1/\beta)^2 = 4.62$, while the actual and reference nozzles have the same value of R. Of course, if the actual nozzle had an identical throat contour to the scaled nozzle there would be no difference. It also follows that, if the elliptical arc which is the generatrix of the throat wall of the scaled nozzle (obtained by scaling the circular arc of the reference nozzle) has identical curvature at the throat to the curvature of the actual nozzle, the difference in the results would be quite small.

In certain cases, the asymptotic analysis may provide a better procedure for the determination of the admittance coefficients than interpolation. This is especially so in the regions of wild behavior as indicated in Figure 21. There the irrotational admittance coefficient α is plotted versus Mach number along the nozzle. It is seen that the asymptotic solution compares favorably with the exact computer solution. Whereas interpolation with the exact results over Mach number increments of 0.025 would obviously produce serious errors in the region of wild behavior, the asymptotic solution more accurately approximates the exact solution.

As noted in Section 17, the constants of integration in the asymptotic solution must be determined by matching the exact solution to the asymptotic solution at some point in the low Mach number range. In Figure 21, the matching point is $M = 0.099$. The comparison between the two solutions is most favorable when the matching point is in or near the region of wild behavior. Whenever the matching point is away from this region the comparison becomes unfavorable.

Furthermore, the comparison for $\hat{\zeta}$ is better than for $\hat{\xi}^{(1)}$ and $\xi^{(2)}$. Note that α depends solely upon $\hat{\zeta}$, while the other admittance coefficients depend upon $\hat{\xi}^{(1)}$ and sometimes $\xi^{(2)}$ as well. Therefore the comparison is more favorable for α than for the other coefficients. It is believed that the unfavorable comparisons for $\hat{\xi}^{(1)}$ and $\xi^{(2)}$ are due to computational difficulties rather than theoretical difficulties. At this time, the asymptotic solutions do provide a substantial amount of insight to the nature of the oscillatory flow but caution must be used whenever they are employed to actually calculate admittance coefficients.

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TABLE I

Values of $s_{\nu h}$

ν \ h	1	2	3
0	0	3.8317	7.0156
1	1.8413	5.3313	8.5263
2	3.0543	6.7060	9.9695

TABLE II

ω	$s_{\nu h}$	R_2	$\bar{q}_e$	$\mathcal{A}$	$\mathcal{B}$	$\mathcal{C}$
5.0	2.0	2.0	0.1	-0.830 +0.000i -0.827 +0.020i -0.831 -0.058i	0.000 -0.030i 0.002 -0.032i 0.000 -0.032i	0.000 +0.000i 0.000 +0.000i 0.000 +0.000i
5.0	2.0	1.5	0.1	-0.841 -0.010i -0.817 +0.020i -0.831 -0.058i	0.000 -0.030i 0.001 -0.033i 0.000 -0.032i	0.000 +0.000i 0.000 +0.000i 0.000 +0.000i
5.0	2.0	2.5	0.1	-0.844 +0.004i -0.843 -0.005i -0.831 -0.058i	0.000 +0.000i 0.001 -0.031i 0.000 -0.032i	0.000 +0.000i 0.000 +0.000i 0.000 +0.000i
5.0	2.0	2.0	0.2	-0.827 -0.002i -0.824 +0.009i -0.835 -0.098i	0.000 -0.085i 0.004 -0.092i -0.001 -0.093i	0.000 +0.000i 0.000 +0.000i 0.000 +0.001i
2.0	2.0	0.5	0.1	-0.619 -0.353i -0.524 -0.242i -0.506 -0.225i	-0.017 -0.026i 0.010 -0.120i 0.033 -0.108i	0 -0.003i 0 +0.003i -0.001 +0.003i
2.0	2.0	1.0	0.1	-0.849 -0.419i -0.542 -0.244i -0.506 -0.225i	-0.002 -0.138i -0.011 -0.124i 0.033 -0.108i	0.001 +0.003i 0.001 +0.002i -0.001 +0.003i
2.0	2.0	2.0	0.1	-1.223 -0.061i -0.572 -0.226i -0.506 -0.225i	-0.048 -0.132i -0.036 -0.110i 0.033 -0.108i	0.004 +0.002i 0.003 +0.001i -0.001 +0.003i
2.0	2.0	2.0	0.2	-0.630 +0.402i -1.145 -0.336i -0.838 -0.604i	0.053 -0.187i 0.016 -0.133i -0.047 -0.157i	-0.004 +0.001i -0.004 -0.004i 0.000 -0.003i
1.25	2.0	2.0	0.2	-0.013 +0.039i 0.062 +0.105i 0.068 +0.103i	-0.145 -0.407i 0.094 -0.196i 0.001 -0.182i	0.015 -0.003i -0.015 -0.005i -0.011 -0.009i

TABLE III REAL PART OF PRESSURE ADMITTANCE COEFFICIENT $S_{\nu h} = 0$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.268	-.236	-.208	-.183	-.163	-.155	-.143	-.136	-.129	-.126	-.126	-.128	-.132	-.141	-.162	-.206	-.342	-1.231
1.0	-.556	-.506	-.464	-.430	-.405	-.396	-.384	-.382	-.399	-.441	-.491	-.569	-.660	-.867	-1.173	-1.160	-.723	-.629
1.5	-.791	-.728	-.676	-.636	-.610	-.601	-.592	-.599	-.651	-.747	-.839	-.939	-.996	-.983	-.827	-.712	-.853	-.798
2.0	-.945	-.872	-.812	-.766	-.735	-.725	-.714	-.724	-.786	-.876	-.926	-.933	-.898	-.811	-.768	-.868	-.830	-.892
2.5	-1.042	-.961	-.896	-.843	-.807	-.794	-.780	-.789	-.848	-.904	-.905	-.865	-.823	-.794	-.857	-.854	-.825	-.798
3.0	-1.104	-1.019	-.948	-.890	-.850	-.835	-.818	-.826	-.875	-.896	-.869	-.828	-.811	-.840	-.865	-.809	-.849	-.847
3.5	-1.146	-1.057	-.982	-.921	-.876	-.860	-.841	-.848	-.887	-.880	-.845	-.822	-.832	-.866	-.824	-.852	-.821	-.839
4.0	-1.175	-1.083	-1.005	-.941	-.894	-.877	-.856	-.862	-.890	-.864	-.835	-.836	-.856	-.851	-.829	-.840	-.849	-.822
4.5	-1.196	-1.101	-1.022	-.955	-.907	-.888	-.867	-.872	-.888	-.853	-.838	-.853	-.862	-.832	-.852	-.826	-.824	-.846
5.0	-1.211	-1.115	-1.034	-.966	-.916	-.897	-.874	-.879	-.884	-.848	-.846	-.862	-.857	-.834	-.844	-.846	-.845	-.829
5.5	-1.223	-1.125	-1.043	-.974	-.922	-.903	-.880	-.883	-.880	-.848	-.856	-.859	-.841	-.847	-.831	-.837	-.829	-.834
6.0	-1.232	-1.133	-1.050	-.980	-.928	-.908	-.884	-.887	-.876	-.852	-.862	-.852	-.839	-.851	-.839	-.832	-.841	-.839
6.5	-1.239	-1.140	-1.055	-.984	-.932	-.912	-.888	-.889	-.872	-.857	-.863	-.845	-.845	-.844	-.846	-.843	-.833	-.829
7.0	-1.215	-1.145	-1.060	-.988	-.935	-.915	-.891	-.891	-.869	-.862	-.860	-.844	-.852	-.838	-.839	-.836	-.837	-.838
7.5	-1.249	-1.149	-1.063	-.991	-.937	-.917	-.893	-.892	-.868	-.865	-.856	-.848	-.853	-.841	-.836	-.835	-.836	-.834
8.0	-1.253	-1.152	-1.066	-.994	-.940	-.919	-.895	-.893	-.867	-.867	-.852	-.853	-.849	.847	-.842	-.842	-.834	-.832
8.5	-1.256	-1.155	-1.069	-.996	-.941	-.921	-.896	-.893	-.867	-.866	-.851	-.855	-.845	-.847	-.843	-.836	-.838	-.838
9.0	-1.259	-1.157	-1.071	-.997	-.943	-.922	-.897	-.894	-.868	-.864	-.852	-.854	-.844	-.842	-.838	-.836	-.833	-.832
9.5	-1.261	-1.159	-1.073	-.999	-.944	-.923	-.898	-.894	-.869	-.862	-.855	-.851	-.847	-.841	-.838	-.841	-.838	-.835
10.0	-1.263	-1.261	-1.074	-1.000	-.945	-.924	-.899	-.894	-.870	-.860	-.857	-.848	-.850	-.844	-.843	-.836	-.833	-.835

TABLE IV IMAGINARY PART OF PRESSURE ADMITTANCE COEFFICIENT $S_{\nu h} = 0$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.395	-.369	-.347	-.330	-.319	-.315	-.311	-.314	-.331	-.358	-.382	-.412	-.441	-.495	-.582	-.711	-.966	-1.509
1.0	-.568	-.532	-.504	-.483	-.471	-.467	-.466	-.476	-.516	-.573	-.619	-.669	-.704	-.714	-.513	-.039	.069	-.312
1.5	-.578	-.541	-.510	-.486	-.470	-.465	-.460	-.469	-.497	-.511	-.492	-.425	-.327	-.147	-.047	-.134	-.261	.011
2.0	-.531	-.494	-.463	-.437	-.417	-.409	-.400	-.402	-.403	-.359	-.286	-.190	-.125	-.094	-.158	-.185	-.032	-.109
2.5	-.474	-.439	-.409	-.382	-.360	-.351	-.340	-.339	-.318	-.243	-.171	-.116	-.106	-.140	-.156	-.056	-.118	-.073
3.0	-.421	-.389	-.360	-.334	-.313	-.304	-.293	-.288	-.253	-.172	-.122	-.109	-.124	-.144	-.076	-.085	-.041	-.028
3.5	-.376	-.346	-.319	-.295	-.274	-.266	-.256	-.250	-.205	-.131	-.107	-.117	-.130	-.101	-.065	-.084	-.070	-.063
4.0	-.338	-.311	-.286	-.263	-.244	-.236	-.227	-.219	-.168	-.109	-.106	-.119	-.112	-.067	-.084	-.044	-.043	-.031
4.5	-.306	-.281	-.258	-.237	-.219	-.212	-.203	-.194	-.141	-.100	-.107	-.108	-.084	-.065	-.070	-.059	-.046	-.032
5.0	-.279	-.256	-.235	-.215	-.199	-.192	-.184	-.174	-.121	-.096	-.104	-.089	-.065	-.073	-.046	-.053	-.041	-.038
5.5	-.256	-.235	-.215	-.197	-.182	-.176	-.168	-.157	-.106	-.095	-.096	-.071	-.061	-.069	-.050	-.035	-.032	-.021
6.0	-.237	-.217	-.198	-.181	-.167	-.162	-.155	-.143	-.095	-.092	-.084	-.061	-.065	-.054	-.055	-.044	-.038	-.030
6.5	-.220	-.201	-.184	-.168	-.155	-.150	-.143	-.131	-.087	-.088	-.072	-.059	-.065	-.044	-.043	-.039	-.025	-.024
7.0	-.205	-.188	-.172	-.157	-.144	-.139	-.133	-.120	-.082	-.082	-.062	-.060	-.059	-.046	-.035	-.029	-.033	-.019
7.5	-.192	-.176	-.161	-.146	-.135	-.130	-.124	-.111	-.078	-.075	-.056	-.060	-.049	-.049	-.039	-.035	-.022	-.025
8.0	-.181	-.165	-.151	-.138	-.127	-.122	-.117	-.103	-.075	-.067	-.054	-.057	-.043	-.044	-.039	-.031	-.028	-.016
8.5	-.171	-.156	-.142	-.130	-.119	-.115	-.110	-.096	-.073	-.060	-.054	-.050	-.042	-.037	-.031	-.025	-.021	-.019
9.0	-.162	-.148	-.135	-.123	-.113	-.109	-.104	-.090	-.071	-.055	-.054	-.044	-.044	-.034	-.029	-.029	-.023	-.019
9.5	-.153	-.140	-.128	-.116	-.107	-.103	-.099	-.084	-.068	-.051	-.053	-.040	-.043	-.036	-.032	-.025	-.020	-.014
10.0	-.146	-.133	-.122	-.111	-.102	-.098	-.094	-.079	-.066	-.049	-.050	-.040	-.040	-.036	-.030	-.022	-.019	-.018

TABLE V REAL PART OF PRESSURE ADMITTANCE COEFFICIENT $S_{\nu h}$ = 1

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.313	-.371	-.334	-.351	-.370	-.370	-.388	-.399	-.418	-.420	-.396	-.339	-.261	-.094	.100	.036	.053	.031
1.0	-.568	-.524	-.485	-.445	-.404	-.382	-.345	-.313	-.257	-.190	-.140	-.107	-.117	-.220	-.323	-.985	-1.726	-.307
1.5	-.787	-.715	-.651	-.590	-.535	-.509	-.474	-.455	-.460	-.525	-.632	-.796	-.937	-1.096	-.920	-.644	-.678	-.927
2.0	-.936	-.853	-.780	-.717	-.669	-.652	-.634	-.639	-.704	-.818	-.898	-.934	-.911	-.803	-.720	-.025	-.854	-.874
2.5	-1.033	-.944	-.868	-.805	-.762	-.748	-.734	-.745	-.810	-.881	-.892	-.856	-.810	-.769	-.834	-.858	-.801	-.783
3.0	-1.097	-1.004	-.926	-.863	-.819	-.804	-.788	-.798	-.852	-.882	-.858	-.816	-.795	-.822	-.861	-.798	-.851	-.851
3.5	-1.140	-1.045	-.965	-.900	-.854	-.838	-.820	-.828	-.871	-.869	-.835	-.811	-.819	-.857	-.819	-.844	-.813	-.831
4.0	-1.170	-1.073	-.992	-.925	-.878	-.860	-.841	-.848	-.878	-.855	-.826	-.826	-.847	-.847	-.822	-.839	-.848	-.821
4.5	-1.191	-1.093	-1.011	-.943	-.894	-.876	-.855	-.861	-.879	-.846	-.830	-.845	-.856	-.828	-.847	-.822	-.821	-.845
5.0	-1.207	-1.108	-1.025	-.955	-.905	-.887	-.865	-.870	-.877	-.842	-.839	-.856	-.848	-.829	-.842	-.843	-.843	-.826
5.5	-1.219	-1.120	-1.035	-.965	-.914	-.895	-.872	-.876	-.874	-.843	-.850	-.855	-.838	-.843	-.829	-.836	-.827	-.834
6.0	-1.229	-1.128	-1.044	-.972	-.920	-.901	-.878	-.881	-.871	-.847	-.858	-.848	-.835	-.849	-.836	-.830	-.839	-.838
6.5	-1.236	-1.139	-1.050	-.978	-.925	-.906	-.882	-.884	-.868	-.853	-.860	-.842	-.842	-.842	-.845	-.842	-.832	-.828
7.0	-1.242	-1.141	-1.055	-.983	-.929	-.909	-.886	-.887	-.866	-.858	-.857	-.841	-.849	-.836	-.838	-.835	-.835	-.838
7.5	-1.247	-1.146	-1.059	-.986	-.933	-.913	-.889	-.888	-.864	-.862	-.853	-.845	-.851	-.839	-.834	-.833	-.836	-.833
8.0	-1.251	-1.149	-1.063	-.990	-.935	-.915	-.891	-.890	-.864	-.864	-.850	-.850	-.847	-.845	-.841	-.841	-.833	-.832
8.5	-1.255	-1.153	-1.066	-.992	-.938	-.917	-.893	-.890	-.864	-.864	-.849	-.853	-.843	-.846	-.842	-.835	-.838	-.837
9.0	-1.257	-1.155	-1.068	-.994	-.940	-.919	-.894	-.891	-.865	-.862	-.850	-.852	-.843	-.841	-.837	-.835	-.832	-.831
9.5	-1.260	-1.157	-1.070	-.996	-.941	-.920	-.896	-.891	-.867	-.860	-.853	-.850	-.846	-.839	-.837	-.840	-.838	-.835
10.0	-1.262	-1.159	-1.072	-.998	-.943	-.922	-.897	-.892	-.868	-.858	-.856	-.847	-.849	-.842	-.842	-.836	-.833	-.835

TABLE VI IMAGINARY PART OF PRESSURE ADMITTANCE COEFFICIENT $S_{\nu h}$ = 1

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.355	-.290	-.226	-.159	-.093	-.060	-.011	.034	.116	.216	.296	.375	.426	.440	.278	.001	.032	-.309
1.0	-.526	-.450	-.379	-.310	-.249	-.224	-.190	-.172	-.163	-.192	-.249	-.352	-.466	-.631	-.799	-1.275	.629	.133
1.5	-.550	-.485	-.430	-.385	-.356	-.349	-.349	-.367	-.431	-.523	-.580	-.584	-.517	-.292	.049	-.009	-.309	.109
2.0	-.513	-.461	-.419	-.387	-.370	-.368	-.372	-.388	-.417	-.399	-.331	-.220	-.128	-.062	-.134	-.221	-.011	-.156
2.5	-.462	-.419	-.384	-.357	-.341	-.337	-.333	-.338	-.327	-.257	-.179	-.114	-.096	-.132	-.168	-.056	-.120	-.061
3.0	-.413	-.376	-.345	-.320	-.303	-.297	-.290	-.288	-.258	-.177	-.122	-.105	-.120	-.146	-.081	-.080	-.044	-.031
3.5	-.370	-.338	-.310	-.286	-.269	-.262	-.254	-.249	-.207	-.132	-.105	-.115	-.130	-.105	-.062	-.088	-.067	-.065
4.0	-.334	-.305	-.279	-.257	-.240	-.234	-.225	-.219	-.170	-.110	-.104	-.119	-.114	-.068	-.083	-.044	-.045	-.028
4.5	-.303	-.277	-.253	-.233	-.217	-.210	-.202	-.194	-.142	-.099	-.106	-.109	-.085	-.064	-.071	-.057	-.044	-.035
5.0	-.277	-.253	-.231	-.212	-.197	-.191	-.183	-.174	-.121	-.096	-.104	-.090	-.065	-.073	-.047	-.055	-.043	-.037
5.5	-.255	-.233	-.213	-.195	-.180	-.175	-.167	-.157	-.106	-.094	-.096	-.071	-.061	-.070	-.049	-.035	-.031	-.020
6.0	-.236	-.215	-.197	-.180	-.166	-.161	-.154	-.143	-.095	-.092	-.085	-.061	-.064	-.054	-.055	-.044	-.038	-.031
6.5	-.219	-.200	-.183	-.167	-.154	-.149	-.143	-.131	-.087	-.008	-.072	-.059	-.065	-.044	-.044	-.040	-.025	-.023
7.0	-.205	-.187	-.170	-.156	-.144	-.139	-.133	-.120	-.082	-.082	-.062	-.060	-.059	-.046	-.035	-.029	-.033	-.019
7.5	-.192	-.175	-.160	-.146	-.134	-.130	-.124	-.111	-.078	-.075	-.056	-.060	-.050	-.049	-.039	-.035	-.022	-.025
8.0	-.181	-.165	-.150	-.137	-.126	-.122	-.117	-.103	-.075	-.067	-.054	-.057	-.043	-.045	-.039	-.031	-.028	-.016
8.5	-.170	-.155	-.142	-.129	-.119	-.115	-.110	-.096	-.073	-.061	-.054	-.050	-.042	-.037	-.031	-.025	-.021	-.019
9.0	-.161	-.147	-.134	-.122	-.113	-.109	-.104	-.090	-.071	-.055	-.054	-.044	-.043	-.034	-.029	-.029	-.023	-.018
9.5	-.153	-.140	-.127	-.116	-.107	-.103	-.099	-.084	-.068	-.051	-.053	-.040	-.043	-.035	-.032	-.025	-.021	-.014
10.0	-.146	-.133	-.121	-.110	-.102	-.098	-.094	-.079	-.066	-.049	-.050	-.040	-.040	-.036	-.030	-.022	-.019	-.018

TABLE VII REAL PART OF PRESSURE ADMITTANCE COEFFICIENT $S\nu h$ = 2

ω \ M	902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.4 5	-.564	-.680	-.793	-.888	-.925	-.969	-1.013	-1.078	-1.081	-1.018	-.882	-.714	-.387	-.008	.066	-.025	.026
1.0	-.606	-.597	-.591	-.582	-.561	-.544	-.507	-.467	-.371	-.224	-.101	.016	.077	.051	-.021	.033	.005	.067
1.5	-.776	-.692	-.609	-.521	-.427	-.378	-.300	-.232	-.120	-.022	.010	-.013	-.054	-.013	-.114	-1.312	-.234	-.104
2.0	-.911	-.802	-.696	-.590	-.489	-.443	-.379	-.339	-.323	-.405	-.547	-.761	-1.019	-1.153	-.659	-.481	-1.107	-.538
2.5	-1.007	-.893	-.788	-.692	-.616	-.589	-.563	-.569	-.652	-.794	-.875	-.880	-.809	-.692	-.714	-.910	-.702	-.773
3.0	-1.074	-.960	-.861	-.776	-.718	-.700	-.687	-.703	-.777	-.841	-.833	-.783	-.747	-.760	-.852	-.770	-.860	-.865
3.5	-1.121	-1.009	-.913	-.835	-.783	-.767	-.754	-.767	-.821	-.837	-.806	-.775	-.778	-.829	-.808	-.815	-.792	-.805
4.0	-1.154	-1.043	-.951	-.875	-.825	-.809	-.795	-.803	-.842	-.830	-.800	-.794	-.818	-.834	-.799	-.835	-.841	-.823
4.5	-1.178	-1.069	-.978	-.903	-.853	-.836	-.818	-.826	-.851	-.824	-.806	-.820	-.839	-.815	-.831	-.810	-.814	-.838
5.0	-1.196	-1.088	-.998	-.924	-.873	-.855	-.835	-.842	-.856	-.823	-.819	-.839	-.837	-.814	-.835	-.832	-.835	-.819
5.5	-1.210	-1.103	-1.013	-.939	-.887	-.869	-.848	-.853	-.856	-.826	-.833	-.843	-.827	-.830	-.821	-.832	-.824	-.834
6.0	-1.221	-1.114	-1.024	-.950	-.898	-.879	-.858	-.862	-.856	-.832	-.844	-.839	-.825	-.840	-.827	-.823	-.837	-.832
6.5	-1.229	-1.123	-1.034	-.959	-.907	-.887	-.865	-.868	-.855	-.840	-.849	-.834	-.832	-.836	-.838	-.836	-.831	-.826
7.0	-1.236	-1.130	-1.041	-.967	-.913	-.894	-.871	-.873	-.855	-.847	-.849	-.833	-.841	-.830	-.834	-.833	-.830	-.836
7.5	-1.242	-1.136	-1.047	-.972	-.919	-.899	-.876	-.877	-.855	-.853	-.846	-.837	-.844	-.833	-.829	-.829	-.834	-.830
8.0	-1.246	-1.141	-1.052	-.977	-.923	-.903	-.880	-.879	-.855	-.856	-.843	-.843	-.842	-.840	-.836	-.837	-.830	-.832
8.5	-1.250	-1.145	-1.056	-.981	-.927	-.907	-.883	-.881	-.856	-.857	-.843	-.848	-.838	-.842	-.839	-.833	-.836	-.835
9.0	-1.254	-1.148	-1.059	-.984	-.930	-.910	-.886	-.883	-.858	-.857	-.845	-.848	-.838	-.838	-.834	-.832	-.830	-.830
9.5	-1.256	-1.151	-1.062	-.987	-.932	-.912	-.888	-.884	-.860	-.855	-.848	-.846	-.841	-.836	-.834	-.838	-.836	-.835
10.0	-1.259	-1.154	-1.065	-.990	-.935	-.914	-.890	-.885	-.862	-.854	-.851	-.843	-.845	-.839	-.839	-.834	-.832	-.833

TABLE VIII IMAGINARY PART OF PRESSURE ADMITTANCE COEFFICIENT $S\nu h$ = 2

.5	-.246	-.093	.048	.185	.312	.373	.464	.561	.770	1.027	1.220	1.402	1.512	1.560	1.347	.911	.831	.518
1.0	-.407	-.222	-.048	.121	.270	.335	.420	.493	.600	.655	.638	.557	.444	.256	.219	.020	-.181	-.839
1.5	-.463	-.319	-.186	-.064	.033	.069	.106	.120	.100	.006	-.105	-.228	-.295	-.413	-.886	-2.960	.840	-.523
2.0	-.458	-.357	-.272	-.206	-.172	-.170	-.187	-.226	-.339	-.490	-.578	-.611	-.546	.026	.179	-.165	-.338	-.149
2.5	-.427	-.356	-.300	-.265	-.258	-.267	-.291	-.325	-.375	-.348	-.253	-.119	-.042	-.065	-.201	-.109	-.078	.028
3.0	-.390	-.337	-.296	-.272	-.267	-.271	-.280	-.291	-.281	-.199	-.125	-.087	-.099	-.153	-.107	-.052	-.066	-.057
3.5	-.355	-.312	-.279	-.259	-.250	-.249	-.249	-.250	-.217	-.138	-.101	-.106	-.128	-.121	-.053	-.099	-.054	-.064
4.0	-.323	-.287	-.259	-.240	-.229	-.226	-.222	-.219	-.175	-.110	-.100	-.116	-.120	-.072	-.078	-.048	-.056	-.021
4.5	-.296	-.265	-.240	-.221	-.209	-.205	-.200	-.194	-.145	-.098	-.103	-.111	-.091	-.060	-.077	-.052	-.037	-.042
5.0	-.272	-.244	-.221	-.204	-.192	-.187	-.181	-.174	-.123	-.094	-.103	-.094	-.067	-.070	-.049	-.059	-.047	-.033
5.5	-.250	-.226	-.205	-.189	-.176	-.172	-.166	-.157	-.107	-.093	-.097	-.074	-.060	-.071	-.046	-.036	-.029	-.021
6.0	-.232	-.210	-.191	-.175	-.163	-.159	-.153	-.143	-.096	-.091	-.086	-.061	-.063	-.057	-.055	-.042	-.040	-.033
6.5	-.216	-.196	-.178	-.163	-.152	-.147	-.142	-.131	-.087	-.088	-.074	-.058	-.065	-.045	-.046	-.042	-.024	-.021
7.0	-.202	-.183	-.167	-.153	-.142	-.137	-.132	-.120	-.082	-.083	-.063	-.059	-.060	-.045	-.035	-.029	-.033	-.021
7.5	-.190	-.172	-.157	-.143	-.133	-.129	-.124	-.111	-.078	-.076	-.057	-.060	-.051	-.048	-.038	-.034	-.023	-.024
8.0	-.179	-.162	-.148	-.135	-.125	-.121	-.116	-.103	-.075	-.068	-.054	-.057	-.043	-.045	-.040	-.032	-.027	-.016
8.5	-.169	-.154	-.140	-.128	-.118	-.114	-.110	-.096	-.073	-.061	-.054	-.051	-.042	-.038	-.032	-.025	-.022	-.020
9.0	-.160	-.146	-.133	-.121	-.112	-.108	-.104	-.090	-.070	-.055	-.054	-.045	-.043	-.034	-.029	-.029	-.022	-.018
9.5	-.152	-.138	-.126	-.115	-.106	-.103	-.098	-.084	-.068	-.051	-.053	-.040	-.043	-.035	-.032	-.026	-.021	-.014
10.0	-.145	-.132	-.120	-.109	-.101	-.098	-.093	-.079	-.066	-.049	-.050	-.040	-.040	-.036	-.030	-.022	-.019	-.018

TABLE IX REAL PART OF PRESSURE ADMITTANCE COEFFICIENT $S_{vh} = 3$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.653	-.924	-1.157	-1.354	-1.499	-1.552	-1.610	-1.674	-1.765	-1.744	-1.627	-1.409	-1.165	-.720	-.213	.018	-.066	-.001
1.0	-.681	-.752	-.825	-.883	-.903	-.896	-.862	-.815	-.680	-.468	-.294	-.123	-.017	.031	-.038	.016	.003	.015
1.5	-.769	-.694	-.634	-.577	-.512	-.473	-.403	-.331	-.189	-.044	.026	.042	.012	.001	.023	.030	.052	15.364
2.0	-.874	-.737	-.607	-.478	-.356	-.297	-.209	-.136	-.028	.031	.025	.007	.031	.050	.101	1.058	.001	.076
2.5	-.966	-.814	-.669	-.525	-.393	-.333	-.253	-.200	-.159	-.202	-.282	-.482	-1.067	-1.445	-.407	-.332	-1.305	-1.039
3.0	-1.037	-.888	-.752	-.625	-.525	-.488	-.454	-.459	-.556	-.754	-.879	-.845	-.717	-.589	-.746	-.848	-.764	-.739
3.5	-1.090	-.948	-.824	-.717	-.646	-.626	-.615	-.638	-.729	-.798	-.774	-.717	-.694	-.759	-.816	-.740	-.780	-.763
4.0	-1.128	-.994	-.879	-.785	-.727	-.711	-.702	-.720	-.779	-.788	-.756	-.739	-.762	-.814	-.764	-.827	-.813	-.837
4.5	-1.156	-1.028	-.920	-.834	-.780	-.764	-.751	-.765	-.804	-.788	-.766	-.777	-.805	-.798	-.793	-.796	-.809	-.818
5.0	-1.177	-1.054	-.951	-.868	-.815	-.799	-.784	-.794	-.818	-.791	-.784	-.807	-.817	-.791	-.822	-.810	-.817	-.812
5.5	-1.194	-1.074	-.974	-.893	-.841	-.824	-.806	-.814	-.826	-.798	-.804	-.822	-.811	-.807	-.810	-.826	-.822	-.833
6.0	-1.207	-1.089	-.991	-.913	-.860	-.842	-.823	-.830	-.831	-.807	-.821	-.823	-.808	-.825	-.811	-.813	-.820	-.822
6.5	-1.218	-1.102	-1.005	-.928	-.874	-.856	-.836	-.841	-.834	-.818	-.831	-.820	-.815	-.826	-.827	-.825	-.828	-.825
7.0	-1.226	-1.112	-1.017	-.939	-.886	-.867	-.846	-.850	-.836	-.828	-.835	-.819	-.826	-.820	-.828	-.828	-.822	-.833
7.5	-1.233	-1.120	-1.026	-.949	-.895	-.876	-.855	-.857	-.838	-.837	-.834	-.824	-.834	-.822	-.822	-.822	-.832	-.825
8.0	-1.238	-1.127	-1.033	-.956	-.902	-.883	-.861	-.862	-.841	-.843	-.833	-.832	-.834	-.830	-.827	-.831	-.825	-.831
8.5	-1.243	-1.132	-1.039	-.963	-.908	-.889	-.867	-.866	-.843	-.846	-.833	-.838	-.831	-.835	-.833	-.830	-.833	-.832
9.0	-1.247	-1.137	-1.045	-.968	-.914	-.894	-.871	-.869	-.846	-.847	-.835	-.840	-.830	-.832	-.830	-.827	-.827	-.828
9.5	-1.251	-1.141	-1.049	-.973	-.918	-.898	-.875	-.872	-.849	-.847	-.839	-.839	-.834	-.830	-.829	-.833	-.833	-.833
10.0	-1.253	-1.144	-1.053	-.976	-.922	-.901	-.878	-.874	-.653	-.846	-.843	-.837	-.838	-.833	-.834	-.831	-.829	-.830

TABLE X IMAGINARY PART OF PRESSURE ADMITTANCE COEFFICIENT $S_{vh} = 3$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.090	.150	.344	.517	.675	.750	.866	1.011	1.347	1.750	2.036	2.289	2.433	2.494	2.265	1.765	1.465	1.117
1.0	-.221	.099	.368	.601	.787	.864	.964	1.064	1.230	1.324	1.317	1.234	1.112	.883	.740	.590	.400	.130
1.5	-.320	-.049	.193	.407	.568	.626	.687	.728	.752	.686	.585	.458	.378	.325	.157	-.019	-.370	-4.704
2.0	-.362	-.173	-.002	.149	.256	.290	.314	.313	.258	.137	.043	-.024	-.083	-.294	-.690	-4.121	.404	-2.132
2.5	-.365	-.240	-.134	-.053	-.015	-.015	-.041	-.090	-.222	-.399	-.550	-.807	-.982	.429	.256	-.338	.587	-.757
3.0	-.350	-.264	-.198	-.160	-.162	-.179	-.223	-.278	-.367	-.361	-.226	-.027	.027	-.079	-.265	.060	-.221	-.187
3.5	-.328	-.265	-.220	-.198	-.206	-.218	-.241	-.263	-.257	-.161	-.087	-.073	-.108	-.160	-.043	-.101	-.008	-.022
4.0	-.305	-.256	-.221	-.205	-.206	-.211	-.219	-.224	-.190	-.113	-.090	-.109	-.128	-.088	-.062	-.066	-.077	-.025
4.5	-.283	-.242	-.214	-.199	-.195	-.196	-.197	-.196	-.153	-.097	-.097	-.114	-.103	-.057	-.084	-.040	-.028	-.052
5.0	-.262	-.228	-.203	-.189	-.182	-.181	-.179	-.175	-.127	-.091	-.101	-.100	-.073	-.065	-.056	-.062	-.053	-.024
5.5	-.243	-.214	-.192	-.177	-.169	-.167	-.164	-.158	-.109	-.090	-.098	-.079	-.059	-.072	-.043	-.039	-.027	-.027
6.0	-.226	-.201	-.181	-.167	-.158	-.155	-.151	-.143	-.097	-.090	-.089	-.063	-.060	-.061	-.053	-.038	-.041	-.033
6.5	-.212	-.188	-.170	-.157	-.148	-.144	-.140	-.131	-.088	-.088	-.077	-.057	-.064	-.046	-.049	-.044	-.025	-.018
7.0	-.199	-.177	-.161	-.148	-.138	-.135	-.131	-.120	-.082	-.083	-.065	-.058	-.062	-.043	-.036	-.030	-.032	-.024
7.5	-.187	-.168	-.152	-.139	-.130	-.127	-.123	-.111	-.077	-.077	-.057	-.059	-.053	-.047	-.036	-.033	-.024	-.023
8.0	-.176	-.159	-.144	-.132	-.123	-.119	-.115	-.103	-.074	-.069	-.054	-.058	-.044	-.047	-.040	-.034	-.026	-.016
8.5	-.167	-.150	-.136	-.125	-.116	-.113	-.109	-.096	-.072	-.062	-.053	-.052	-.041	-.039	-.034	-.025	-.023	-.021
9.0	-.158	-.143	-.130	-.119	-.110	-.107	-.103	-.090	-.070	-.056	-.053	-.046	-.042	-.034	-.028	-.028	-.021	-.016
9.5	-.151	-.136	-.124	-.113	-.105	-.102	-.098	-.084	-.068	-.052	-.053	-.041	-.043	-.034	-.031	-.027	-.022	-.015
10.0	-.144	-.130	-.118	-.108	-.100	-.097	-.093	-.079	-.066	-.049	-.050	-.039	-.041	-.036	-.031	-.022	-.018	-.018

TABLE XI REAL PART OF PRESSURE ADMITTANCE COEFFICIENT $S_{\nu h} = 4$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.921	-1.351	-1.681	-1.937	-2.112	-2.172	-2.245	-2.317	-2.427	-2.369	-2.194	-1.899	-1.588	-1.050	-.441	-.076	-.081	-.021
1.0	-.800	-.988	-1.148	-1.256	-1.291	-1.281	-1.235	-1.172	-.987	-.709	-.489	-.275	-.133	-.021	-.042	-.006	-.002	.003
1.5	-.778	-.758	-.767	-.768	-.730	-.695	-.620	-.534	-.350	-.158	-.053	0.000	-.002	-.021	.008	.011	.013	.020
2.0	-.833	-.691	-.586	-.502	-.419	-.374	-.297	-.222	-.090	.002	.017	0.000	.001	.024	.027	.030	.069	.049
2.5	-.913	-.724	-.558	-.409	-.283	-.227	-.147	-.083	0.000	.025	.016	.029	.045	.046	.091	3.790	.045	.188
3.0	-.987	-.794	-.615	-.445	-.298	-.234	-.151	-.094	-.042	-.038	-.032	-.049	-.180	-4.722	-.138	-.112	-.333	-.984
3.5	-1.046	-.864	-.699	-.546	-.423	-.377	-.329	-.322	-.404	-.676	-.963	-.881	-.633	-.466	-.816	-.661	-1.063	-1.099
4.0	-1.091	-.922	-.775	-.647	-.561	-.537	-.528	-.558	-.679	-.768	-.723	-.650	-.641	-.768	-.749	-.770	-.714	-.827
4.5	-1.125	-.969	-.835	-.726	-.660	-.645	-.641	-.666	-.736	-.741	-.709	-.706	-.747	-.784	-.742	-.794	-.821	-.777
5.0	-1.150	-1.004	-.882	-.784	-.726	-.711	-.703	-.721	-.763	-.747	-.733	-.758	-.786	-.763	-.798	-.776	-.787	-.818
5.5	-1.171	-1.032	-.917	-.826	-.771	-.755	-.744	-.757	-.782	-.759	-.761	-.789	-.789	-.774	-.799	-.812	-.819	-.822
6.0	-1.188	-1.054	-.944	-.857	-.803	-.787	-.772	-.782	-.794	-.772	-.786	-.800	-.786	-.800	-.791	-.803	-.804	-.810
6.5	-1.201	-1.072	-.965	-.881	-.827	-.810	-.794	-.802	-.803	-.787	-.804	-.801	-.792	-.811	-.808	-.808	-.823	-.825
7.0	-1.211	-1.086	-.982	-.899	-.846	-.828	-.810	-.816	-.809	-.801	-.814	-.801	-.805	-.808	-.818	-.821	-.813	-.824
7.5	-1.220	-1.097	-.995	-.914	-.860	-.842	-.823	-.828	-.815	-.814	-.817	-.806	-.817	-.808	-.812	-.813	-.826	-.820
8.0	-1.227	-1.107	-1.007	-.926	-.872	-.854	-.834	-.837	-.820	-.823	-.817	-.815	-.822	-.816	-.815	-.821	-.819	-.829
8.5	-1.233	-1.115	-1.016	-.936	-.882	-.863	-.843	-.844	-.825	-.830	-.818	-.824	-.820	-.825	-.825	-.825	-.827	-.826
9.0	-1.238	-1.121	-1.024	-.945	-.890	-.871	-.850	-.850	-.829	-.833	-.821	-.828	-.820	-.825	-.824	-.821	-.823	-.826
9.5	-1.243	-1.127	-1.030	-.952	-.897	-.878	-.856	-.855	-.834	-.834	-.826	-.829	-.823	-.822	-.822	-.827	-.828	-.831
10.0	-1.246	-1.131	-1.036	-.958	-.903	-.883	-.861	-.859	-.839	-.835	-.832	-.828	-.829	-.825	-.827	-.828	-.827	-.827

TABLE XII IMAGINARY PART OF PRESSURE ADMITTANCE COEFFICIENT $S_{\nu h} = 4$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	.086	.392	.617	.816	1.000	1.090	1.231	1.428	1.900	2.443	2.809	3.116	3.284	3.353	3.115	2.582	2.087	1.655
1.0	.009	.449	.772	1.030	1.230	1.313	1.423	1.551	1.773	1.897	1.895	1.809	1.683	1.431	1.200	1.031	.807	.538
1.5	-.125	.289	.617	.871	1.040	1.097	1.156	1.205	1.246	1.183	1.077	.935	.928	.730	.584	.432	.219	-.183
2.0	-.224	.090	.368	.594	.737	.778	.807	.813	.770	.649	.544	.461	.414	.296	.148	-.077	-.656	.592
2.5	-.274	-.060	.138	.307	.416	.444	.453	.436	.358	.239	.169	.099	.003	-.162	-.560	-4.546	.147	1.840
3.0	-.289	-.148	-.024	.073	.121	.124	.101	.056	-.061	-.211	-.355	-.645	-1.132	2.195	.399	-.434	.867	1.630
3.5	-.288	-.190	-.114	-.070	-.073	-.095	-.151	-.223	-.371	-.467	-.287	.101	.144	-.088	-.377	.175	-.195	.035
4.0	-.277	-.206	-.156	-.135	-.152	-.172	-.212	-.248	-.256	-.134	-.049	-.059	-.122	-.165	.002	-.142	-.074	-.110
4.5	-.263	-.208	-.172	-.158	-.169	-.180	-.197	-.207	-.172	-.094	-.082	-.113	-.125	-.059	-.082	-.027	-.030	-.039
5.0	-.247	-.203	-.174	-.163	-.167	-.171	-.177	-.178	-.136	-.088	-.096	-.109	-.086	-.054	-.072	-.057	-.049	-.016
5.5	-.232	-.196	-.171	-.160	-.159	-.160	-.162	-.159	-.114	-.087	-.098	-.088	-.061	-.070	-.041	-.050	-.032	-.038
6.0	-.217	-.187	-.165	-.154	-.150	-.150	-.149	-.144	-.099	-.087	-.092	-.068	-.056	-.067	-.048	-.032	-.036	-.026
6.5	-.205	-.178	-.159	-.147	-.142	-.140	-.139	-.132	-.089	-.087	-.081	-.057	-.062	-.050	-.053	-.045	-.030	-.018
7.0	-.194	-.169	-.152	-.140	-.134	-.132	-.130	-.121	-.082	-.084	-.068	-.056	-.063	-.042	-.039	-.033	-.028	-.028
7.5	-.183	-.161	-.145	-.133	-.127	-.124	-.122	-.112	-.077	-.078	-.059	-.058	-.056	-.045	-.034	-.030	-.028	-.019
8.0	-.173	-.153	-.138	-.127	-.120	-.117	-.114	-.103	-.074	-.071	-.054	-.058	-.046	-.047	-.040	-.035	-.023	-.018
8.5	-.164	-.146	-.132	-.121	-.114	-.111	-.108	-.096	-.071	-.064	-.052	-.054	-.041	-.041	-.036	-.026	-.026	-.022
9.0	-.156	-.139	-.126	-.115	-.108	-.106	-.102	-.090	-.069	-.057	-.053	-.047	-.041	-.034	-.028	-.027	-.020	-.015
9.5	-.149	-.133	-.120	-.110	-.103	-.100	-.097	-.084	-.068	-.052	-.053	-.041	-.043	-.033	-.030	-.028	-.024	-.017
10.0	-.142	-.127	-.115	-.105	-.098	-.096	-.093	-.079	-.066	-.049	-.051	-.039	-.042	-.036	-.032	-.022	-.017	-.017

TABLE XIII REAL PART OF PRESSURE ADMITTANCE COEFFICIENT $S\nu h = 5$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-1.230	-1.806	-2.217	-2.518	-2.715	-2.780	-2.846	-2.946	-3.069	-2.968	-2.733	-2.366	-1.996	-1.374	-.674	-.196	-.094	-.035
1.0	-.962	-1.277	-1.506	-1.642	-1.676	-1.660	-1.599	-1.517	-1.278	-.937	-.677	-.427	-.256	-.089	-.052	-.025	-.010	-.002
1.5	-.816	-.890	-.972	-1.007	-.971	-.930	-.843	-.736	-.507	-.271	-.138	-.053	-.029	-.032	-.007	-.001	.003	.007
2.0	-.800	-.698	-.661	-.630	-.567	-.523	-.440	-.350	-.184	-.055	-.015	-.014	-.016	.004	.007	.011	.014	.052
2.5	-.854	-.653	-.518	-.424	-.340	-.296	-.224	-.154	-.048	.001	-.001	.001	.014	.013	.020	.031	.196	.028
3.0	-.926	-.691	-.498	-.344	-.228	-.179	-.111	-.058	.004	.013	.016	.031	.030	.044	.083	7.374	.039	.035
3.5	-.991	-.760	-.555	-.370	-.222	-.162	-.088	-.039	.005	.019	.039	.053	.090	.874	.037	.030	.007	-.012
4.0	-1.043	-.831	-.640	-.465	-.324	-.270	-.208	-.180	-.202	-.397	-1.012	-1.218	-.549	-.290	-.873	-.409	-1.228	-.494
4.5	-1.084	-.891	-.721	-.573	-.470	-.440	-.426	-.459	-.622	-.771	-.686	-.573	-.573	-.799	-.660	-.855	-.776	-.838
5.0	-1.116	-.939	-.787	-.663	-.588	-.572	-.573	-.608	-.697	-.696	-.660	-.674	-.735	-.747	-.737	-.740	-.761	-.833
5.5	-1.142	-.977	-.839	-.730	-.667	-.654	-.652	-.675	-.723	-.707	-.701	-.738	-.762	-.734	-.786	-.779	-.803	-.791
6.0	-1.163	-1.008	-.879	-.779	-.722	-.708	-.701	-.717	-.745	-.726	-.737	-.767	-.761	-.762	-.772	-.797	-.792	-.807
6.5	-1.180	-1.032	-.910	-.817	-.761	-.746	-.735	-.748	-.761	-.746	-.766	-.776	-.763	-.789	-.781	-.787	-.809	-.822
7.0	-1.193	-1.051	-.935	-.845	-.791	-.775	-.761	-.771	-.774	-.766	-.784	-.778	-.777	-.793	-.802	-.808	-.806	-.811
7.5	-1.203	-1.067	-.955	-.868	-.814	-.797	-.782	-.789	-.784	-.783	-.794	-.783	-.794	-.791	-.802	-.806	-.815	-.818
8.0	-1.212	-1.080	-.972	-.886	-.832	-.815	-.798	-.804	-.792	-.797	-.798	-.793	-.805	-.798	-.801	-.807	-.814	-.825
8.5	-1.220	-1.091	-.985	-.901	-.847	-.829	-.811	-.815	-.800	-.808	-.800	-.804	-.807	-.810	-.812	-.818	-.818	-.818
9.0	-1.227	-1.100	-.996	-.914	-.859	-.841	-.822	-.825	-.807	-.814	-.804	-.813	-.807	-.814	-.817	-.814	-.820	-.825
9.5	-1.232	-1.108	-1.006	-.924	-.869	-.851	-.831	-.832	-.814	-.818	-.810	-.817	-.810	-.813	-.813	-.818	-.820	-.826
10.0	-1.236	-1.115	-1.014	-.933	-.878	-.859	-.839	-.838	-.821	-.820	-.817	-.817	-.816	-.814	-.817	-.823	-.824	-.823

TABLE XIV IMAGINARY PART OF PRESSURE ADMITTANCE COEFFICIENT $S\nu h = 5$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	.269	.620	.872	1.096	1.309	1.414	1.580	1.835	2.448	3.125	3.561	3.913	4.098	4.173	3.927	3.368	2.718	2.173
1.0	.261	.788	1.143	1.420	1.636	1.726	1.847	2.005	2.282	2.430	2.430	2.341	2.210	1.944	1.646	1.429	1.165	.861
1.5	.110	.647	1.013	1.280	1.448	1.504	1.561	1.620	1.677	1.615	1.504	1.353	1.228	1.083	.926	.758	.549	.255
2.0	-.045	.410	.762	1.006	1.142	1.177	1.197	1.205	1.164	1.037	.919	.813	.748	.638	.495	.327	.073	-.726
2.5	-.149	.187	.486	.711	.832	.857	.859	.841	.760	.629	.545	.477	.408	.290	.120	-.143	-1.360	-.082
3.0	-.207	.023	.242	.426	.531	.551	.546	.517	.425	.315	.248	.157	.066	-.109	-.524	2.955	-.083	.301
3.5	-.233	-.077	.064	.180	.243	.250	.232	.192	.090	-.031	-.149	-.343	-.612	-2.884	.598	-.400	.585	.470
4.0	-.240	-.132	-.044	.011	.016	-.003	-.058	-.133	-.314	-.604	-.740	.354	.394	-.049	-.700	.222	.490	.304
4.5	-.237	-.158	-.102	-.078	-.100	-.127	-.184	-.243	-.289	-.117	.008	-.034	-.141	-.183	.031	-.141	-.172	.081
5.0	-.227	-.168	-.129	-.117	-.136	-.154	-.182	-.199	-.161	-.075	-.075	-.119	-.123	-.033	-.100	-.021	-.010	-.051
5.5	-.217	-.170	-.140	-.131	-.142	-.151	-.163	-.166	-.123	-.081	-.095	-.103	-.071	-.058	-.052	-.066	-.053	-.041
6.0	-.206	-.168	-.143	-.134	-.139	-.143	-.148	-.147	-.104	-.083	-.095	-.078	-.054	-.071	-.038	-.032	-.024	-.015
6.5	-.196	-.163	-.142	-.133	-.134	-.135	-.137	-.133	-.091	-.084	-.086	-.060	-.057	-.058	-.053	-.040	-.039	-.027
7.0	-.187	-.157	-.139	-.129	-.128	-.128	-.128	-.122	-.082	-.083	-.073	-.054	-.062	-.043	-.045	-.040	-.023	-.027
7.5	-.177	-.152	-.134	-.125	-.122	-.121	-.120	-.112	-.077	-.080	-.061	-.056	-.059	-.042	-.033	-.027	-.032	-.015
8.0	-.168	-.146	-.130	-.120	-.116	-.115	-.113	-.104	-.073	-.073	-.054	-.058	-.049	-.047	-.037	-.035	-.020	-.022
8.5	-.160	-.140	-.125	-.115	-.110	-.109	-.107	-.097	-.071	-.066	-.051	-.055	-.042	-.044	-.038	-.029	-.028	-.020
9.0	-.152	-.134	-.120	-.111	-.165	-.104	-.101	-.090	-.069	-.059	-.052	-.049	-.040	-.036	-.030	-.024	-.019	-.014
9.5	-.146	-.129	-.116	-.106	-.101	-.099	-.096	-.085	-.067	-.053	-.052	-.043	-.042	-.032	-.028	-.029	-.024	-.019
10.0	-.140	-.123	-.111	-.102	-.096	-.094	-.092	-.079	-.065	-.049	-.051	-.039	-.042	-.035	-.032	-.023	-.018	-.015

TABLE XV REAL PART OF PRESSURE ADMITTANCE COEFFICIENT $S_{\nu h} = 7$

ω \ ν	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-1.910	-2.741	-3.287	-3.666	-3.900	-3.973	-4.042	-4.179	-4.315	-4.113	-3.762	-3.260	-2.780	-2.004	-1.138	-.473	-.149	-.056
1.0	-1.382	-1.916	-2.239	-2.405	-2.428	-2.396	-2.304	-2.180	-1.829	-1.370	-1.039	-.726	-.504	-.249	-.103	-.057	-.025	-.009
1.5	-.993	-1.273	-1.444	-1.499	-1.445	-1.388	-1.270	-1.120	-.803	-.492	-.312	-.176	-.107	-.060	-.032	-.016	-.006	0.000
2.0	-.802	-.876	-.951	-.954	-.884	-.827	-.722	-.597	-.364	-.177	-.095	-.054	-.041	-.023	-.010	0.000	.002	.005
2.5	-.756	-.656	-.654	-.630	-.558	-.509	-.420	-.323	-.162	-.063	-.036	-.026	-.015	-.006	.001	.005	.008	.018
3.0	-.790	-.563	-.481	-.431	[illegible]	-.318	-.247	-.173	-.068	-.024	-.017	-.006	0.000	.005	.009	.012	.025	.040
3.5	-.853	-.560	-.397	-.309	-.238	-.201	-.142	-.089	-.024	-.008	0.000	.007	.009	.014	.020	.039	.238	7.603
4.0	-.917	-.611	-.386	-.249	-.163	-.128	-.079	-.040	-.003	.006	.015	.018	.023	.032	.077	.389	.078	.152
4.5	-.973	-.685	-.440	-.252	-.133	-.092	.045	-.013	.012	.023	.032	.042	.059	.207	.160	.072	.044	.066
5.0	-1.021	-.759	-.524	-.320	-.171	-.118	-.057	-.021	.010	.036	.063	.227	2.573	.023	.049	.005	-.015	-.032
5.5	-1.062	-.824	-.612	-.426	-.289	-.241	-.193	-.184	-.290	-1.021	-.859	-.356	-.293	-.934	-.369	-1.287	-1.213	-.674
6.0	-1.095	-.877	-.688	-.531	-.430	-.406	-.409	-.465	-.652	-.634	-.542	-.586	-.746	-.651	-.794	-.668	-.774	-.718
6.5	-1.121	-.920	-.751	-.616	-.540	-.528	-.539	-.582	-.651	-.626	-.637	-.705	-.712	-.689	-.730	-.779	-.754	-.769
7.0	-1.141	-.955	-.800	-.681	-.617	-.606	-.610	-.637	-.673	-.663	-.691	-.720	-.707	-.742	-.737	-.751	-.793	-.810
7.5	-1.157	-.984	-.840	-.730	-.671	-.660	-.657	-.676	-.696	-.694	-.723	-.727	-.725	-.755	-.770	-.783	-.782	-.805
8.0	-1.172	-1.007	-.872	-.769	-.712	-.659	-.692	-.707	-.716	-.721	-.742	-.735	-.750	-.755	-.773	-.782	-.801	-.800
8.5	-1.184	-1.026	-.898	-.800	-.744	-.730	-.720	-.732	-.732	-.744	-.752	-.750	-.767	-.765	-.774	-.787	-.796	-.814
9.0	-1.195	-1.043	-.919	-.825	-.770	-.755	-.743	[illegible]	-.746	-.761	-.759	-.766	-.774	-.781	-.789	-.799	-.807	-.814
9.5	-1.203	-1.056	-.937	-.846	-.791	-.775	-.761	-.768	-.759	-.773	-.767	-.789	-.776	-.789	-.795	-.796	-.804	-.811
10.0	-1.210	-1.068	-.953	-.863	-.808	-.792	-.777	-.782	-.771	-.781	-.776	-.787	-.702	-.789	-.794	-.805	-.812	-.819

TABLE XVI IMAGINARY PART OF PRESSURE ADMITTANCE COEFFICIENT $S_{\nu h} = 7$

.5	.615	1.044	1.354	1.636	1.908	2.045	2.262	2.643	3.546	4.474	5.030	5.457	5.673	5.755	5.485	4.872	3.998	3.193
1.0	.771	1.412	1.825	2.149	2.403	2.510	2.652	2.876	3.256	3.439	3.438	3.340	3.199	2.907	2.517	2.183	1.826	1.432
1.5	.635	1.320	1.731	2.011	2.187	2.244	2.301	2.384	2.467	2.400	2.279	2.113	1.963	1.747	1.529	1.313	1.064	.763
2.0	.410	1.069	1.463	1.704	1.828	1.856	1.865	1.879	1.841	1.701	1.567	1.426	1.324	1.183	1.012	0.000	.619	.315
2.5	.200	.787	1.170	1.390	1.486	1.499	1.485	1.465	1.376	1.224	1.113	1.014	.937	.813	.663	.495	.258	-.262
3.0	.041	.522	.888	1.103	1.186	1.192	1.166	1.130	1.023	.886	802	.719	.645	.530	.375	.179	-.226	.540
3.5	-.064	.298	.624	.839	.922	.926	.896	.852	.744	.629	.556	.470	.397	.269	.073	-.298	1.512	8.270
4.0	-.125	.132	.388	.590	.678	.684	.656	.611	.508	.405	.325	.230	.141	-.042	-.504	1.675	-.668	-1.416
4.5	-.156	.022	.198	.355	.438	.446	.423	.380	.281	.169	.066	-.072	-.244	-.919	.848	-.431	.085	-.756
5.0	-.168	-.045	.066	.159	.203	.201	.170	.119	-.003	-.193	-.448	-1.259	4.891	.310	-.763	.329	.725	-.726
5.5	-.172	-.084	-.014	.028	.019	-.007	-.076	-.169	-.413	-.517	.372	.173	-.120	-.596	.116	.303	.393	-.482
6.0	-.173	-.106	-.059	-.043	-.072	-.102	-.163	-.218	-.186	-.004	-.042	-.163	-.150	.032	-.131	-.116	-.140	-.100
6.5	-.171	-.117	-.083	-.076	-.100	-.120	-.148	-.159	-.103	-.066	-.100	-.094	-.039	-.077	-.017	-.022	-.010	-.008
7.0	-.167	-.122	-.096	-.091	-.107	-.118	-.131	-.132	-.088	-.078	-.090	-.060	-.048	-.064	-.052	-.037	-.040	-.015
7.5	-.161	-.124	-.102	-.097	-.106	-.112	-.120	-.118	-.079	-.079	-.075	-.050	-.059	-.041	-.046	-.040	-.021	-.030
8.0	-.154	-.124	-.104	-.099	-.104	-.107	-.112	-.107	-.073	-.077	-.062	-.052	-.059	-.039	-.030	-.025	-.030	-.018
8.5	-.148	-.122	.105	-.099	-.101	-.103	-.105	-.099	-.069	-.072	-.052	-.055	-.049	-.045	-.036	-.035	-.021	-.015
9.0	-.143	-.119	-.104	-.097	-.097	-.098	-.100	-.092	-.067	-.065	-.049	-.054	-.040	-.043	-.037	-.026	-.026	-.022
9.5	-.138	-.116	-.102	-.095	-.094	-.094	-.095	-.086	-.065	-.057	-.049	-.048	-.038	-.034	-.028	-.025	-.019	-.016
10.0	-.133	-.113	-.100	-.093	-.091	-.091	-.090	-.080	-.064	-.051	-.050	-.041	-.041	-.031	-.028	-.028	-.023	-.014

TABLE XVII REAL PART OF PRESSURE ADMITTANCE COEFFICIENT $S\nu h$ = 9

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-2.619	-3.681	-4.352	-4.802	-5.069	-5.148	-5.216	-5.393	-5.530	-5.219	-4.75[illegible]	-4.124	-3.541	-2.616	-1.590	-.766	-.245	-.080
1.0	-1.865	-2.572	-2.968	-3.156	-3.164	-3.116	-2.989	-2.819	-2.352	-1.782	-1.387	-1.016	-.749	-.423	-.185	-.088	-.039	-.015
1.5	-1.273	-1.705	-1.924	-1.982	-1.906	-1.831	-1.682	-1.486	-1.081	-.705	-.486	-.309	-.203	-.106	-.055	-.028	-.012	-.004
2.0	-.918	-1.153	-1.274	-1.280	-1.190	-1.120	-.991	-.830	-.536	-.301	-.186	-.111	-.075	-.044	-.023	-.011	-.004	0.000
2.5	-.743	-.815	-.877	-.861	-.775	-.716	-.609	-.484	-.275	-.135	-.081	-.052	-.036	-.020	-.009	-.003	.001	.003
3.0	-.690	-.612	-.627	-.599	-.523	-.472	-.386	-.289	-.144	-.065	-.042	-.026	-.017	-.008	-.002	.002	.005	.010
3.5	-.709	-.500	-.466	-.430	-.361	-.319	-.248	-.175	-.075	-.035	-.022	-.011	-.006	0.000	.004	.007	.011	.563
4.0	-.754	-.461	-.364	-.316	-.254	-.218	-.160	-.105	-.040	-.018	-.008	-.002	.002	.006	.009	.014	.092	.052
4.5	-.830	-.480	-.310	-.239	-.180	-.149	-.103	-.061	-.019	-.006	.002	.006	.009	.013	.019	.061	.035	.028
5.0	-.894	-.539	-.304	-.192	-.129	-.102	-.064	-.033	-.006	.005	.010	.014	.018	.026	.078	.101	5.265	.037
5.5	-.952	-.615	-.345	-.180	-.098	-.071	-.057	-.013	.005	.016	.021	.028	.035	.108	.167	.092	.055	.076
6.0	-1.001	-.692	-.42[illegible]	-.214	-.095	-.060	-.023	0.000	.017	.031	.045	.083	.298	.174	.082	.063	.047	.157
6.5	-1.039	-.759	-.509	-.296	-.148	-.098	-.045	-.014	.015	.052	.241	.133	.018	.034	.004	-.235	-.046	-.132
7.0	-1.069	-.816	-.592	-.401	-.267	-.224	-.189	-.201	-.477	-.869	-.366	-.314	-.711	-.484	-1.152	-.653	-1.199	- 777
7.5	-1.094	-.863	-.663	-.499	-.401	-.381	-.398	-.473	-.628	-.531	-.544	-.713	-.669	-.649	-.629	-.675	-.768	-.858
8.0	-1.116	-.902	-.720	-.578	-.503	-.494	-.512	-.559	-.607	-.597	-.651	-.676	-.654	-.725	-.733	-.757	-.752	-.804
8.5	-1.136	-.934	-.767	-.641	-.576	-.567	-.576	-.605	-.633	-.644	-.681	-.681	-.693	-.716	-.746	-.758	-.784	-.784
9.0	-1.152	-.961	-.806	-.689	-.630	-.619	-.621	-.642	-.659	-.677	-.700	-.697	-.722	-.724	-.742	-.760	-.776	-.799
9.5	-1.164	-.984	-.838	-.729	-.671	-.659	-.656	-.673	-.681	-.703	-.711	-.719	-.737	-.746	-.761	-.779	-.793	-.808
10.0	-1.175	-1.003	-.864	-.761	-.704	-.691	-.685	-.69[illegible]	-.700	-.722	-.722	-.739	-.742	-.760	-.772	-.776	-.789	-.802

TABLE XVIII IMAGINARY PART OF PRESSURE ADMITTANCE COEFFICIENT $S\nu h$ = 9

.5	.935	1.448	1.821	2.164	2.498	2.665	2.933	3.454	4.652	5.810	6.475	6.969	7.211	7.297	6.992	6.316	5.276	4.205
1.0	1.252	1.993	2.474	2.851	3.147	3.269	3.433	3.728	4.203	4.411	4.406	4.295	4.141	3.820	3.364	2.920	2.460	1.964
1.5	1.156	1.928	2.387	2.696	2.887	2.947	3.002	3.110	3.211	3.134	3.002	2.820	2.652	2.386	2.097	1.824	1.519	1.167
2.0	.912	1.662	2.080	2.332	2.454	2.478	2.479	2.499	2.458	2.303	2.155	1.990	1.859	1.673	1.464	1.255	1.010	.709
2.5	.642	1.361	1.751	1.966	2.051	2.057	2.031	2.010	1.913	1.743	1.611	1.482	1.384	1.236	1.063	.884	.661	.352
3.0	.397	1.072	1.449	1.644	1.709	1.706	1.667	1.627	1.507	1.348	1.241	1.138	1.054	.926	.770	.600	.367	-.072
3.5	.201	.804	1.178	1.367	1.421	1.414	1.370	1.320	1.196	1.057	.968	.876	.800	.681	.528	.346	.042	2.957
4.0	.065	.562	.932	1.123	1.176	1.167	1.121	1.067	.947	.827	.747	.659	.586	.466	.299	.063	-.879	-.683
4.5	-.022	.356	.701	.903	.960	.952	.907	.853	.740	.632	.553	.466	.389	.253	.031	-.526	.216	-.084
5.0	-.073	.197	.487	.694	.762	.756	.715	.663	.557	.452	.368	.273	.181	-.007	-.53[illegible]	.723	-5.618	.338
5.5	-.103	.085	.300	.490	.569	.570	.534	.484	.382	.268	.171	.044	-.101	-.592	.890	-.618	-.398	.852
6.0	-.122	.013	.156	.296	.371	.376	.347	.299	.190	.046	-.104	-.380	-1.051	.811	-.426	.324	.224	1.464
6.5	-.133	-.033	.059	.137	.171	.166	.128	.066	-.092	-.425	-1.391	1.412	.290	-.530	.476	5.214	.864	1.313
7.0	-.137	-.061	-.001	.033	.017	-.013	-.092	-.204	-.481	.251	.165	-.191	-.515	.274	-.374	-.459	.299	.417
7.5	-.137	-.078	-.036	-.024	-.056	-.088	-.150	-.193	-.092	-.023	-.127	-.112	.027	-.134	-.010	.028	-.117	.031
8.0	-.135	-.088	-.057	-.052	-.079	-.099	-.126	-.130	-.072	-.078	-.091	-.040	-.050	-.044	-.062	-.055	-.009	-.039
8.5	-.132	-.093	-.069	-.066	-.085	-.096	-.110	-.109	-.070	-.077	-.067	-.045	-.061	-.032	-.028	-.019	-.032	-.023
9.0	-.130	-.096	-.076	-.073	-.085	-.092	-.101	-.098	-.067	-.073	-.053	-.051	-.053	-.042	-.032	-.034	-.019	-.010
9.5	-.128	-.097	-.080	-.076	-.084	-.089	-.094	-.090	-.064	-.066	-.047	-.054	-.040	-.044	-.038	-.026	-.025	-.019
10.0	-.124	-.097	-.082	-.078	-.083	-.086	-.089	-.083	-.062	-.058	-.046	-.049	-.036	-.034	-.027	-.024	-.019	-.019

TABLE XIX REAL PART OF RADIAL VELOCITY ADMITTANCE COEFFICIENT Sνh = 1

ω \ ν	902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	.636	.625	.609	.583	.544	.519	.472	.428	.342	.222	.114	-.012	-.113	-.216	-.130	.138	-.114	.111
1.0	.434	.400	.355	.292	.213	.169	.098	.036	-.059	-.122	-.119	-.051	.043	.091	-.133	-.018	-.005	-.056
1.5	.281	.241	.192	.130	.063	.032	-.008	-.031	-.035	.005	.037	.003	-.040	.036	-.014	.003	.002	.008
2.0	.187	.153	.112	.066	.026	.011	-.003	-.006	.002	.002	-.006	-.001	.004	-.003	-.004	.004	.001	-.002
2.5	.131	.103	.073	.041	.017	.009	.003	.001	0.000	-.001	-.001	-.001	-.001	-.001	-.001	-.001	0.000	0.000
3.0	.095	.074	.051	.029	.013	.008	.003	.001	0.000	0.000	-.001	0.000	-.001	-.001	0.000	0.000	0.000	0.000
3.5	.072	.055	.038	.021	.010	.006	.002	.001	0.000	-.001	0.000	-.001	0.000	0.000	0.000	0.000	0.000	0.000
4.0	.056	.043	.029	.016	.008	.005	.002	.001	0.000	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.5	.045	.034	.023	.013	.006	.004	.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.0	.037	.028	.019	.011	.005	.003	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.5	.031	.023	.016	.009	.004	.003	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.0	.026	.019	.013	.007	.004	.002	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.5	.022	.017	.011	.006	.003	.002	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.0	.019	.014	.010	.005	.003	.002	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	.017	.013	.009	.005	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	.015	.011	.007	.004	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.5	.013	.010	.007	.004	.002	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.0	.012	.009	.006	.003	.002	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.5	.011	.008	.005	.003	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	.010	.007	.005	.003	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XX IMAGINARY PART OF RADIAL VELOCITY ADMITTANCE COEFFICIENT Sνh = 1

ω \ ν	902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.271	-.294	-.322	-.357	-.395	-.413	-.440	-.466	-.513	-.546	-.545	-.506	-.434	-.245	.022	-.038	.011	-.006
1.0	-.374	-.390	-.407	-.422	-.427	-.423	-.406	-.382	-.314	-.198	-.096	-.008	.004	-.141	-.010	-.251	.248	.084
1.5	-.368	-.366	-.360	-.343	-.308	-.283	-.236	-.190	-.114	-.070	-.084	-.110	-.071	-.035	-.005	.002	.012	-.042
2.0	-.328	-.316	-.297	-.265	-.219	-.193	-.153	-.124	-.093	-.083	-.069	-.050	-.051	-.034	-.025	-.022	-.009	-.003
2.5	-.288	-.271	-.246	-.212	-.170	-.150	-.121	-.102	-.080	-.063	-.053	-.045	-.038	-.031	-.022	-.016	-.010	-.006
3.0	-.253	-.234	-.209	-.177	-.141	-.124	-.101	-.086	-.067	-.053	-.045	-.037	-.032	-.026	-.019	-.013	-.009	-.005
3.5	-.224	-.205	-.181	-.152	-.121	-.107	-.087	-.074	-.057	-.046	-.038	-.032	-.028	-.022	-.016	-.012	-.007	-.004
4.0	-.201	-.182	-.160	-.133	-.106	-.093	-.076	-.065	-.050	-.040	-.034	-.028	-.024	-.019	-.014	-.010	-.006	-.004
4.5	-.181	-.164	-.143	-.118	-.094	-.083	-.068	-.058	-.045	-.035	-.030	-.025	-.021	-.017	-.013	-.009	-.006	-.003
5.0	-.165	-.148	-.129	-.107	-.085	-.075	-.061	-.052	-.040	-.032	-.027	-.023	-.019	-.015	-.011	-.008	-.005	-.003
5.5	-.151	-.136	-.118	-.097	-.077	-.068	-.056	-.047	-.037	-.029	-.024	-.020	-.018	-.014	-.010	-.007	-.005	-.003
6.0	-.139	-.125	-.108	-.089	-.071	-.062	-.051	-.043	-.034	-.027	-.022	-.019	-.016	-.013	-.009	-.007	-.004	-.002
6.5	-.129	-.116	-.100	-.082	-.065	-.058	-.047	-.040	-.031	-.024	-.021	-.017	-.015	-.012	-.009	-.006	-.004	-.002
7.0	-.120	-.108	-.093	-.077	-.061	-.054	-.044	-.037	-.029	-.023	-.019	-.016	-.014	-.011	-.008	-.006	-.004	-.002
7.5	-.113	-.101	-.087	-.071	-.057	-.0 0	-.041	-.035	-.027	-.021	-.018	-.015	-.013	-.010	-.008	-.005	-.003	-.002
8.0	-.106	-.095	-.082	-.067	-.053	-.047	-.038	-.032	-.025	-.020	-.017	-.014	-.012	-.010	-.007	-.005	-.003	-.002
8.5	-.100	-.089	-.077	-.063	-.050	-.044	-.036	-.031	-.024	-.019	-.016	-.013	-.011	-.009	-.007	-.005	-.003	-.002
9.0	-.095	-.084	-.073	-.060	-.047	-.042	-.034	-.029	-.022	-.018	-.015	-.013	-.011	-.009	-.006	-.004	-.003	-.002
9.5	-.090	-.080	-.069	-.056	-.045	-.039	-.032	-.027	-.021	-.017	-.014	-.012	-.010	-.008	-.006	-.004	-.003	-.002
10.0	-.085	-.076	-.065	-.054	-.042	-.037	-.031	-.026	-.020	-.016	-.013	-.011	-.010	-.008	-.006	-.004	-.003	-.001

TABLE XXI REAL PART OF RADIAL VELOCITY ADMITTANCE COEFFICIENT $S_{\nu h}$ = 2

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	2.543	2.488	2.407	2.280	2.104	1.999	1.820	1.666	1.383	1.010	.688	.328	.045	-.282	-.325	.048	-.109	.022
1.0	1.741	1.615	1.459	1.251	.996	.857	.632	.429	.090	-.195	-.304	-.286	-.160	.102	-.071	.096	.016	.133
1.5	1.128	.979	.801	.580	.338	.220	.051	-.075	-.200	-.158	-.023	.105	.052	-.142	.045	.031	-.220	-.024
2.0	.750	.616	.460	.280	.107	.035	-.044	-.075	-.039	.041	.004	-.070	.044	-.078	-.057	.029	-.066	-.041
2.5	.523	.414	.292	.163	.056	.021	-.007	-.009	.004	-.009	-.006	.003	-.011	-.002	-.008	0.000	-.005	.006
3.0	.361	.296	.203	.111	.044	[illegible]	.007	.002	-.002	-.002	-.003	-.002	-.003	-.001	-.001	0.000	0.000	0.000
3.5	.288	.221	.150	.083	.036	[illegible]	.008	.002	-.001	-.002	-.002	-.002	-.001	-.001	-.001	0.000	0.000	0.000
4.0	.226	.171	.115	.064	.029	.011	[illegible]	.002	0.000	-.002	-.001	-.002	-.001	-.001	-.001	-.001	0.000	-.001
4.5	.181	.137	.092	.051	.024	.015	[illegible]	.001	0.000	-.002	-.001	-.001	-.001	0.000	0.000	-.001	0.000	0.000
5.0	.148	.111	.075	.042	.019	.012	.005	.001	0.000	-.001	-.001	-.001	-.001	-.001	0.000	0.000	0.000	0.000
5.5	.123	.092	.062	.035	.016	.010	.004	.001	0.000	-.001	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000
6.0	.104	.078	.052	.029	.014	.009	.004	.001	0.000	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.5	.089	.066	.044	.025	.012	.008	.003	.001	0.000	-.001	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000
7.0	.077	.057	.038	.022	.010	.007	.003	.001	0.000	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000
7.5	.068	.050	.034	.019	.009	.006	.002	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	.059	.044	.030	.017	.008	.005	.002	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.5	.053	.039	.026	.015	.007	.005	.002	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.0	.047	.035	.023	.013	.006	.004	.002	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.5	.042	.032	.021	.012	.006	.004	.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	.039	.029	.019	.011	.005	.003	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XXII IMAGINARY PART OF RADIAL VELOCITY ADMITTANCE COEFFICIENT $S_{\nu h}$ = 2

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-1.079	-1.144	-1.209	-1.277	-1.340	-1.369	-1.410	-1.472	-1.609	-1.709	-1.711	-1.617	-1.450	-1.040	-.439	-.175	-.244	-.067
1.0	-1.494	-1.544	-1.587	-1.614	-1.606	-1.585	-1.525	-1.463	-1.301	-1.008	-.728	-.423	-.231	-.208	-.341	-.142	-.197	-.048
1.5	-1.470	-1.466	-1.445	-1.390	-1.282	-1.206	-1.059	-.911	-.624	-.332	-.216	-.261	-.366	-.152	-.330	-.691	-.033	.118
2.0	-1.314	-1.270	-1.199	-1.085	-.922	-.825	-.662	-.523	-.342	-.297	-.318	-.206	-.132	-.159	-.124	-.046	-.033	-.050
2.5	-1.152	-1.086	-.993	-.861	-.697	-.611	-.485	-.399	-.315	-.258	-.206	-.185	-.155	-.129	-.093	-.072	-.037	-.022
3.0	-1.012	-.938	-.841	-.712	-.568	-.499	-.404	-.342	-.267	-.211	-.180	-.149	-.130	-.102	-.075	-.053	-.034	-.019
3.5	-.897	-.822	-.727	-.609	-.485	-.427	-.349	-.295	-.228	-.182	-.153	-.129	-.110	-.087	-.064	-.047	-.029	-.017
4.0	-.803	-.729	-.641	-.533	-.424	-.374	-.306	-.259	-.201	-.159	-.134	-.112	-.097	-.076	-.057	-.041	-.026	-.015
4.5	-.724	-.655	-.572	-.474	-.377	-.332	-.272	-.230	-.179	-.141	-.120	-.100	-.086	-.068	-.051	-.036	-.023	-.013
5.0	-.658	-.593	-.517	-.427	-.339	-.299	-.245	-.207	-.161	-.127	-.107	-.090	-.078	-.061	-.045	-.032	-.021	-.012
5.5	-.603	-.542	-.472	-.389	-.309	-.272	-.223	-.188	-.147	-.116	-.098	-.082	-.071	-.056	-.041	-.029	-.019	-.011
6.0	-.557	-.499	-.433	-.357	-.283	-.250	-.204	-.173	-.135	-.106	-.090	-.075	-.065	-.051	-.038	-.027	-.017	-.010
6.5	-.517	-.462	-.400	-.330	-.261	-.231	-.189	-.159	-.124	-.098	-.083	-.069	-.060	-.047	-.035	-.025	-.016	-.009
7.0	-.482	-.430	-.372	-.306	-.243	-.214	-.175	-.148	-.115	-.091	-.077	-.064	-.055	-.044	-.032	-.023	-.015	-.008
7.5	-.451	-.403	-.348	-.286	-.227	-.200	-.164	-.138	-.108	-.085	-.072	-.060	-.052	-.041	-.030	-.022	-.014	-.008
8.0	-.423	-.378	-.326	-.268	-.212	-.187	-.153	-.130	-.101	-.080	-.067	-.056	-.049	-.038	-.028	-.020	-.013	-.007
8.5	-.399	-.356	-.307	-.252	-.200	-.176	-.144	-.122	-.095	-.075	-.063	-.053	-.046	-.036	-.027	-.019	-.012	-.007
9.0	-.378	-.337	-.290	-.238	-.189	-.167	-.136	-.115	-.090	-.071	-.060	-.050	-.043	-.034	-.025	-.018	-.012	-.006
9.5	-.359	-.319	-.275	-.226	-.179	-.158	-.129	-.109	-.085	-.067	-.057	-.047	-.041	-.032	-.024	-.017	-.011	-.006
10.0	-.341	-.304	-.262	-.215	-.170	-.150	-.123	-.104	-.081	-.064	-.054	-.045	-.039	-.031	-.023	-.016	-.010	-.006

TABLE XXIII REAL PART OF RADIAL VELOCITY ADMITTANCE COEFFICIENT $S_{\nu h}$ = 3

M \	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	5.716	5.551	5.309	4.951	4.498	4.247	3.840	3.518	2.954	2.235	1.638	.995	.499	-.101	-.380	-.112	-.054	-.017
1.0	3.933	3.670	3.351	2.924	2.406	2.127	1.684	1.284	.612	.028	-.252	-.367	-.313	-.050	.002	.008	.019	.007
1.5	2.551	2.247	1.903	1.485	1.027	.796	.456	.173	-.194	-.317	-.222	-.029	.065	-.079	.048	.043	.026	.431
2.0	1.694	1.410	1.088	.720	.355	.191	-.022	-.159	-.225	-.072	.049	-.006	-.106	.072	.051	.417	.064	-.088
2.5	1.177	.940	.672	.380	.121	.020	-.080	-.109	-.037	.023	-.060	-.011	.068	.116	.023	.037	-.046	-.088
3.0	.856	.667	.457	.242	.074	.021	-.017	-.017	-.001	-.016	.004	-.014	0.000	-.005	.003	-.006	-.006	.004
3.5	.649	.497	.334	.177	.066	.034	.008	0.000	-.005	-.004	-.006	-.005	-.004	-.003	-.003	0.000	-.001	.001
4.0	.508	.385	.257	.139	.058	.034	.012	.001	-.002	-.005	-.003	-.004	-.003	-.003	-.002	-.001	-.001	-.001
4.5	.408	.306	.204	.112	.049	.030	.011	.002	-.001	-.004	-.003	-.002	-.003	-.001	-.001	-.001	-.001	0.000
5.0	.333	.250	.166	.092	.041	.026	.010	.002	-.001	-.002	-.003	-.002	-.001	-.002	0.000	-.001	-.001	0.000
5.5	.277	.207	.138	.077	.035	.022	.009	.002	-.001	-.002	-.002	-.002	-.002	-.001	-.001	0.000	0.000	0.000
6.0	.234	.175	.116	.065	.030	.019	.008	.002	-.001	-.002	-.001	-.001	-.001	-.001	-.001	0.000	0.000	0.000
6.5	.201	.149	.100	.056	.026	.016	.007	.002	-.001	-.002	-.001	-.001	-.001	-.001	-.001	0.000	0.000	0.000
7.0	.174	.129	.086	.048	.023	.014	.006	.002	-.001	-.001	-.001	-.001	-.001	-.001	0.000	0.000	0.000	0.000
7.5	.152	.113	.075	.042	.020	.013	.005	.001	-.001	-.001	-.001	-.001	-.001	-.001	0.000	0.000	0.000	0.000
8.0	.134	.099	.066	.037	.018	.011	.005	.001	-.001	-.001	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000
8.5	.118	.088	.059	.033	.016	.010	.004	.001	-.001	-.001	-.001	-.001	-.001	-.001	0.000	0.000	0.000	0.000
9.0	.106	.079	.052	.030	.014	.009	.004	.001	0.000	-.001	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000
9.5	.096	.071	.047	.027	.013	.008	.003	.001	0.000	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000
10.0	.087	.064	.043	.024	.011	.007	.003	.001	0.000	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XXIV IMAGINARY PART OF RADIAL VELOCITY ADMITTANCE COEFFICIENT $S_{\nu h}$ = 3

.5	-2.411	-2.484	-2.540	-2.586	-2.625	-2.643	-2.066	-2.770	-3.038	-3.229	-3.231	-3.070	-2.801	-2.179	-1.272	-.618	-.534	-.243
1.0	-3.346	-3.410	-3.428	-3.388	-3.281	-3.203	-3.050	-2.935	-2.674	-2.206	-1.759	-1.258	-.887	-.568	-.592	-.320	-.248	-.117
1.5	-3.303	-3.289	-3.226	-3.085	-2.848	-2.694	-2.416	-2.155	-1.649	-1.077	-.735	-.557	-.572	-.523	-.337	-.233	-.111	1.085
2.0	-2.958	-2.868	-2.727	-2.506	-2.194	-2.008	-1.690	-1.400	-.931	-.605	-.565	-.589	-.447	-.331	-.153	-.531	-.210	.196
2.5	-2.594	-2.455	-2.264	-1.994	-1.653	-1.465	-1.169	-.933	-.657	-.585	-.515	-.315	-.425	-.261	-.258	-.107	-.187	-.031
3.0	-2.280	-2.118	-1.909	-1.630	-1.307	-1.143	-.912	-.755	-.596	-.471	-.396	-.344	-.285	-.221	-.176	-.129	-.085	-.051
3.5	-2.020	-1.853	-1.646	-1.382	-1.098	-.963	-.782	-.660	-.514	-.409	-.345	-.290	-.248	-.195	-.146	-.103	-.066	-.037
4.0	-1.806	-1.643	-1.4[illegible]	-1.204	-.956	-.842	-.687	-.581	-.451	-.358	-.302	-.252	-.219	-.172	-.128	-.092	-.058	-.033
4.5	-1.629	-1.474	-1.[illegible]	-1.070	-.849	-.748	-.612	-.517	-.402	-.317	-.269	-.225	-.193	-.153	-.114	-.081	-.052	-.029
5.0	-1.481	-1.336	-1.[illegible]	-.963	-.764	-.674	-.551	-.466	-.363	-.286	-.242	-.203	-.175	-.137	-.102	-.072	-.047	-.026
5.5	-1.357	-1.221	-1.061	-.876	-.694	-.613	-.501	-.424	-.330	-.260	-.220	-.184	-.159	-.125	-.092	-.066	-.043	-.024
6.0	-1.253	-1.123	-.974	-.803	-.637	-.562	-.460	-.389	-.303	-.239	-.201	-.169	-.145	-.114	-.085	-.060	-.039	-.022
6.5	-1.163	-1.040	-.901	-.742	-.588	-.519	-.424	-.359	-.280	-.220	-.187	-.156	-.135	-.106	-.078	-.056	-.036	-.020
7.0	-1.084	-.969	-.837	-.689	-.546	-.482	-.394	-.333	-.259	-.204	-.173	-.145	-.124	-.098	-.073	-.052	-.033	-.019
7.5	-1.014	-.906	-.782	-.643	-.510	-.450	-.368	-.311	-.242	-.191	-.161	-.135	-.117	-.092	-.068	-.048	-.031	-.017
8.0	-.953	-.851	-.734	-.603	-.478	-.422	-.345	-.292	-.227	-.179	-.151	-.127	-.109	-.086	-.063	-.045	-.029	-.016
8.5	-.899	-.802	-.691	-.568	-.450	-.397	-.325	-.275	-.214	-.169	-.143	-.119	-.103	-.081	-.060	-.043	-.028	-.016
9.0	-.851	-.758	-.653	-.537	-.425	-.375	-.307	-.259	-.202	-.159	-.135	-.113	-.097	-.077	-.057	-.040	-.026	-.015
9.5	-.807	-.719	-.619	-.508	-.403	-.355	-.291	-.246	-.191	-.151	-.127	-.107	-.092	-.072	-.054	-.038	-.025	-.014
10.0	-.768	-.683	-.589	-.483	-.382	-.338	-.276	-.233	-.182	-.143	-.121	-.101	-.087	-.069	-.051	-.036	-.023	-.013

TABLE XXV REAL PART OF RADIAL VELOCITY ADMITTANCE COEFFICIENT $S_{\nu}h$ = 4

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	10.145	9.765	9.236	8.502	7.632	7.171	6.446	5.904	4.979	3.823	2.893	1.918	1.175	.261	-.307	-.229	-.036	-.025
1.0	7.018	6.564	5.996	5.232	4.332	3.860	3.127	2.475	1.392	.453	-.024	-.301	-.352	-.180	-.008	-.038	-.003	-.005
1.5	4.563	4.081	3.543	2.870	2.123	1.745	1.104	.701	.033	-.310	-.317	-.165	-.027	-.040	-.007	-.004	-.008	.003
2.0	3.025	2.566	2.067	1.503	.926	.654	.277	-.009	-.278	-.217	-.059	.007	-.061	-.011	-.028	.015	-.035	-.037
2.5	2.096	1.698	1.257	.782	.343	.158	-.065	-.191	-.193	-.025	-.006	-.082	-.018	-.054	-.065	.408	-.003	.041
3.0	1.523	1.194	.827	.444	.122	.004	-.108	-.134	-.050	-.019	-.074	.026	-.049	-.263	.022	.019	.072	.046
3.5	1.154	.885	.591	.300	.079	.011	-.038	-.036	-.010	-.020	.010	-.016	-.003	-.002	.002	-.009	.012	-.006
4.0	.904	.683	.451	.231	.079	.035	.003	-.006	-.006	-.008	-.007	-.008	-.006	-.004	-.002	-.001	-.001	0.000
4.5	.726	.543	.357	.189	.075	.042	.012	-.001	-.003	-.007	-.006	-.004	-.005	-.002	-.001	-.002	-.002	-.001
5.0	.592	.442	.292	.157	.067	.040	.013	.001	-.003	-.005	-.006	-.004	-.003	-.003	-.001	-.002	-.001	.001
5.5	.491	.367	.243	.132	.058	.035	.013	.002	-.002	-.004	-.004	-.004	-.003	-.002	-.001	-.001	0.000	0.000
6.0	.415	.309	.205	.113	.051	.031	.012	.002	-.002	-.003	-.003	-.002	-.002	-.002	-.002	0.000	0.000	0.000
6.5	.356	.264	.175	.097	.045	.027	.011	.002	-.002	-.003	-.002	-.002	-.002	-.001	-.001	0.000	0.000	0.000
7.0	.310	.229	.152	.085	.039	.024	.010	.002	-.002	-.003	-.003	-.002	-.002	-.001	-.001	0.000	0.000	0.000
7.5	.271	.200	.133	.074	.035	.021	.009	.002	-.002	-.002	-.002	-.001	-.001	-.001	0.000	-.001	0.000	0.000
8.0	.237	.176	.117	.065	.031	.019	.008	.002	-.001	-.002	-.002	-.002	-.001	-.001	-.001	0.000	0.000	0.000
8.5	.210	.157	.104	.058	.027	.017	.007	.002	-.001	-.001	-.001	-.001	-.001	-.001	-.001	0.000	0.000	0.000
9.0	.188	.140	.093	.052	.024	.016	.006	.002	-.001	-.001	-.002	-.001	-.001	-.001	-.001	0.000	0.000	0.000
9.5	.170	.126	.084	.047	.022	.014	.006	.002	0.000	-.001	-.001	-.001	-.001	-.001	0.000	0.000	0.000	0.000
10.0	.154	.114	.076	.042	.020	.013	.005	.002	0.000	-.001	-.001	-.001	-.001	-.001	0.000	0.000	0.000	0.000

TABLE XXVI IMAGINARY PART OF RADIAL VELOCITY ADMITTANCE COEFFICIENT $S_{\nu}h$ = 4

.5	-4.246	-4.271	-4.270	-4.255	-4.237	-4.227	-4.212	-4.373	-4.817	-5.112	-5.103	-4.853	-4.464	-3.607	-2.375	-1.321	-.890	-.467
1.0	-5.910	-5.927	-5.834	-5.639	-5.359	-5.192	-4.903	-4.721	-4.349	-3.683	-3.060	-2.361	-1.817	-1.200	-.946	-.629	-.415	-.230
1.5	-5.860	-5.801	-5.619	-5.286	-4.820	-4.548	-4.086	-3.695	-2.959	-2.116	-1.564	-1.156	-.998	-.879	-.585	-.417	-.269	-.150
2.0	-5.261	-5.110	-4.860	-4.464	-3.928	-3.620	-3.110	-2.660	-1.903	-1.262	-1.013	-.933	-.831	-.580	-.428	-.318	-.216	-.111
2.5	-4.617	-4.388	-4.081	-3.647	-3.097	-2.792	-2.302	-1.886	-1.290	-.970	-.901	-.734	-.575	-.508	-.366	-.287	-.116	-.224
3.0	-4.058	-3.784	-3.440	-2.983	-2.442	-2.158	-1.729	-1.398	-1.023	-.872	-.705	-.558	-.578	-.639	-.256	-.170	-.151	.035
3.5	-3.595	-3.306	-2.953	-2.500	-1.996	-1.748	-1.398	-1.158	-.908	-.708	-.617	-.507	-.455	-.357	-.246	-.192	-.123	-.079
4.0	-3.213	-2.928	-2.585	-2.160	-1.712	-1.501	-1.218	-1.026	-.800	-.636	-.538	-.450	-.388	-.306	-.227	-.162	-.104	-.059
4.5	-2.896	-2.625	-2.301	-1.910	-1.513	-1.331	-1.086	-.918	-.715	-.564	-.479	-.399	-.344	-.272	-.202	-.144	-.093	-.051
5.0	-2.633	-2.377	-2.074	-1.716	-1.359	-1.198	-.980	-.827	-.645	-.508	-.430	-.361	-.311	-.244	-.181	-.129	-.084	-.047
5.5	-2.414	-2.171	-1.888	-1.559	-1.235	-1.090	-.891	-.753	-.587	-.463	-.390	-.327	-.282	-.223	-.164	-.117	-.076	-.043
6.0	-2.228	-1.998	-1.733	-1.429	-1.132	-.999	-.817	-.691	-.538	-.424	-.358	-.300	-.258	-.204	-.151	-.107	-.069	-.039
6.5	-2.067	-1.850	-1.602	-1.319	-1.045	-.922	-.754	-.638	-.497	-.392	-.331	-.278	-.239	-.189	-.139	-.099	-.064	-.036
7.0	-1.926	-1.723	-1.489	-1.226	-.971	-.857	-.701	-.592	-.461	-.363	-.307	-.257	-.221	-.174	-.129	-.093	-.059	-.034
7.5	-1.802	-1.611	-1.391	-1.144	-.906	-.799	-.654	-.553	-.430	-.339	-.287	-.240	-.207	-.163	-.121	-.086	-.055	-.031
8.0	-1.693	-1.513	-1.305	-1.073	-.850	-.750	-.614	-.518	-.403	-.318	-.269	-.225	-.194	-.153	-.113	-.081	-.052	-.029
8.5	-1.598	-1.426	-1.229	-1.010	-.800	-.705	-.577	-.498	-.380	-.300	-.253	-.212	-.183	-.144	-.106	-.076	-.049	-.028
9.0	-1.513	-1.348	-1.162	-.954	-.755	-.666	-.545	-.461	-.359	-.283	-.239	-.200	-.173	-.136	-.101	-.072	-.046	-.026
9.5	-1.436	-1.278	-1.101	-.904	-.716	-.632	-.517	-.437	-.340	-.268	-.226	-.190	-.163	-.129	-.095	-.068	-.044	-.024
10.0	-1.365	-1.215	-1.047	-.859	-.680	-.600	-.491	-.415	-.323	-.254	-.215	-.180	-.155	-.123	-.090	-.065	-.042	-.024

TABLE XXVII REAL PART OF RADIAL VELOCITY ADMITTANCE COEFFICIENT $S_{\nu h}$ = 5

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	15.813	15.094	14.149	12.901	11.480	10.747	9.617	8.305	7.437	5.751	4.428	3.068	2.045	.772	-.125	-.281	-.056	-.028
1.0	10.996	10.268	9.340	8.116	6.721	6.006	4.917	3.959	2.385	1.042	.342	-.118	-.292	-.255	-.059	-.058	-.018	-.007
1.5	7.173	6.486	5.682	4.665	3.539	2.979	2.151	1.430	.411	-.182	-.315	-.248	-.123	-.048	-.041	-.022	-.011	-.003
2.0	4.751	4.116	3.426	2.620	1.769	1.361	.785	.320	-.197	-.284	-.159	-.043	-.052	-.047	-.032	-.008	-.007	.002
2.5	3.285	2.716	2.108	1.454	.818	.532	.158	-.101	-.262	-.118	-.032	-.067	-.057	-.029	-.021	.003	.020	-.013
3.0	2.383	1.890	1.357	.810	.330	.136	-.084	-.195	-.156	-.032	-.062	-.046	-.009	-.005	.008	.051	-.015	.011
3.5	1.805	1.388	.938	.486	.122	-.006	-.123	-.148	-.062	-.051	-.056	-.003	-.055	-.139	-.048	-.004	-.030	.018
4.0	1.413	1.065	.699	.341	.075	-.007	-.065	-.064	-.027	-.031	.016	.010	-.027	-.019	-.019	-.011	-.003	-.013
4.5	1.134	.845	.550	.272	.082	.028	-.009	-.016	-.009	-.008	-.011	-.009	-.008	-.004	-.003	-.002	-.002	0.000
5.0	.925	.687	.448	.230	.085	.044	.010	-.004	-.005	-.008	-.010	-.007	-.004	-.005	-.001	-.003	0.000	.001
5.5	.767	.571	.373	.197	.080	.046	.014	0.000	-.005	-.006	-.007	-.006	-.005	-.003	-.002	-.001	0.000	0.000
6.0	.648	.481	.316	.170	.072	.043	.014	.002	-.004	-.006	-.005	-.004	-.004	-.003	-.002	-.001	0.000	-.001
6.5	.557	.412	.271	.147	.065	.039	.014	.002	-.004	-.005	-.004	-.003	-.003	-.002	-.001	0.000	-.001	0.000
7.0	.484	.356	.235	.129	.058	.035	.013	.002	-.004	-.005	-.004	-.004	-.003	-.002	-.001	0.000	-.001	0.000
7.5	.423	.312	.206	.113	.052	.031	.012	.002	-.003	-.003	-.003	-.002	-.002	-.002	-.001	-.001	-.001	-.001
8.0	.370	.275	.182	.101	.046	.028	.011	.003	-.002	-.003	-.002	-.002	-.002	-.001	-.001	-.001	0.000	0.000
8.5	.328	.244	.161	.090	.041	.026	.010	.003	-.002	-.002	-.002	-.002	-.002	-.001	-.001	-.001	0.000	0.000
9.0	.294	.219	.144	.080	.037	.024	.009	.002	-.001	-.002	-.002	-.002	-.002	-.001	-.001	0.000	0.000	0.000
9.5	.266	.196	.130	.072	.034	.021	.009	.002	-.001	-.002	-.002	-.002	-.002	-.001	0.000	0.000	0.000	0.000
10.0	.241	.177	.118	.065	.031	.019	.008	.002	-.001	-.002	-.001	-.002	-.001	-.001	0.000	0.000	0.000	0.000

TABLE XXVIII IMAGINARY PART OF RADIAL VELOCITY ADMITTANCE COEFFICIENT $S_{\nu h}$ = 5

.5	-6.561	-6.489	-6.400	-6.293	-6.186	-6.134	-6.057	-6.294	-6.961	-7.371	-7.336	-6.971	-6.438	-5.309	-3.714	-2.241	-1.358	-.739
1.0	-9.160	-9.057	-8.781	-8.365	-7.849	-7.564	-7.094	-6.834	-6.328	-5.431	-4.610	-3.697	-2.971	-2.052	-1.453	-1.022	-.653	-.365
1.5	-9.127	-8.953	-8.553	-7.935	-7.163	-6.739	-6.054	-5.507	-4.512	-3.382	-2.620	-1.982	-1.634	-1.330	-.940	-.671	-.434	-.242
2.0	-8.221	-7.969	-7.525	-6.845	-5.992	-5.529	-4.788	-4.160	-3.109	-2.160	-1.693	-1.429	-1.269	-.949	-.707	-.500	-.329	-.179
2.5	-7.225	-6.887	-6.418	-5.740	-4.898	-4.442	-3.724	-3.118	-2.202	-1.578	-1.367	-1.170	-.959	-.783	-.575	-.401	-.271	-.144
3.0	-6.351	-5.947	-5.450	-4.784	-3.986	-3.563	-2.910	-2.381	-1.686	-1.335	-1.157	-.916	-.820	-.619	-.487	-.062	-.236	-.146
3.5	-5.625	-5.190	-4.674	-4.014	-3.265	-2.884	-2.320	-1.893	-1.414	-1.157	-.934	-.812	-.671	-.586	-.413	-.255	-.159	-.073
4.0	-5.025	-4.5[illegible]	4.076	-3.434	-2.739	-2.402	-1.928	-1.597	-1.243	-.962	-.843	-.691	-.614	-.473	-.362	-.262	-.142	-.096
4.5	-4.527	-4.[illegible]	.615	-3.011	-2.383	-2.090	-1.693	-1.424	-1.111	-.882	-.749	-.626	-.537	-.425	-.315	-.224	-.145	-.080
5.0	-4.115	-3.[illegible]	-3.252	-2.693	-2.130	-1.873	-1.528	-1.289	-1.006	-.794	-.672	-.564	-.485	-.382	-.282	-.202	-.132	-.072
5.5	-3.772	-3.[illegible]57	-2.957	-2.442	-1.933	-1.703	-1.391	-1.175	-.917	-.723	-.610	-.511	-.441	-.348	-.256	-.183	-.119	-.067
6.0	-3.482	-3.125	-2.712	-2.236	-1.770	-1.561	-1.276	-1.078	-.841	-.663	-.560	-.469	-.404	-.318	-.235	-.168	-.108	-.061
6.5	-3.231	-2.893	-2.506	-2.063	-1.634	-1.442	-1.179	-.996	-.777	-.612	-.517	-.434	-.374	-.295	-.218	-.155	-.100	-.056
7.0	-3.010	-2.693	-2.329	-1.916	-1.517	-1.339	-1.095	-.925	-.721	-.568	-.481	-.402	-.346	-.272	-.202	-.145	-.093	-.053
7.5	-2.816	-2.518	-2.175	-1.789	-1.416	-1.249	-1.022	-.864	-.672	-.530	-.448	-.375	-.324	-.255	-.188	-.135	-.087	-.049
8.0	-2.646	-2.364	-2.041	-1.677	-1.328	-1.171	-.959	-.810	-.630	-.497	-.420	-.352	-.303	-.239	-.176	-.126	-.082	-.045
8.5	-2.497	-2.228	-1.922	-1.579	-1.250	-1.102	-.902	-.762	-.593	-.468	-.396	-.331	-.285	-.225	-.166	-.119	-.077	-.043
9.0	-2.364	-2.106	-1.816	-1.491	-1.181	-1.041	-.852	-.720	-.560	-.442	-.374	-.313	-.270	-.212	-.157	-.112	-.072	-.041
9.5	-2.243	-1.997	-1.721	-1.413	-1.118	-.987	-.807	-.682	-.531	-.419	-.354	-.297	-.255	-.201	-.149	-.106	-.068	-.038
10.0	-2.132	-1.898	-1.636	-1.342	-1.062	-.938	-.767	-.648	-.505	-.398	-.336	-.281	-.243	-.191	-.141	-.101	-.065	-.037

TABLE XXIX REAL PART OF RADIAL VELOCITY ADMITTANCE COEFFICIENT $S_{\nu h}$ = 7

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	30.782	29.031	26.886	24.206	21.288	19.823	17.620	16.125	13.615	10.584	8.294	6.007	4.312	2.181	.512	-.195	-.135	-.046
1.0	21.570	20.005	18.036	15.548	12.836	11.489	9.483	7.743	4.948	2.639	1.421	.534	.074	-.226	-.163	-.067	-.038	-.012
1.5	14.184	12.921	11.333	9.350	7.232	6.204	4.710	3.395	1.534	.359	-.072	-.235	-.221	-.121	-.078	-.040	-.017	-.005
2.0	9.426	8.402	7.165	5.653	4.072	3.322	2.260	1.358	.255	-.206	-.237	-.156	-.104	-.081	-.044	0.000	-.009	-.003
2.5	6.509	5.618	4.632	3.473	2.291	1.746	1.001	.409	-.167	-.223	-.133	-.090	-.084	-.047	-.027	-.014	-.006	-.002
3.0	4.708	3.886	3.062	2.153	1.263	.868	.357	-.008	-.234	-.128	-.084	-.081	-.051	-.037	-.018	-.010	-.005	-.002
3.5	3.556	2.801	2.074	1.336	.662	.381	.044	-.157	-.180	-.079	-.082	-.050	-.042	-.025	-.015	-.005	.001	.019
4.0	2.778	2.108	1.458	.831	.316	.120	-.088	-.180	-.115	-.075	-.063	-.039	-.033	-.020	-.017	-.021	-.010	.007
4.5	2.221	1.651	1.082	.539	.133	-.003	-.124	-.153	-.078	-.072	-.037	-.041	-.019	-.018	-.011	-.011	-.001	0.000
5.0	1.806	1.335	.852	.390	.064	-.035	-.108	-.112	-.061	-.053	-.033	-.009	.004	-.021	-.023	-.004	.015	-.012
5.5	1.498	1.106	.703	.323	.066	-.007	-.057	-.058	-.033	.013	-.004	-.019	-.019	.004	-.006	-.008	.009	.006
6.0	1.267	.933	.595	.286	.084	.030	-.010	-.016	-.011	-.014	-.013	-.009	-.006	-.007	-.005	-.002	0.000	-.002
6.5	1.091	.799	.513	.257	.092	.047	.007	-.005	-.011	-.013	-.010	-.008	-.007	-.004	-.003	-.001	-.001	0.000
7.0	.949	.693	.447	.231	.091	.051	.013	-.002	-.009	-.010	-.008	-.007	-.006	-.004	-.002	-.001	-.001	0.000
7.5	.827	.607	.393	.207	.086	.049	.015	0.000	-.008	-.009	-.008	-.006	-.004	-.003	-.002	-.001	-.001	0.000
8.0	.724	.536	.348	.186	.080	.047	.016	.001	-.006	-.006	-.005	-.005	-.005	-.003	-.002	-.002	0.000	-.001
8.5	.641	.476	.310	.167	.073	.043	.015	.002	-.005	-.005	-.005	-.005	-.003	-.003	-.002	-.001	0.000	0.000
9.0	.575	.426	.278	.151	.067	.041	.014	.003	-.004	-.005	-.005	-.004	-.003	-.002	-.001	-.001	0.000	0.000
9.5	.521	.383	.251	.137	.061	.038	.014	.003	-.003	-.004	-.004	-.003	-.003	-.002	-.001	-.001	-.001	0.000
10.0	.471	.345	.227	.124	.056	.035	.013	.003	-.002	-.004	-.003	-.004	-.002	-.001	-.001	0.000	-.001	0.000

TABLE XXX IMAGINARY PART OF RADIAL VELOCITY ADMITTANCE COEFFICIENT $S_{\nu h}$ = 7

.5	-12.576	-12.223	-11.877	-11.497	-11.119	-10.934	-10.664	-11.125	12.384	-13.035	-12.895	-12.220	-11.322	-9.519	-7.047	-4.642	-2.691	-1.455
1.0	-17.656	-17.127	-16.317	-15.278	-14.101	-13.483	-12.513	-12.060	-11.210	-9.730	-8.434	-7.018	-5.881	-4.312	-2.935	-2.044	-1.294	-.718
1.5	-17.731	-17.076	-16.010	-14.601	-13.001	-12.171	-10.887	-9.956	-8.331	-6.538	-5.314	-4.196	-3.446	-2.613	-1.893	-1.335	-.856	-.477
2.0	-16.093	-15.399	-14.285	-12.791	-11.101	-10.234	-8.905	-7.851	-6.128	-4.528	-3.608	-2.901	-2.474	-1.921	-1.403	0.000	-.640	-.358
2.5	-14.204	-13.487	-12.424	-10.978	-9.339	-8.503	-7.230	-6.196	-4.607	-3.346	-2.741	-2.288	-1.950	-1.520	-1.115	-.792	-.511	-.286
3.0	-12.500	-11.751	-10.764	-9.413	-7.876	-7.094	-5.914	-4.959	-3.594	-2.682	-2.268	-1.876	-1.601	-1.256	-.924	-.659	-.425	-.239
3.5	-11.061	-10.285	-9.355	-8.110	-6.691	-5.972	-4.900	-4.051	-2.940	-2.284	-1.923	-1.586	-1.372	-1.077	-.789	-.563	-.361	-.242
4.0	-9.867	-9.085	-8.183	-7.030	-5.733	-5.082	-4.126	-3.393	-2.513	-1.989	-1.652	-1.391	-1.185	-.931	-.692	-.488	-.317	-.189
4.5	-8.882	-8.116	-7.226	-6.139	-4.955	-4.372	-3.535	-2.915	-2.215	-1.742	-1.461	-1.222	-1.058	-.840	-.601	-.430	-.274	-.145
5.0	-8.072	-7.329	-6.460	-5.419	-4.329	-3.808	-3.079	-2.557	-1.978	-1.538	-1.314	-1.060	-1.093	-.762	-.553	-.409	-.255	-.154
5.5	-7.401	-6.682	-5.845	-4.656	-3.847	-3.377	-2.734	-2.286	-1.775	-1.408	-1.203	-1.001	-.864	-.679	-.498	-.353	-.234	-.132
6.0	-6.831	-6.141	-5.345	-4.416	-3.489	-3.066	-2.494	-2.100	-1.643	-1.303	-1.099	-.918	-.790	-.624	-.461	-.328	-.212	-.120
6.5	-6.336	-5.681	-4.929	-4.061	-3.209	-2.825	-2.307	-1.947	-1.520	-1.199	-1.013	-.849	-.732	-.577	-.426	-.304	-.196	-.110
7.0	-5.901	-5.285	-4.575	-3.765	-2.977	-2.624	-2.144	-1.812	-1.412	-1.113	-.941	-.789	-.679	-.535	-.395	-.282	-.183	-.102
7.5	-5.519	-4.940	-4.270	-3.511	-2.778	-2.450	-2.002	-1.692	-1.318	-1.038	-.878	-.735	-.633	-.500	-.368	-.264	-.171	-.096
8.0	-5.187	-4.637	-4.004	-3.291	-2.604	-2.297	-1.878	-1.586	-1.235	-.974	-.823	-.691	-.595	-.468	-.345	-.248	-.160	-.089
8.5	-4.896	-4.369	-3.770	-3.097	-2.451	-2.161	-1.768	-1.494	-1.163	-.917	-.776	-.649	-.559	-.441	-.326	-.232	-.151	-.084
9.0	-4.636	-4.129	-3.561	-2.924	-2.315	-2.041	-1.670	-1.411	-1.098	-.866	-.733	-.613	-.529	-.416	-.308	-.220	-.142	-.080
9.5	-4.397	-3.915	-3.375	-2.771	-2.193	-1.934	-1.582	-1.337	-1.040	-.821	-.694	-.581	-.500	-.395	-.291	-.208	-.134	-.075
10.0	-4.179	-3.721	-3.207	-2.632	-2.083	-1.837	-1.503	-1.270	-.989	-.779	-.659	-.552	-.476	-.375	-.276	-.198	-.128	-.071

TABLE XXXI REAL PART OF RADIAL VELOCITY ADMITTANCE COEFFICIENT $S\nu h$ = 9

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	50.499	47.300	43.478	38.833	33.887	31.442	27.809	25.445	21.442	16.688	13.191	9.758	7.237	4.059	1.467	.115	-.181	-.073
1.0	35.545	32.782	29.364	25.164	20.701	18.522	15.325	12.570	8.228	4.746	2.905	1.511	.715	.016	-.190	-.124	-.059	-.019
1.5	23.540	21.414	18.722	15.442	12.024	10.393	8.053	5.978	3.079	1.217	.435	-.009	-.162	-.173	-.111	-.062	-.027	-.009
2.0	15.762	14.154	12.111	9.645	7.121	5.938	4.274	2.826	1.020	.111	-.138	-.188	-.159	-.113	-.068	-.036	-.016	-.005
2.5	10.933	9.640	8.074	6.204	4.329	3.452	2.257	1.259	.179	-.172	-.181	-.139	-.115	-.078	-.045	-.024	-.010	-.003
3.0	7.906	6.770	5.535	4.088	2.654	2.008	1.144	.466	-.126	-.180	-.134	-.109	-.086	-.056	-.032	-.017	-.007	-.002
3.5	5.946	4.891	3.880	2.735	1.628	1.144	.523	.076	-.195	-.135	-.110	-.085	-.064	-.041	-.024	-.012	-.005	-.002
4.0	4.615	3.637	2.769	1.844	.982	.620	.181	-.098	-.174	-.108	-.093	-.064	-.052	-.033	-.018	-.009	-.004	-.001
4.5	3.667	2.792	2.012	1.246	.572	.304	.003	-.157	-.135	-.096	-.072	-.054	-.039	-.025	-.014	-.007	-.003	0.000
5.0	2.974	2.218	1.501	.842	.314	.118	-.081	-.163	-.105	-.085	-.056	-.044	-.034	-.022	-.012	-.009	-.016	-.002
5.5	2.467	1.818	1.166	.579	.155	.014	-.113	-.148	-.088	-.069	-.050	-.033	-.028	-.015	-.005	-.009	.002	0.000
6.0	2.093	1.527	.951	.424	.070	-.035	-.118	-.127	-.079	-.053	-.047	-.033	-.016	-.013	-.009	-.001	.002	.005
6.5	1.804	1.306	.808	.348	.045	-.040	-.101	-.101	-.071	-.040	-.033	-.025	-.012	-.015	.003	-.009	-.009	-.013
7.0	1.566	1.133	.704	.314	.061	-.007	-.056	-.053	-.025	-.012	-.027	-.021	-.015	-.010	.004	.002	-.006	.004
7.5	1.362	.993	.622	.293	.084	.029	-.012	-.014	-.012	-.017	-.014	-.008	-.008	-.006	-.003	-.002	-.001	-.001
8.0	1.191	.877	.554	.272	.095	.047	.006	-.007	-.014	-.014	-.011	-.008	-.007	-.005	-.003	-.001	-.001	0.000
8.5	1.057	.780	.497	.252	.097	.052	.012	-.004	-.011	-.010	-.008	-.009	-.007	-.004	-.003	-.002	-.001	-.001
9.0	.951	.697	.448	.232	.094	.052	.015	-.001	-.008	-.009	-.009	-.007	-.005	-.004	-.002	-.002	-.001	0.000
9.5	.861	.627	.405	.213	.089	.051	.015	.001	-.007	-.008	-.007	-.006	-.005	-.004	-.002	-.001	-.001	0.000
10.0	.778	.566	.368	.196	.083	.049	.016	.001	-.006	-.007	-.007	-.005	-.004	-.003	-.002	-.001	-.001	0.000

TABLE XXXII IMAGINARY PART OF RADIAL VELOCITY ADMITTANCE COEFFICIENT $S\nu h$ = 9

.5	-20.413	-19.709	-18.998	-18.218	[illegible]	-17.059	-16.503	-17.311	-19.351	-20.246	-19.929	-18.832	-17.460	-14.797	-11.228	-7.723	-4.549	-2.419
1.0	-28.796	-27.633	-26.077	-24.165	-22.166	-20.987	-19.328	-18.642	-17.334	-15.104	-13.213	-11.180	-9.546	-7.232	-4.991	-3.412	-2.150	-1.189
1.5	-29.048	-27.616	-25.622	-23.129	[illegible]	-19.022	-16.935	-15.514	-13.093	-10.490	-8.724	-7.064	-5.877	-4.416	-3.163	-2.219	-1.419	-.790
2.0	-26.507	-25.020	-22.956	-20.364	-17.551	-16.147	-14.045	-12.455	-9.914	-7.569	-6.154	-4.952	-4.164	-3.212	-2.331	-1.649	-1.060	-.592
2.5	-23.502	-22.064	-20.097	-17.611	-14.926	-13.595	-11.613	-10.059	-7.691	-5.735	-4.675	-3.829	-3.259	-2.530	-1.850	-1.314	-.847	-.473
3.0	-20.727	-19.383	-17.568	-15.254	-12.756	-11.522	-9.693	-8.243	-6.140	-4.582	-3.797	-3.141	-2.678	-2.093	-1.535	-1.093	-.705	-.394
3.5	-18.340	-17.091	-15.437	-13.305	-10.999	-9.864	-8.190	-6.869	-5.056	-3.840	-3.214	-2.659	-2.279	-1.785	-1.312	-.935	-.604	-.359
4.0	-16.346	-15.165	-13.654	-11.699	-9.576	-8.535	-7.012	-5.826	-4.296	-3.325	-2.781	-2.312	-1.983	-1.556	-1.146	-.817	-.527	-.295
4.5	-14.708	-13.558	-12.157	-10.366	-8.415	-7.462	-6.084	-5.032	-3.751	-2.930	-2.450	-2.046	-1.755	-1.380	-1.017	-.725	-.470	-.263
5.0	-13.367	-12.225	-10.894	-9.247	-7.458	-6.589	-5.348	-4.421	-3.342	-2.612	-2.196	-1.830	-1.576	-1.237	-.910	-.655	-.404	-.235
5.5	-12.255	-11.120	-9.831	-8.297	-6.660	-5.872	-4.758	-3.944	-3.017	-2.354	-1.990	-1.659	-1.424	-1.123	-.837	-.591	-.383	-.220
6.0	-11.307	-10.199	-8.946	-7.493	-5.988	-5.274	-4.278	-3.560	-2.747	-2.145	-1.810	-1.517	-1.299	-1.048	-.754	-.551	-.357	-.185
6.5	-10.481	-9.421	-8.214	-6.824	-5.424	-4.773	-3.875	-3.238	-2.511	-1.966	-1.612	-1.450	-1.215	-.940	-.705	-.450	-.335	-.193
7.0	-9.755	-8.755	-7.602	-6.279	-4.967	-4.365	-3.545	-2.971	-2.301	-1.849	-1.560	-1.299	-1.115	-.882	-.649	-.463	-.308	-.165
7.5	-9.124	-8.178	-7.082	-5.832	-4.606	-4.051	-3.301	-2.784	-2.177	-1.718	-1.449	-1.216	-1.050	-.827	-.609	-.437	-.283	-.159
8.0	-8.578	[illegible]	-6.633	-5.454	-4.309	-3.796	-3.100	-2.620	-2.043	-1.610	-1.361	-1.141	-.983	-.774	-.572	-.409	-.264	-.148
8.5	-8.098	-7.226	-6.241	-5.127	-4.053	-3.573	-2.921	-2.468	-1.922	-1.515	-1.282	-1.074	-.924	-.730	-.539	-.385	-.248	-.138
9.0	-7.665	-6.829	-5.893	-4.840	-3.828	-3.375	-2.760	-2.331	-1.815	-1.432	-1.211	-1.013	-.874	-.688	-.509	-.363	-.234	-.132
9.5	-7.268	-6.473	-5.583	-4.584	-3.626	-3.197	-2.615	-2.210	-1.721	-1.357	-1.147	-.961	-.827	-.652	-.482	-.344	-.222	-.124
10.0	-6.907	-6.154	-5.304	-4.354	-3.445	-3.037	-2.484	-2.100	-1.634	-1.289	-1.089	-.913	-.786	-.619	-.457	-.326	-.212	-.118

TABLE XXXIII REAL PART OF ENTROPY ADMITTANCE COEFFICIENT $S_{\nu h}$ = 0

ω \ ν	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.366	-.317	-.264	-.207	-.151	-.125	-.088	-.060	-.019	.018	.039	.054	.057	.034	-.031	-.015	-.027	.025
1.0	-.247	-.197	-.144	-.089	-.040	-.021	.002	.016	.026	.016	-.004	-.025	-.025	.017	-.019	.014	-.015	-.009
1.5	-.159	-.117	-.076	-.038	-.012	-.005	.001	.002	-.001	-.006	-.004	.002	.003	-.003	-.001	-.002	-.002	.001
2.0	-.105	-.074	-.045	-.022	-.009	-.006	-.004	-.002	0.000	-.002	-.002	.001	0.000	.001	0.000	0.000	.001	0.000
2.5	-.073	-.050	-.030	-.015	-.007	-.005	-.003	-.001	.001	-.002	.001	0.000	-.001	-.001	-.001	-.001	0.000	0.000
3.0	-.054	-.036	-.022	-.011	-.005	-.004	-.002	0.000	0.000	0.000	.001	-.001	.001	.001	-.001	0.000	0.000	0.000
3.5	-.041	-.027	-.016	-.008	-.004	-.003	-.002	0.000	-.001	.001	-.001	.001	-.001	-.001	0.000	0.000	0.000	0.000
4.0	-.032	-.021	-.013	-.007	-.003	-.002	-.001	0.000	-.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.5	-.025	-.017	-.010	-.005	-.003	-.002	-.001	0.000	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.0	-.021	-.014	-.008	-.004	-.002	-.002	-.001	0.000	0.000	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.5	-.017	-.011	-.007	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.0	-.015	-.010	-.006	-.003	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.5	-.013	-.008	-.005	-.003	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.0	-.011	-.007	-.004	-.002	-.001	-.001	0.000	0.000	.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	-.010	-.006	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	-.008	-.005	-.003	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.5	-.007	-.005	-.003	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.0	-.007	-.004	-.003	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.5	-.006	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	-.005	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XXXIV IMAGINARY PART OF ENTROPY ADMITTANCE COEFFICIENT $S_{\nu h}$ = 0

.5	.159	.156	.153	.147	.139	.134	.125	.117	.103	.083	.062	.032	.004	-.038	-.036	.043	-.037	.060
1.0	.217	.201	.181	.154	.121	.103	.076	.053	.018	-.012	-.021	-.009	.015	.021	-.016	-.016	.001	-.006
1.5	.211	.185	.153	.115	.077	.059	.036	.022	.008	.006	.009	.007	0.000	.003	-.002	-.001	-.001	.001
2.0	.188	.158	.124	.088	.056	.043	.028	.020	.010	.005	.005	.003	.001	.002	.001	-.001	0.000	0.000
2.5	.165	.135	.103	.071	.045	.036	.024	.018	.007	.006	.005	.001	.003	0.000	.001	0.000	.001	-.001
3.0	.144	.116	.088	.060	.039	.030	.021	.015	.006	.006	.002	.002	.001	.001	0.000	.001	.001	0.000
3.5	.128	.102	.076	.052	.034	.026	.018	.013	.005	.004	.002	.002	.001	.001	0.000	.001	0.000	0.000
4.0	.114	.091	.068	.046	.030	.023	.016	.011	.005	.003	.003	.001	.001	0.000	0.000	0.000	0.000	0.000
4.5	.103	.081	.060	.041	.026	.021	.014	.009	.005	.002	.002	.002	0.000	.001	.001	0.000	0.000	0.000
5.0	.094	.074	.055	.037	.024	.019	.013	.008	.005	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
5.5	.086	.067	.050	.034	.022	.017	.012	.007	.005	.003	.001	.001	0.000	.001	0.000	0.000	0.000	0.000
6.0	.079	.062	.046	.031	.020	.016	.011	.007	.004	.002	.002	.001	.001	0.000	0.000	0.000	0.000	0.000
6.5	.074	.058	.042	.029	.019	.015	.010	.006	.003	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
7.0	.069	.054	.040	.027	.017	.014	.009	.006	.003	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
7.5	.064	.050	.037	.025	.016	.013	.009	.005	.003	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
8.0	.060	.047	.035	.024	.015	.012	.008	.005	.003	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
8.5	.057	.044	.033	.022	.014	.011	.008	.005	.003	.002	.001	0.000	.001	0.000	0.000	0.000	0.000	0.000
9.0	.054	.042	.031	.021	.013	.011	.007	.004	.003	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
9.5	.051	.040	.029	.020	.013	.010	.007	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
10.0	.049	.038	.028	.019	.012	.010	.007	.004	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XXXV REAL PART OF ENTROPY ADMITTANCE COEFFICIENT $S_{\nu h} = 1$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.366	-.318	-.266	-.210	-.157	-.133	-.098	-.071	-.031	.005	.026	.042	.048	.038	0.000	-.010	.005	-.010
1.0	-.247	-.197	-.145	-.091	-.042	-.022	.003	.020	.036	.032	.015	-.011	-.027	-.008	.014	.027	-.028	-.003
1.5	-.159	-.117	-.075	-.037	-.008	.001	.007	.009	.003	-.009	-.010	.002	.008	-.009	-.001	-.003	-.002	.001
2.0	-.105	-.074	-.044	-.020	-.006	-.003	-.002	-.002	-.001	-.002	-.001	.001	0.000	.001	0.000	0.000	0.000	0.000
2.5	-.073	-.050	-.029	-.014	-.006	-.005	-.003	-.001	0.000	-.002	.001	0.000	-.001	-.001	-.001	-.001	0.000	0.000
3.0	-.053	-.036	-.021	-.011	-.005	-.004	-.002	0.000	0.000	0.000	.001	-.001	.001	.001	-.001	-.001	0.000	0.000
3.5	-.041	-.027	-.016	-.008	-.004	-.003	-.002	0.000	-.001	.001	-.001	.001	-.001	-.001	0.000	0.000	0.000	0.000
4.0	-.032	-.021	-.012	-.006	-.003	-.002	-.001	0.000	-.001	.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.5	-.025	-.017	-.010	-.005	-.003	-.002	-.001	0.000	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.0	-.021	-.014	-.008	-.004	-.002	-.002	-.001	0.000	0.000	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.5	-.017	-.011	-.007	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.0	-.015	-.010	-.006	-.003	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.5	-.013	-.008	-.005	-.003	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.0	-.011	-.007	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	-.009	-.006	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	-.008	-.005	-.003	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.5	-.007	-.005	-.003	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.0	-.007	-.004	-.003	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.5	-.006	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	-.005	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XXXVI IMAGINARY PART OF ENTROPY ADMITTANCE COEFFICIENT $S_{\nu h} = 1$

.5	.159	.156	.152	.146	.139	.134	.125	.119	.107	.089	.071	.047	.024	-.012	-.029	.010	-.015	.006
1.0	.217	.202	.183	.158	.128	.112	.087	.065	.029	-.006	-.024	-.026	-.009	.027	-.025	.029	-.035	-.015
1.5	.212	.185	.155	.118	.080	.062	.037	.020	.002	0.000	.009	.012	0.000	.003	-.006	-.004	-.004	.004
2.0	.188	.158	.125	.088	.055	.042	.026	.018	.009	.006	.005	.003	.001	.001	.001	0.000	0.000	0.000
2.5	.165	.135	.103	.071	.045	.035	.023	.017	.007	.005	.005	.001	.003	0.000	.001	0.000	.001	-.001
3.0	.144	.116	.088	.060	.038	.030	.021	.015	.006	.006	.002	.002	.001	.001	0.000	.001	.001	0.000
3.5	.128	.102	.076	.052	.033	.026	.018	.012	.005	.004	.002	.002	.001	.001	0.000	.001	0.000	0.000
4.0	.114	.091	.067	.046	.029	.023	.016	.011	.005	.003	.002	.001	.001	0.000	0.000	0.000	0.000	0.000
4.5	.103	.081	.060	.041	.026	.021	.014	.009	.005	.002	.002	.002	0.000	.001	.001	0.000	0.000	0.000
5.0	.094	.074	.055	.037	.024	.019	.013	.008	.005	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
5.5	.086	.067	.050	.034	.022	.017	.012	.007	.005	.003	.001	.001	0.000	.001	0.000	0.000	0.000	0.000
6.0	.079	.062	.046	.031	.020	.016	.011	.007	.004	.002	.002	.001	.001	0.000	0.000	0.000	0.000	0.000
6.5	.074	.057	.042	.029	.018	.015	.010	.006	.003	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
7.0	.069	.054	.039	.027	.017	.014	.009	.006	.003	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
7.5	.064	.050	.037	.025	.016	.013	.009	.005	.003	.002	.00.	.001	.001	0.000	0.000	0.000	0.000	0.000
8.0	.060	.047	.035	.024	.015	.012	.008	.005	.003	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
8.5	.057	.044	.033	.022	.014	.011	.008	.005	.003	.002	.001	0.000	.001	0.000	0.000	0.000	0.000	0.000
9.0	.054	.042	.031	.021	.013	.011	.007	.004	.003	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
9.5	.051	.040	.029	.020	.013	.010	.007	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
10.0	.049	.038	.028	.019	.012	.010	.007	.004	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XXXVII REAL PART OF ENTROPY ADMITTANCE COEFFICIENT $S\nu h = 2$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.367	-.320	-.270	-.218	-.168	-.145	-.113	-.087	-.047	-.013	.008	.024	.031	.030	.013	-.006	.004	-.002
1.0	-.248	-.199	-.149	-.098	-.052	-.032	-.007	.012	.035	.040	.033	.016	0.000	-.012	.009	-.006	.007	-.005
1.5	-.159	-.117	-.075	-.034	-.002	.010	.021	.026	.023	.005	-.010	-.013	.001	.006	.004	.021	.010	-.003
2.0	-.105	-.073	-.042	-.015	.002	.006	.007	.005	-.002	-.007	-.001	.003	-.006	.006	.004	-.002	.003	.002
2.5	-.073	-.049	-.027	-.011	-.002	-.001	-.002	-.002	-.001	-.002	0.000	0.000	0.000	0.000	-.001	0.000	0.000	0.000
3.0	-.053	-.035	-.020	-.009	-.004	-.003	-.002	-.001	0.000	-.001	.001	-.001	.001	.001	-.001	-.001	-.001	0.000
3.5	-.040	-.027	-.015	-.007	-.004	-.003	-.002	0.000	-.001	.001	-.001	.001	-.001	-.001	0.000	0.000	0.000	0.000
4.0	-.032	-.021	-.012	-.006	-.003	-.002	-.001	0.000	-.001	.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.5	-.025	-.017	-.010	-.005	-.002	-.002	-.001	0.000	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.0	-.021	-.014	-.008	-.004	-.002	-.001	-.001	0.000	0.000	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.5	-.017	-.011	-.007	-.003	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.0	-.015	-.010	-.006	-.003	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.5	-.013	-.008	-.005	-.003	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.0	-.011	-.007	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	-.009	-.006	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	-.008	-.005	-.003	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.5	-.007	-.005	-.003	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.0	-.007	-.004	-.003	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.5	-.006	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	-.005	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XXXVIII IMAGINARY PART OF ENTROPY ADMITTANCE COEFFICIENT $S\nu h = 2$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	.160	.156	.151	.144	.135	.130	.122	.116	.106	.090	.075	.055	.036	.009	-.011	-.003	-.001	-.001
1.0	.218	.204	.187	.166	.141	.128	.107	.088	.054	.019	-.003	-.016	-.017	.001	.004	.003	.005	.003
1.5	.212	.188	.160	.127	.092	.075	.050	.029	.001	-.014	-.010	.006	.013	-.010	.014	.036	-.009	-.007
2.0	.189	.159	.127	.091	.057	.041	.022	.010	.001	.005	.008	0.000	-.001	0.000	0.000	-.001	-.002	0.000
2.5	.165	.135	.104	.071	.042	.032	.020	.014	.007	.005	.004	.001	.002	0.000	.001	0.000	0.000	0.000
3.0	.144	.116	.088	.059	.036	.028	.019	.014	.006	.005	.002	.002	.001	.001	0.000	0.000	0.000	0.000
3.5	.123	.102	.076	.051	.032	.025	.017	.012	.005	.004	.001	.002	.001	.001	0.000	.001	0.000	0.000
4.0	.114	.090	.067	.045	.029	.023	.016	.010	.005	.003	.002	.001	.001	0.000	0.000	0.000	0.000	0.000
4.5	.103	.081	.060	.041	.026	.020	.014	.009	.005	.002	.002	.001	0.000	.001	.001	0.000	0.000	0.000
5.0	.094	.074	.054	.037	.024	.019	.013	.008	.005	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
5.5	.086	.067	.050	.034	.021	.017	.012	.007	.004	.003	.001	.001	0.000	.001	0.000	0.000	0.000	0.000
6.0	.079	.062	.046	.031	.020	.016	.011	.007	.004	.002	.002	.001	.001	0.000	0.000	0.000	0.000	0.000
6.5	.073	.057	.042	.029	.018	.014	.010	.006	.003	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
7.0	.069	.053	.039	.027	.017	.014	.009	.006	.003	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
7.5	.064	.050	.037	.025	.016	.013	.009	.005	.003	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
8.0	.060	.047	.035	.023	.015	.012	.008	.005	.003	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
8.5	.057	.044	.033	.022	.014	.011	.008	.005	.002	.002	.001	0.000	.001	0.000	0.000	0.000	0.000	0.000
9.0	.054	.042	.031	.021	.013	.011	.007	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
9.5	.051	.040	.029	.020	.013	.010	.007	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
10.0	.049	.038	.028	.019	.012	.010	.007	.004	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XXXIX REAL PART OF ENTROPY ADMITTANCE COEFFICIENT $S_{\nu h} = 3$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.369	-.322	-.275	-.224	-.176	-.154	-.122	-.096	-.057	-.023	-.003	.012	.020	.023	.014	0.000	.002	0.000
1.0	-.249	-.202	-.156	-.109	-.066	-.047	-.023	-.002	.023	.033	.031	.021	.011	-.002	.002	-.001	0.000	0.000
1.5	-.159	-.117	-.076	-.037	-.005	.007	.021	.029	.032	.020	.006	-.004	-.004	.005	-.002	-.002	-.002	-.031
2.0	-.105	-.072	-.039	-.010	.011	.018	.022	.022	.012	-.003	-.006	.002	.004	-.005	-.005	-.008	0.000	-.002
2.5	-.073	-.048	-.024	-.005	.007	.008	.007	.004	-.003	-.004	.001	-.003	-.003	-.005	0.000	-.002	.002	.002
3.0	-.053	-.034	-.018	-.005	0.000	0.000	-.001	-.001	-.001	-.001	0.000	0.000	.001	0.000	0.000	0.000	0.000	0.000
3.5	-.040	-.026	-.014	-.003	-.002	-.002	-.002	0.000	-.001	.001	0.000	.001	-.001	-.001	0.000	0.000	0.000	0.000
4.0	-.032	-.020	-.011	-.005	-.002	-.002	-.001	0.000	-.001	.001	-.001	0.000	0.000	.001	0.000	0.000	0.000	0.000
4.5	-.025	-.016	-.009	-.004	-.002	-.002	-.001	0.000	-.001	0.000	0.000	-.001	0.000	0.000	0.000	0.000	0.000	0.000
5.0	-.021	-.013	-.008	-.004	-.002	-.001	-.001	0.000	0.000	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.5	-.017	-.011	-.006	-.003	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.0	-.015	-.009	-.005	-.003	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.5	-.013	-.008	-.005	-.002	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.0	-.011	-.007	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	-.009	-.006	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	-.008	-.005	-.003	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.5	-.007	-.005	-.003	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.0	-.007	-.004	-.003	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.5	-.006	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	.005	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XL IMAGINARY PART OF ENTROPY ADMITTANCE COEFFICIENT $S_{\nu h} = 3$

.5	.160	.155	.148	.140	.130	.125	.116	.111	.102	.087	.073	.055	.040	.018	-.001	-.004	.001	0.000
1.0	.219	.206	.190	.171	.148	.135	.116	.098	.067	.034	.014	-.002	-.008	-.004	.003	-.001	.001	0.000
1.5	.213	.191	.167	.139	.108	.093	.070	.049	.018	-.005	-.010	-.004	.003	.001	.002	.001	-.001	-.020
2.0	.189	.161	.132	.099	.067	.052	.031	.015	-.004	-.006	.002	.006	-.002	.002	-.002	.023	.004	-.008
2.5	.165	.136	.105	.073	.043	.030	.015	.006	.001	.005	.003	-.002	.007	.002	.003	-.001	.002	-.001
3.0	.145	.117	.088	.058	.034	.024	.015	.010	.005	.005	.003	.002	.001	.001	0.000	0.000	0.000	0.000
3.5	.128	.102	.076	.050	.030	.023	.016	.011	.005	.004	.001	.002	0.000	0.000	0.000	.001	0.000	0.000
4.0	.114	.090	.067	.044	.027	.021	.015	.010	.005	.003	.002	.001	.001	0.000	0.000	0.000	0.000	0.000
4.5	.103	.081	.060	.040	.025	.020	.014	.009	.005	.002	.002	.001	0.000	.001	.001	0.000	0.000	0.000
5.0	.094	.073	.054	.036	.023	.018	.012	.008	.005	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
5.5	.086	.067	.049	.033	.021	.017	.011	.007	.004	.002	.001	.001	0.000	.001	0.000	0.000	0.000	0.000
6.0	.079	.062	.046	.031	.020	.015	.011	.007	.004	.002	.002	.001	.001	0.000	0.000	0.000	0.000	0.000
6.5	.073	.057	.042	.028	.018	.014	.010	.006	.003	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
7.0	.068	.053	.039	.027	.017	.013	.009	.006	.003	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
7.5	.064	.050	.037	.025	.016	.013	.009	.005	.003	.001	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
8.0	.060	.047	.034	.023	.015	.012	.008	.005	.002	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
8.5	.057	.044	.033	.022	.014	.011	.008	.005	.002	.002	.001	0.000	.001	0.000	0.000	0.000	0.000	0.000
9.0	.054	.042	.031	.021	.013	.011	.007	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
9.5	.051	.040	.029	.020	.013	.010	.007	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
10.0	.049	.038	.028	.019	.012	.009	.007	.004	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XLI REAL PART OF ENTROPY ADMITTANCE COEFFICIENT S_{vh} = 4

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.370	-.324	-.278	-.228	-.181	-.152	-.128	-.102	-.062	-.028	-.009	.005	.013	.018	.013	.004	.001	0.000
1.0	-.251	-.206	-.163	-.118	-.077	-.059	-.034	-.013	.013	.026	.026	.021	.014	.003	.001	0.000	0.000	0.000
1.5	-.160	-.119	-.081	-.044	-.014	-.002	.022	.023	.030	.023	.013	.003	0.000	.002	0.000	0.000	0.000	0.000
2.0	-.104	-.071	-.039	-.009	.012	.019	[illegible]	.027	.021	.007	-.001	0.000	.002	-.001	0.000	-.001	.001	.001
2.5	-.072	-.046	-.021	.001	.015	.019	.019	.017	.007	-.002	0.000	.002	-.001	.002	.002	-.012	-.001	.001
3.0	-.053	-.033	-.014	.001	.009	.010	.008	.005	-.001	-.001	.001	-.003	.001	.013	-.001	-.001	-.002	-.001
3.5	-.040	-.025	-.011	-.002	.002	.002	0.000	-.001	-.002	-.001	0.000	.001	0.000	0.000	0.000	0.000	0.000	0.000
4.0	-.031	-.019	-.010	-.003	-.001	-.001	-.001	0.000	-.001	.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.5	-.025	-.016	-.008	-.003	-.002	-.001	-.001	0.000	-.001	0.000	0.000	-.001	0.000	0.000	0.000	0.000	0.000	0.000
5.0	-.021	-.013	-.007	-.003	-.002	-.001	-.001	0.000	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.5	-.017	-.011	-.006	-.003	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.0	-.015	-.009	-.005	-.003	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.5	-.012	-.008	-.005	-.002	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.0	-.011	-.007	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	-.009	-.006	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	-.008	-.005	-.003	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.5	-.007	-.005	-.003	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.0	-.007	-.004	-.003	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.5	-.006	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	-.005	-.003	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XLII IMAGINARY PART OF ENTROPY ADMITTANCE COEFFICIENT S_{vh} = 4

.5	.160	.153	.146	.136	.126	.120	.112	.107	.098	.084	.071	.055	.041	.022	.005	-.002	.001	0.000
1.0	.219	.207	.191	.171	.148	.136	.118	.101	.071	.041	.022	.007	0.000	-.003	.001	-.001	0.000	0.000
1.5	.214	.194	.173	.147	.118	.104	.082	.062	.030	.006	-.003	-.004	0.000	.002	0.000	0.000	0.000	0.000
2.0	.190	.165	.139	.110	.081	.066	.046	.028	.005	-.005	-.002	.003	.001	-.001	-.001	0.000	0.000	-.001
2.5	.165	.138	.109	.079	.051	.038	.021	.009	-.003	0.000	.003	0.000	-.001	0.000	-.001	.007	-.001	.003
3.0	.145	.117	.089	.059	.034	.023	.011	.004	0.000	.003	0.000	0.000	.003	.008	-.001	-.001	.001	-.002
3.5	.128	.102	.075	.049	.027	.019	.011	.007	.004	.004	.002	.002	0.000	0.000	0.000	0.000	0.000	0.000
4.0	.114	.090	.066	.043	.025	.019	.013	.009	.004	.003	.002	.001	.001	.001	.001	.001	0.000	0.000
4.5	.103	.081	.059	.039	.024	.016	.013	.008	.005	.002	.002	.001	0.000	.001	.001	0.000	0.000	0.000
5.0	.094	.073	.054	.035	.022	.017	.012	.008	.005	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
5.5	.086	.067	.049	.033	.021	.016	.011	.007	.004	.002	.001	.001	0.000	.001	0.000	0.000	0.000	0.000
6.0	.079	.062	.045	.030	.019	.015	.010	.006	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
6.5	.073	.057	.042	.028	.018	.014	.010	.006	.003	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
7.0	.068	.053	.039	.026	.017	.013	.009	.005	.003	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	.064	.050	.037	.025	.016	.012	.008	.005	.003	.001	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
8.0	.060	.047	.034	.023	.015	.012	.008	.005	.002	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
8.5	.057	.044	.032	.022	.014	.011	.008	.005	.002	.002	.001	0.000	.001	0.000	0.000	0.000	0.000	0.000
9.0	.054	.042	.031	.021	.013	.010	.007	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
9.5	.051	.040	.029	.020	.013	.010	.007	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
10.0	.049	.033	.028	.019	.012	.009	.006	.004	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XLIII REAL PART OF ENTROPY ADMITTANCE COEFFICIENT $S\nu h = 5$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.371	-.326	-.280	-.231	-.184	-.162	-.131	-.105	-.065	-.032	-.013	.001	.008	.014	.012	.005	.001	0.000
1.0	-.252	-.210	-.168	-.125	-.085	-.067	.043	-.021	.006	.020	.022	.019	.014	.006	.001	.001	0.000	0.000
1.5	-.160	-.122	-.087	-.052	-.023	-.011	.005	.016	.026	.022	.015	.007	.003	.002	.001	0.000	0.000	0.000
2.0	-.106	-.071	-.041	-.014	.007	.014	.021	.026	.023	.011	.004	.001	.002	.001	0.000	0.000	0.000	0.000
2.5	-.072	-.045	-.019	.003	.017	.021	.023	.022	.013	.002	.001	.002	.001	.001	0.000	0.000	0.000	0.000
3.0	-.052	-.031	-.010	.007	.016	.018	.017	.014	.004	0.000	.002	0.000	0.000	-.001	0.000	-.005	0.000	0.000
3.5	-.040	-.023	-.008	.004	.010	.010	.008	.005	0.000	0.000	0.000	-.001	0.000	.002	.001	0.000	0.000	0.000
4.0	-.031	-.018	-.007	.001	.003	.003	.001	0.000	-.002	-.001	-.002	.001	.001	0.000	0.000	0.000	0.000	0.000
4.5	-.025	-.015	-.007	-.001	0.000	0.000	-.001	-.001	-.002	0.000	0.000	0.000	0.000	-.001	0.000	0.000	0.000	0.000
5.0	-.020	-.012	-.006	-.002	-.001	-.001	-.001	0.000	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.5	-.017	-.011	-.005	-.002	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.0	-.014	-.009	-.005	-.002	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.5	-.012	-.008	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.0	-.011	-.007	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	-.009	-.006	-.003	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	-.008	-.005	-.003	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.5	-.007	-.004	-.003	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.0	-.007	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.5	-.006	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	-.005	-.003	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XLIV IMAGINARY PART OF ENTROPY ADMITTANCE COEFFICIENT $S\nu h = 5$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	.160	.152	.143	.133	.122	.117	.109	.104	.095	.081	.069	.054	.042	.024	.008	0.000	0.000	0.000
1.0	.219	.207	.191	.170	.147	.136	.118	.102	.073	.044	.027	.013	.005	-.001	0.000	0.000	0.000	0.000
1.5	.215	.197	.177	.151	.123	.109	.087	.068	.037	.013	.003	-.001	-.001	.001	0.000	0.000	0.000	0.000
2.0	.191	.168	.145	.119	.090	.076	.056	.038	.012	-.001	-.002	.001	.001	0.000	0.000	0.000	0.000	0.000
2.5	.166	.140	.115	.088	.062	.049	.032	.018	.001	-.001	.002	.001	-.001	0.000	0.000	0.000	0.000	0.000
3.0	.145	.118	.092	.065	.041	.030	.016	.007	-.001	.001	.001	-.001	.001	0.000	.001	-.007	0.000	0.000
3.5	.128	.102	.076	.050	.028	.019	.009	.003	0.000	.002	-.001	.001	-.001	.001	0.00	-.001	-.001	0.000
4.0	.114	.090	.066	.042	.023	.015	.008	.005	.002	.002	.002	.001	.001	0.000	.001	0.000	0.000	0.000
4.5	.103	.081	.059	.037	.021	.016	.010	.007	.004	.002	.002	.001	0.000	0.000	0.000	.001	0.000	0.000
5.0	.094	.073	.053	.034	.021	.016	.011	.007	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
5.5	.086	.067	.048	.032	.020	.015	.010	.007	.004	.002	.001	.001	.001	.001	0.000	0.000	0.000	0.000
6.0	.079	.062	.045	.030	.018	.014	.010	.006	.004	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
6.5	.073	.057	.042	.028	.017	.014	.009	.006	.003	.002	.001	.001	.001	.001	0.000	0.000	0.000	0.000
7.0	.068	.053	.039	.026	.016	.013	.009	.005	.003	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	.064	.050	.036	.024	.015	.012	.008	.005	.003	.001	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
8.0	.060	.047	.034	.023	.014	.011	.008	.005	.002	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
8.5	.057	.044	.032	.022	.014	.011	.007	.004	.002	.002	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.0	.054	.042	.031	.021	.013	.010	.007	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
9.5	.051	.040	.029	.019	.012	.010	.007	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
10.0	.049	.038	.028	.019	.012	.009	.006	.004	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XLV REAL PART OF ENTROPY ADMITTANCE COEFFICIENT $S_{\nu h} = 7$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.372	-.328	-.283	-.234	-.187	-.166	-.135	-.109	-.069	-.036	-.019	-.005	.007	.008	.009	.006	.002	0.000
1.0	-.255	-.215	-.176	-.134	-.095	-.077	-.053	-.031	-.003	.011	.015	.015	.013	.008	.003	.001	0.000	0.000
1.5	.162	-.128	-.096	-.064	-.035	-.023	-.007	.006	.018	.019	.015	.010	.005	.003	.001	0.000	0.000	0.000
2.0	-.104	-.075	-.049	-.024	-.004	.004	.013	.020	.021	.014	.009	.004	.003	.002	.001	0.000	0.000	0.000
2.5	-.071	-.045	-.022	-.003	.011	.016	.020	.022	.017	.008	.004	.003	.002	.001	0.000	0.000	0.000	0.000
3.0	-.051	-.028	-.008	.007	.017	.019	.020	.019	.011	.004	.002	.002	.001	0.000	0.000	0.000	0.000	0.000
3.5	-.039	-.019	-.002	.011	.017	.018	.017	.014	.006	.002	.002	.001	.001	0.000	0.000	0.000	0.000	0.000
4.0	-.030	-.015	-.001	.010	.015	.015	.013	.009	.003	.002	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.5	-.024	-.013	-.001	.007	.011	.010	.008	.005	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.0	-.020	-.011	-.003	.004	.006	.005	.004	.002	0.000	0.000	-.001	-.002	.002	0.000	0.000	0.000	0.000	0.000
5.5	-.017	-.009	-.003	.001	.002	.001	0.000	0.000	-.002	-.002	.001	.001	0.000	-.001	0.000	0.000	0.000	0.000
6.0	-.014	-.008	-.003	0.000	0.000	0.000	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.5	-.012	-.007	-.003	-.001	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.0	-.011	-.006	-.003	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	-.009	-.006	-.003	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	-.008	-.005	-.003	-.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.5	-.007	-.005	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.0	-.007	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.5	-.006	-.004	-.002	-.001	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	-.005	-.003	-.002	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XLVI IMAGINARY PART OF ENTROPY ADMITTANCE COEFFICIENT $S_{\nu h} = 7$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	.158	.149	.140	.129	.118	.112	.104	.099	.091	.077	.065	.052	.041	.026	.012	.003	0.000	0.000
1.0	.219	.206	.188	.167	.144	.133	.116	.100	.073	.047	.037	.019	.011	.003	.001	0.000	0.000	0.000
1.5	.217	.200	.179	.153	.126	.112	.092	.074	.044	.021	.011	.004	.001	.001	0.000	0.000	0.000	0.000
2.0	.193	.174	.153	.127	.099	.086	.066	.048	.022	.007	.002	.001	.001	0.000	0.000	0.000	0.000	0.000
2.5	.168	.147	.126	.101	.075	.063	.045	.030	.010	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
3.0	.146	.123	.107	.079	.055	.045	.029	.017	.003	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.5	.128	.105	[illegible]	.061	.040	.031	.019	.009	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.0	.114	.091	.069	.047	.029	.021	.012	.005	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.5	.103	.080	.059	.038	.021	.015	.007	.003	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.0	.093	.073	.052	.032	.017	.011	.006	.003	.001	0.000	.001	-.001	-.001	0.000	0.000	0.000	0.000	0.000
5.5	.086	.066	.047	.029	.015	.011	.006	.004	.002	.003	.001	0.000	.001	0.000	0.000	0.000	0.000	0.000
6.0	.079	.061	.043	.027	.016	.012	.008	.005	.004	.002	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.5	.073	.057	.040	.026	.015	.012	.008	.005	.003	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
7.0	.068	.053	.038	.025	.015	.012	.008	.005	.003	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
7.5	.064	.049	.036	.023	.014	.011	.008	.005	.002	.001	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
8.0	.060	.046	.034	.022	.014	.011	.007	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
8.5	.057	.044	.032	.021	.013	.010	.007	.004	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.0	.054	.042	.030	.020	.013	.010	.007	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
9.5	.051	.039	.029	.019	.012	.009	.006	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
10.0	.048	.037	.027	.018	.011	.009	.006	.004	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XLVII REAL PART OF ENTROPY ADMITTANCE COEFFICIENT $S\nu h$ = 9

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	[illegible]	.099	.074	.050
.5	-.373	-.329	-.284	-.236	-.189	-.168	-.138	-.111	-.070	-.039	-.022	-.009	-.001	.00[illegible]	.007	.006	.002	0.000
1.0	-.256	-.219	-.180	-.139	-.100	-.083	-.059	-.037	-.008	.006	.010	.012	.011	.008	.004	.002	0.000	0.000
1.5	-.164	-.133	-.103	-.071	-.043	-.031	-.015	-.001	.013	.015	.013	.010	.007	.004	.002	.001	0.000	0.000
2.0	-.106	-.080	-.055	-.032	-.012	-.004	.006	.014	.018	.014	.010	.006	.004	.002	.001	0.000	0.000	0.000
2.5	-.071	-.048	-.028	-.010	.004	.010	.015	.019	.016	.010	.006	.004	.002	.00[illegible]	.001	0.000	0.000	0.000
3.0	-.050	-.029	-.012	.002	.012	.015	.018	.019	.013	.006	.004	.002	.002	.001	0.000	0.000	0.000	0.000
3.5	-.058	-.018	-.003	.009	.016	.017	.018	.016	.009	.004	.003	.002	.001	0.000	0.000	0.000	0.000	0.000
4.0	-.029	-.012	.002	.012	.016	.017	.016	.013	.006	.003	.002	.001	.001	0.000	0.000	0.000	0.000	0.000
4.5	-.023	-.009	.004	.012	.015	.015	.013	.010	.004	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
5.0	-.019	-.008	.003	.011	.013	.012	.010	.007	.003	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.5	-.016	-.007	.002	.008	.010	.009	.007	.005	.002	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.0	-.014	-.007	0.000	.005	.007	.006	.005	.003	.001	0.000	0.000	0.000	-.001	0.000	0.000	0.000	0.000	0.000
6.5	-.012	-.006	-.001	.003	.004	.003	.002	.001	0.000	-.001	-.002	.001	0.000	0.000	0.000	.001	0.000	0.000
7.0	-.010	-.006	-.002	.001	.001	.001	0.000	-.001	-.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	-.009	-.005	-.002	0.000	0.000	0.000	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	-.008	-.005	-.002	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.5	-.007	-.004	-.002	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.0	.007	-.004	-.002	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.5	-.006	-.004	-.002	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	-.005	-.003	-.002	-.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XLVIII IMAGINARY PART OF ENTROPY ADMITTANCE COEFFICIENT $S\nu h$ = 9

.5	.157	.147	.137	.126	.115	.109	.101	.096	.088	.075	.063	.051	.041	.027	.014	.005	.001	0.000
1.0	.219	.204	.186	.164	.142	.130	.114	.099	.072	.048	.034	.022	.014	.006	.002	0.000	0.000	0.000
1.5	.218	.200	.179	.153	.126	.113	.094	.075	.047	.026	.015	.008	.004	.001	0.000	0.000	0.000	0.000
2.0	.196	.177	.155	.129	.102	.090	.071	.053	.028	.012	.006	.003	.001	.001	0.000	0.000	0.000	0.000
2.5	.171	.152	.131	.106	.081	.069	.052	.036	.016	.005	.002	.001	.001	0.000	0.000	0.000	0.000	0.000
3.0	.148	.129	.109	.087	.063	.053	.037	.024	.008	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
3.5	.129	.110	.091	.070	.049	.040	.027	.015	.004	.002	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.0	.114	.094	.076	.057	.038	.030	.019	.010	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.5	.103	.082	.066	.046	.029	.022	.013	.006	.002	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.0	.093	.073	.054	.037	.072	.016	.009	.004	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.5	.086	.066	.047	.031	.017	.012	.007	.003	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.0	.079	.060	.042	.026	.014	.010	.005	.002	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.5	.073	.056	.039	.024	.012	.008	.004	.002	.001	0.000	-.001	0.000	0.000	0.000	0.000	-.001	0.000	0.000
7.0	.068	.052	.037	.022	.012	.008	.005	.003	.003	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
7.5	.064	.049	.035	.021	.012	.009	.006	.004	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	.060	.046	.033	.021	.012	.009	.006	.004	002	.001	.001	.001	.001	0.000	0.000	0.000	0.000	0.000
8.5	.057	.043	.031	.020	.012	.009	.006	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
9.0	.054	.041	.029	.019	.012	.009	.006	.004	.002	.001	.001	0.000	.001	0.000	0.000	0.000	0.000	0.000
9.5	.051	.039	.028	.019	.011	.009	.006	.004	.002	.001	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000
10.0	.048	.037	.027	.018	.011	.009	.006	.004	.002	.001	.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE XLIX REAL PART OF IRROTATIONAL ADMITTANCE COEFFICIENT $S_{\nu h} = 0$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	.321	.283	.249	.220	.196	.186	.172	.163	.155	.151	.151	.154	.158	.169	.194	.247	.410	1.477
1.0	.667	.607	.557	.516	.486	.475	.461	.459	.478	.529	.569	.682	.793	1.040	1.408	1.392	.868	.755
1.5	.950	.873	.811	.763	.732	.721	.711	.719	.781	.896	1.007	1.126	1.196	1.179	.993	.855	1.023	.958
2.0	1.134	1.046	.975	.919	.882	.870	.857	.869	.943	1.051	1.111	1.119	1.077	.973	.921	1.042	.996	1.071
2.5	1.250	1.154	1.075	1.012	.968	.953	.936	.947	1.017	1.084	1.085	1.038	.988	.953	1.029	1.025	.990	.957
3.0	1.325	1.222	1.137	1.068	1.020	1.002	.981	.991	1.050	1.076	1.043	.994	.973	1.008	1.038	.971	1.019	1.016
3.5	1.375	1.268	1.178	1.105	1.052	1.032	1.009	1.017	1.064	1.056	1.014	.987	.999	1.039	.989	1.023	.985	1.007
4.0	1.410	1.299	1.206	1.129	1.073	1.052	1.027	1.035	1.067	1.037	1.002	1.003	1.027	1.021	.995	1.008	1.019	.987
4.5	1.435	1.322	1.226	1.146	1.088	1.066	1.040	1.046	1.066	1.024	1.005	1.023	1.034	.998	1.022	.991	.989	1.015
5.0	1.453	1.338	1.241	1.159	1.099	1.076	1.049	1.054	1.061	1.018	1.015	1.034	1.022	1.001	1.012	1.016	1.014	.995
5.5	1.467	1.350	1.252	1.168	1.107	1.084	1.056	1.060	1.056	1.018	1.027	1.031	1.009	1.017	.998	1.005	.994	1.001
6.0	1.478	1.360	1.260	1.176	1.113	1.089	1.061	1.064	1.051	1.022	1.034	1.022	1.007	1.021	1.007	.998	1.009	1.007
6.5	1.487	1.368	1.267	1.181	1.118	1.094	1.065	1.067	1.046	1.028	1.036	1.014	1.014	1.012	1.016	1.012	1.000	.995
7.0	1.493	1.374	1.272	1.186	1.122	1.098	1.069	1.069	1.043	1.034	1.032	1.013	1.022	1.006	1.007	1.004	1.004	1.006
7.5	1.499	1.379	1.276	1.189	1.125	1.100	1.071	1.071	1.041	1.038	1.027	1.017	1.023	1.010	1.003	1.002	1.004	1.001
8.0	1.503	1.383	1.280	1.192	1.127	1.103	1.074	1.072	1.040	1.040	1.023	1.023	1.018	1.016	1.011	1.010	1.001	.999
8.5	1.507	1.386	1.283	1.195	1.130	1.105	1.075	1.072	1.040	1.039	1.021	1.026	1.013	1.016	1.012	1.003	1.006	1.005
9.0	1.510	1.389	1.285	1.197	1.131	1.106	1.077	1.073	1.041	1.037	1.023	1.025	1.013	1.011	1.006	1.003	1.000	.998
9.5	1.513	1.391	1.287	1.199	1.133	1.108	1.078	1.073	1.043	1.034	1.026	1.021	1.017	1.009	1.006	1.009	1.006	1.002
10.0	1.516	1.393	1.289	1.200	1.134	1.109	1.079	1.073	1.044	1.032	1.029	1.018	1.020	1.012	1.011	1.003	1.000	1.002

TABLE L IMAGINARY PART OF IRROTATIONAL ADMITTANCE COEFFICIENT $S_{\nu h} = 0$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	.474	.442	.417	.397	.383	.379	.374	.377	.397	.429	.458	.494	.529	.594	.699	.654	1.160	1.811
1.0	.681	.638	.604	.579	.565	.561	.559	.572	.620	.687	.743	.803	.845	.857	.615	.047	-.083	.374
1.5	.694	.649	.612	.583	.564	.558	.552	.562	.596	.614	.591	.510	.393	.177	.056	.161	.313	-.013
2.0	.638	.593	.556	.524	.500	.491	.480	.482	.484	.431	.344	.229	.149	.112	.190	.222	.038	.131
2.5	.569	.527	.490	.458	.432	.422	.409	.406	.382	.292	.205	.140	.127	.168	.187	.067	.142	.088
3.0	.505	.466	.432	.401	.375	.365	.351	.346	.304	.206	.146	.130	.149	.172	.092	.102	.050	.034
3.5	.451	.415	.383	.354	.329	.319	.307	.300	.246	.157	.128	.140	.156	.121	.078	.101	.084	.076
4.0	.405	.373	.343	.315	.293	.283	.272	.263	.202	.131	.127	.143	.134	.080	.101	.053	.051	.037
4.5	.367	.337	.309	.284	.263	.254	.244	.233	.169	.120	.128	.130	.100	.078	.083	.070	.055	.039
5.0	.335	.307	.282	.258	.238	.231	.221	.209	.145	.115	.125	.106	.078	.088	.056	.064	.049	.046
5.5	.308	.282	.258	.236	.218	.211	.202	.189	.127	.114	.115	.085	.074	.083	.059	.042	.039	.025
6.0	.284	.260	.238	.218	.201	.194	.185	.172	.114	.111	.101	.073	.078	.064	.066	.053	.045	.036
6.5	.264	.242	.221	.202	.186	.179	.172	.157	.105	.106	.086	.071	.078	.053	.052	.047	.030	.029
7.0	.246	.225	.206	.188	.173	.167	.160	.144	.098	.099	.074	.072	.071	.055	.042	.035	.040	.023
7.5	.231	.211	.193	.176	.162	.156	.149	.133	.094	.090	.068	.072	.059	.059	.047	.042	.026	.030
8.0	.217	.199	.181	.165	.152	.147	.140	.124	.090	.081	.065	.068	.052	.053	.047	.037	.034	.020
8.5	.205	.187	.171	.156	.143	.138	.132	.115	.088	.072	.065	.060	.050	.044	.037	.030	.025	.023
9.0	.194	.177	.162	.147	.135	.131	.125	.108	.085	.066	.065	.053	.052	.040	.035	.035	.028	.022
9.5	.184	.168	.153	.140	.128	.124	.118	.101	.082	.062	.063	.048	.052	.043	.039	.030	.024	.017
10.0	.175	.160	.146	.133	.122	.118	.113	.095	.079	.059	.060	.048	.048	.043	.036	.026	.023	.021

TABLE LI REAL PART OF IRROTATIONAL ADMITTANCE COEFFICIENT $S_{\nu h}$ = 1

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.657	-.617	-.569	-.507	-.436	-.399	-.343	-.303	-.252	-.212	-.187	-.164	-.146	-.122	-.097	-.075	-.056	-.040
1.0	.090	.077	.069	.065	.064	.064	.066	.068	.076	.088	.101	.121	.144	.204	.375	1.067	2.173	.394
1.5	.515	.474	.442	.421	.412	.411	.415	.430	.487	.596	.720	.905	1.092	1.302	1.103	.773	.816	1.104
2.0	.798	.742	.701	.675	.668	.670	.679	.707	.803	.949	1.051	1.103	1.076	.953	.857	.985	1.023	1.048
2.5	.989	.923	.873	.839	.825	.824	.827	.853	.943	1.036	1.054	1.015	.962	.916	.996	1.027	.960	.939
3.0	1.121	1.045	.986	.943	.919	.913	.908	.928	1.002	1.043	1.019	.970	.947	.982	1.030	.955	1.020	1.021
3.5	1.213	1.129	1.061	1.010	.978	.967	.956	.972	1.030	1.032	.994	.966	.978	1.025	.980	1.012	.975	.997
4.0	1.279	1.188	1.114	1.055	1.016	1.002	.987	1.000	1.042	1.018	.985	.986	1.012	1.013	.984	1.005	1.017	.985
4.5	1.327	1.231	1.151	1.087	1.043	1.027	1.008	1.019	1.046	1.009	.991	1.010	1.024	.991	1.015	.985	.984	1.013
5.0	1.364	1.263	1.179	1.111	1.062	1.044	1.024	1.033	1.045	1.005	1.003	1.024	1.015	.993	1.009	1.011	1.011	.991
5.5	1.391	1.287	1.200	1.128	1.077	1.057	1.035	1.042	1.043	1.007	1.017	1.024	1.003	1.010	.993	1.002	.992	1.000
6.0	1.413	1.306	1.216	1.142	1.087	1.067	1.044	1.049	1.040	1.013	1.026	1.016	1.001	1.017	1.003	.995	1.006	1.005
6.5	1.431	1.321	1.229	1.152	1.096	1.075	1.050	1.054	1.037	1.020	1.029	1.009	1.009	1.009	1.013	1.009	.999	.994
7.0	1.445	1.333	1.239	1.161	1.103	1.081	1.056	1.058	1.035	1.027	1.027	1.008	1.017	1.002	1.005	1.002	1.002	1.005
7.5	1.456	1.343	1.248	1.167	1.108	1.086	1.060	1.061	1.034	1.032	1.022	1.013	1.020	1.006	1.000	1.000	1.003	.999
8.0	1.466	1.351	1.255	1.173	1.113	1.090	1.064	1.063	1.034	1.035	1.018	1.019	1.016	1.014	1.008	1.008	.999	.998
8.5	1.473	1.358	1.260	1.178	1.117	1.094	1.066	1.065	1.034	1.035	1.017	1.023	1.011	1.014	1.010	1.002	1.005	1.004
9.0	1.480	1.364	1.265	1.182	1.120	1.097	1.069	1.066	1.036	1.033	1.019	1.022	1.010	1.009	1.004	1.002	.999	.998
9.5	1.486	1.369	1.269	1.185	1.123	1.099	1.071	1.067	1.038	1.031	1.023	1.019	1.014	1.007	1.004	1.008	1.005	1.002
10.0	1.491	1.373	1.273	1.188	1.125	1.101	1.073	1.067	1.040	1.029	1.026	1.016	1.018	1.010	1.010	1.003	.999	1.002

TABLE LII IMAGINARY PART OF IRROTATIONAL ADMITTANCE COEFFICIENT $S_{\nu h}$ = 1

.5	-.493	-.511	-.531	-.550	-.558	-.555	-.541	-.524	-.494	-.458	-.428	-.392	-.359	-.300	-.217	-.113	.042	.306
1.0	-.031	-.016	.005	.035	.073	.094	.128	.161	.225	.303	.369	.455	.535	.702	1.011	1.523	-.757	-.144
1.5	.248	.259	.279	.309	.348	.370	.404	.444	.528	.620	.676	.694	.635	.335	-.054	.010	.370	-.133
2.0	.362	.363	.372	.380	.409	.421	.438	.460	.495	.475	.397	.263	.152	.075	.162	.264	.012	.187
2.5	.394	.386	.384	.385	.389	.391	.393	.400	.390	.308	.215	.136	.116	.158	.202	.068	.144	.073
3.0	.390	.376	.366	.358	.351	.348	.343	.343	.308	.211	.147	.126	.144	.176	.097	.096	.052	.037
3.5	.372	.355	.340	.326	.314	.309	.302	.297	.248	.159	.126	.138	.156	.127	.074	.105	.081	.078
4.0	.349	.330	.313	.297	.283	.276	.268	.261	.203	.131	.125	.142	.137	.081	.100	.053	.054	.034
4.5	.326	.306	.288	.271	.256	.249	.241	.232	.170	.119	.127	.131	.103	.077	.086	.069	.052	.042
5.0	.305	.284	.266	.248	.233	.227	.219	.208	.145	.115	.125	.108	.079	.087	.056	.066	.051	.045
5.5	.284	.265	.246	.229	.214	.208	.200	.188	.127	.113	.116	.086	.073	.084	.058	.042	.039	.025
6.0	.266	.247	.229	.212	.198	.192	.184	.171	.114	.110	.101	.073	.077	.065	.066	.053	.046	.037
6.5	.249	.231	.214	.197	.183	.178	.171	.157	.105	.106	.087	.070	.078	.053	.053	.048	.030	.028
7.0	.235	.217	.200	.184	.171	.166	.159	.144	.098	.099	.075	.072	.071	.055	.042	.035	.040	.023
7.5	.221	.204	.188	.173	.160	.155	.149	.133	.094	.090	.068	.072	.060	.058	.047	.042	.027	.030
8.0	.209	.193	.177	.163	.151	.146	.140	.123	.090	.081	.065	.068	.052	.054	.047	.037	.033	.019
8.5	.198	.182	.168	.154	.142	.137	.132	.115	.087	.073	.065	.060	.050	.044	.038	.030	.025	.023
9.0	.188	.173	.159	.146	.135	.130	.125	.107	.085	.066	.065	.053	.052	.040	.035	.035	.028	.022
9.5	.179	.165	.151	.138	.128	.123	.118	.101	.082	.062	.063	.048	.052	.042	.038	.030	.025	.017
10.0	.171	.157	.144	.132	.121	.117	.112	.095	.079	.059	.060	.048	.048	.043	.036	.026	.023	.021

TABLE LIII REAL PART OF IRROTATIONAL ADMITTANCE COEFFICIENT $S_{\gamma h}$ = 2

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-1.424	-1.432	-1.450	-1.466	-1.452	-1.423	-1.341	-1.251	-1.091	-.925	-.806	-.686	-.593	-.462	-.331	-.225	-.136	-.070
1.0	-.865	-.845	-.814	-.760	-.679	-.630	-.549	-.484	-.398	-.325	-.280	-.239	-.208	-.167	-.127	-.097	-.075	-.093
1.5	-.339	-.329	-.312	-.283	-.246	-.226	-.196	-.173	-.143	-.117	-.100	-.083	-.068	-.039	.038	1.363	.276	.148
2.0	.077	.073	.079	.094	.118	.132	.157	.184	.254	.381	.546	.841	1.180	1.335	.760	.567	1.322	.638
2.5	.392	.377	.376	.390	.421	.442	.482	.534	.674	.871	.988	1.004	.931	.801	.838	1.079	.837	.925
3.0	.627	.602	.592	.601	.628	.648	.683	.732	.853	.952	.955	.906	.869	.892	1.009	.916	1.028	1.036
3.5	.804	.767	.747	.745	.761	.774	.796	.835	.926	.962	.935	.905	.913	.980	.960	.972	.947	.965
4.0	.938	.890	.860	.846	.849	.855	.866	.897	.964	.963	.935	.934	.966	.990	.951	.997	1.006	.987
4.5	1.041	.983	.943	.918	.910	.910	.913	.938	.985	.964	.947	.969	.994	.970	.991	.968	.974	1.005
5.0	1.120	1.054	1.005	.972	.954	.950	.947	.967	.997	.967	.967	.994	.994	.970	.997	.995	1.001	.982
5.5	1.183	1.110	1.053	1.012	.987	.979	.972	.988	1.003	.974	.987	1.002	.984	.990	.981	.996	.988	1.000
6.0	1.233	1.154	1.091	1.043	1.012	1.001	.990	1.004	1.007	.984	1.002	.998	.983	1.003	.989	.985	.997	.998
6.5	1.273	1.189	1.121	1.067	1.031	1.019	1.005	1.016	1.009	.995	1.009	.993	.992	.999	1.003	1.001	.996	.991
7.0	1.307	1.218	1.145	1.087	1.047	1.033	1.017	1.025	1.010	1.006	1.010	.993	1.004	.992	.998	.998	.995	1.003
7.5	1.334	1.241	1.165	1.103	1.060	1.044	1.026	1.032	1.012	1.014	1.008	.999	1.009	.996	.993	.993	1.001	.995
8.0	1.357	1.261	1.181	1.116	1.070	1.053	1.034	1.038	1.015	1.019	1.006	1.007	1.007	1.005	1.001	1.003	.995	.998
8.5	1.376	1.277	1.195	1.127	1.079	1.061	1.040	1.043	1.017	1.021	1.006	1.013	1.003	1.008	1.005	.999	1.003	1.002
9.0	1.393	1.291	1.206	1.136	1.086	1.067	1.045	1.046	1.021	1.021	1.009	1.014	1.002	1.003	1.000	.998	.996	.996
9.5	1.407	1.303	1.216	1.144	1.092	1.073	1.050	1.049	1.024	1.020	1.013	1.011	1.007	1.001	1.000	1.004	1.003	1.001
10.0	1.419	1.313	1.225	1.151	1.097	1.077	1.054	1.051	1.027	1.019	1.017	1.009	1.011	1.005	1.005	1.000	.997	1.000

TABLE LIV IMAGINARY PART OF IRROTATIONAL ADMITTANCE COEFFICIENT $S_{\gamma h}$ = 2

.5	-.327	-.406	-.514	-.672	-.874	-.983	-1.146	-1.263	-1.396	-1.467	-1.485	-1.476	-1.451	-1.384	-1.267	-1.116	-.909	-.633
1.0	-.338	-.393	-.459	-.534	-.598	-.620	-.635	-.635	-.615	-.576	-.536	-.486	-.438	-.352	-.226	-.063	.211	.967
1.5	-.179	-.188	-.194	-.191	-.173	-.158	-.126	-.092	-.026	.058	.133	.228	.327	.540	1.037	3.478	-.954	.633
2.0	-.018	-.002	.025	.068	.127	.162	.222	.284	.408	.560	.677	.749	.636	-.007	-.200	.192	.418	.185
2.5	.096	.118	.150	.197	.254	.285	.332	.377	.438	.414	.303	.141	.053	.079	.242	.131	.095	-.034
3.0	.165	.183	.208	.243	.280	.297	.320	.339	.332	.237	.149	.104	.119	.183	.129	.062	.079	.069
3.5	.202	.214	.231	.252	.271	.279	.288	.294	.258	.165	.121	.127	.153	.145	.064	.118	.065	.076
4.0	.221	.227	.235	.245	.254	.256	.258	.258	.209	.132	.120	.140	.143	.087	.094	.057	.067	.026
4.5	.228	.228	.231	.234	.235	.235	.234	.230	.173	.118	.124	.133	.110	.073	.092	.062	.045	.051
5.0	.228	.225	.223	.221	.218	.216	.213	.206	.147	.113	.124	.112	.081	.084	.059	.070	.057	.040
5.5	.224	.218	.213	.208	.203	.200	.196	.187	.128	.111	.117	.089	.072	.085	.055	.043	.034	.025
6.0	.217	.210	.203	.196	.189	.186	.181	.170	.114	.109	.103	.074	.075	.068	.066	.050	.048	.039
6.5	.210	.201	.193	.184	.176	.173	.168	.156	.105	.106	.088	.069	.078	.054	.055	.050	.029	.025
7.0	.202	.192	.183	.174	.165	.162	.157	.143	.098	.099	.076	.071	.072	.054	.042	.035	.040	.025
7.5	.194	.184	.174	.164	.156	.152	.147	.133	.093	.091	.068	.072	.061	.058	.046	.041	.027	.029
8.0	.187	.176	.166	.156	.147	.143	.138	.123	.090	.082	.065	.069	.052	.055	.048	.038	.033	.019
8.5	.179	.168	.158	.148	.139	.135	.130	.115	.087	.073	.065	.061	.050	.045	.039	.030	.026	.024
9.0	.172	.161	.151	.141	.132	.128	.124	.107	.084	.066	.065	.054	.052	.040	.034	.035	.027	.021
9.5	.165	.154	.144	.134	.125	.122	.117	.100	.082	.062	.063	.049	.052	.042	.038	.031	.025	.017
10.0	.159	.148	.138	.128	.120	.116	.112	.095	.079	.059	.060	.047	.049	.043	.036	.026	.022	.021

TABLE LV REAL PART OF IRROTATIONAL ADMITTANCE COEFFICIENT $S_{\nu h}$ = 3

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-1.593	-1.626	-1.687	-1.780	-1.884	-1.928	-1.964	-1.958	-1.882	-1.727	-1.577	-1.397	-1.238	-.992	-.719	-.487	-.285	[illegible]
1.0	-1.303	-1.312	-1.316	-1.300	-1.240	-1.187	-1.078	-.974	-.814	-.664	-.565	-.472	-.402	-.309	-.219	-.149	-.097	[illegible]
1.5	-.930	-.930	-.914	-.868	-.782	-.726	-.631	-.553	-.447	-.358	-.304	-.255	-.219	-.173	-.130	-.099	-.088	[illegible]
2.0	-.553	-.551	-.538	-.505	-.452	-.420	-.368	-.326	-.271	-.225	-.198	-.174	-.158	-.141	-.154	-1.323	-.039	[illegible]
2.5	-.213	-.210	-.197	-.171	-.134	-.114	-.081	-.052	.002	.086	.203	.497	1.170	1.665	.438	.380	1.537	.243
3.0	.074	.077	.092	.125	.175	.206	.263	.329	.503	.781	.956	.935	.800	.665	.865	.998	.906	.882
3.5	.311	.309	.323	.358	.416	.451	.514	.588	.747	.865	.856	.804	.788	.879	.958	.874	.928	.912
4.0	.503	.493	.502	.532	.582	.612	.662	.722	.835	.875	.852	.844	.880	.951	.900	.982	.971	1.001
4.5	.658	.640	.639	.660	.697	.719	.755	.803	.884	.889	.875	.898	.939	.938	.945	.947	.967	.980
5.0	.784	.756	.747	.756	.780	.795	.819	.858	.916	.904	.905	.941	.958	.933	.977	.965	.977	.973
5.5	.887	.850	.831	.829	.842	.851	.866	.898	.937	.920	.935	.963	.955	.956	.964	.986	.903	.999
6.0	.971	.926	.898	.887	.889	.893	.902	.929	.951	.937	.960	.968	.954	.979	.966	.971	.981	.985
6.5	1.041	.989	.953	.932	.926	.926	.930	.952	.961	.954	.976	.967	.964	.982	.986	.986	.992	.989
7.0	1.100	1.040	.997	.969	.956	.953	.952	.970	.969	.970	.983	.969	.980	.976	.988	.991	.985	.999
7.5	1.149	1.083	1.034	.999	.980	.974	.969	.985	.976	.984	.985	.976	.990	.979	.981	.983	.996	.989
8.0	1.190	1.120	1.065	1.024	.999	.992	.984	.996	.983	.993	.985	.987	.992	.990	.989	.994	.988	.996
8.5	1.226	1.150	1.091	1.045	1.016	1.006	.996	1.005	.989	.999	.987	.996	.989	.996	.997	.994	.998	.997
9.0	1.256	1.176	1.113	1.063	1.030	1.018	1.006	1.013	.995	1.002	.991	.999	.989	.994	.993	.990	.991	.993
9.5	1.283	1.199	1.131	1.078	1.042	1.029	1.014	1.019	1.001	1.003	.997	.999	.994	.992	.992	.998	.999	1.000
10.0	1.306	1.219	1.148	1.091	1.052	1.038	1.022	1.024	1.006	1.003	1.003	.998	1.000	.995	.998	.996	.994	.996

TABLE LVI IMAGINARY PART OF IRROTATIONAL ADMITTANCE COEFFICIENT $S_{\nu h}$ = 3

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.193	-.264	-.362	-.511	-.727	-.863	-1.102	-1.314	-1.625	-1.882	-2.022	[illegible]	-2.171	-2.179	-2.098	-1.934	-1.675	-1.321
1.0	-.283	-.377	-.499	-.660	-.842	-.929	-1.045	-1.117	-1.179	-1.187	-1.167	-1.127	-1.081	-.992	-.861	-.702	-.484	-.157
1.5	-.260	-.331	-.420	-.525	-.619	-.654	-.684	-.691	-.672	-.625	-.578	-.517	-.459	-.355	-.200	.011	.437	5.698
2.0	-.179	-.210	-.246	-.283	-.304	-.306	-.293	-.270	-.214	-.136	-.064	.029	.126	.330	.813	4.820	-.495	2.575
2.5	-.089	-.089	-.083	-.062	-.020	.009	.066	.129	.263	.456	.658	.957	1.142	-.540	-.309	.399	-.696	.920
3.0	-.011	.004	.030	.076	.142	.182	.250	.317	.424	.427	.265	.036	-.033	.096	.317	-.071	.266	.274
3.5	.047	.067	.097	.143	.199	.227	.269	.300	.300	.191	.105	.088	.130	.192	.052	.121	.009	.027
4.0	.089	.108	.134	.170	.209	.226	.246	.258	.223	.135	.109	.130	.153	.106	.075	.079	.093	.030
4.5	.117	.133	.154	.180	.204	.214	.224	.228	.180	.116	.116	.136	.124	.068	.101	.049	.033	.062
5.0	.136	.147	.162	.180	.195	.200	.205	.204	.151	.109	.121	.120	.088	.076	.068	.075	.064	.029
5.5	.147	.155	.165	.176	.185	.188	.190	.185	.130	.108	.118	.095	.071	.087	.051	[illegible]	.032	.032
6.0	.154	.158	.164	.171	.175	.176	.176	.169	.115	.107	.107	.076	.072	.074	.064	[illegible]	.039	.039
6.5	.157	.159	.162	.164	.165	.165	.164	.155	.104	.105	.092	.068	.077	.055	.059	[illegible]	.030	.022
7.0	.157	.157	.157	.158	.157	.155	.154	.143	.097	.100	.078	.069	.074	.052	.043	.036	.039	.029
7.5	.156	.154	.153	.151	.148	.147	.144	.132	.092	.092	.068	.071	.063	.057	.044	.039	.029	.027
8.0	.154	.151	.148	.145	.141	.139	.136	.122	.089	.083	.064	.069	.053	.056	.048	.040	.031	.019
8.5	.151	.147	.143	.138	.134	.132	.129	.114	.086	.074	.064	.063	.049	.047	.040	.030	.028	.026
9.0	.148	.143	.138	.133	.128	.125	.122	.107	.084	.067	.064	.055	.051	.040	.034	.034	.025	.020
9.5	.144	.139	.133	.127	.122	.119	.116	.100	.081	.062	.063	.049	.052	.041	.037	.032	.027	.018
10.0	.141	.134	.128	.122	.116	.114	.110	.094	.079	.059	.060	.047	.049	.043	.037	.026	.022	.021

TABLE LVII REAL PART OF IRROTATIONAL ADMITTANCE COEFFICIENT $S\nu h = 4$

ω \ μ	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-1.647	-1.689	-1.766	-1.892	-2.060	-2.149	-2.277	-2.361	-2.423	-2.377	-2.276	-2.112	-1.939	-1.627	-1.228	-.851	-.504	-.242
1.0	-1.486	-1.512	-1.545	-1.580	-1.590	-1.574	-1.508	-1.423	-1.256	-1.065	-.926	-.783	-.673	-.518	-.364	-.243	-.142	-.071
1.5	-1.246	-1.264	-1.265	-1.236	-1.158	-1.097	-.981	-.875	-.717	-.576	-.486	-.402	-.341	-.261	-.184	-.126	-.079	-.053
2.0	-.963	-.978	-.973	-.928	-.835	-.774	-.670	-.584	-.466	-.370	-.311	-.258	-.221	-.172	-.129	-.100	-.121	-.075
2.5	-.671	-.681	-.678	-.647	-.582	-.539	-.468	-.410	-.332	-.268	-.231	-.199	-.177	-.156	-.176	-4.539	-.070	-.253
3.0	-.393	-.396	-.388	-.365	-.325	-.300	-.260	-.228	-.182	-.140	-.106	-.048	.105	5.486	.126	.111	.380	1.184
3.5	-.144	-.140	-.124	-.089	-.038	-.006	.051	.114	.284	.656	1.022	.957	.679	.503	.944	.769	1.262	1.312
4.0	.074	.080	.102	.148	.218	.263	.344	.438	.645	.796	.770	.705	.709	.877	.870	.905	.847	.988
4.5	.261	.265	.287	.335	.408	.452	.526	.606	.745	.790	.773	.788	.848	.906	.867	.939	.977	.929
5.0	.420	.419	.438	.480	.545	.581	.640	.704	.802	.816	.817	.860	.903	.888	.939	.919	.937	.978
5.5	.555	.548	.559	.593	.645	.674	.720	.772	.843	.844	.861	.905	.914	.906	.943	.965	.978	.985
6.0	.670	.655	.659	.683	.722	.745	.779	.823	.873	.870	.899	.926	.916	.940	.937	.955	.960	.970
6.5	.767	.745	.740	.754	.783	.799	.825	.862	.895	.896	.927	.931	.926	.957	.959	.963	.984	.989
7.0	.851	.821	.808	.813	.831	.843	.861	.893	.912	.920	.943	.935	.946	.955	.972	.979	.972	.988
7.5	.922	.885	.865	.861	.870	.878	.891	.918	.926	.940	.952	.945	.963	[illegible]	.966	.971	.988	.983
8.0	.984	.940	.913	.901	.903	.907	.915	.937	.938	.956	.956	.958	.970	[illegible]	.971	.980	.980	.994
8.5	1.037	.987	.954	.935	.930	.931	.934	.953	.949	.967	.960	.971	.970	.980	.983	.986	.991	.990
9.0	1.083	1.028	.988	.964	.953	.951	.951	.967	.959	.974	.966	.979	.971	.981	.983	.981	.986	.990
9.5	1.124	1.063	1.018	.988	.972	.968	.965	.978	.968	.978	.974	.981	.977	.979	.981	.989	.992	.996
10.0	1.159	1.094	1.044	1.009	.989	.983	.977	.987	.977	.981	.982	.967	.985	.982	.987	.990	.991	.991

TABLE LVIII IMAGINARY PART OF IRROTATIONAL ADMITTANCE COEFFICIENT $S\nu h = 4$

ω \ μ	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.133	-.193	-.272	-.394	-.581	-.705	-.940	-1.169	-1.551	-1.932	-2.186	-2.417	-2.570	-2.722	-2.759	-2.645	-2.371	-1.945
1.0	-.219	-.318	-.443	-.617	-.842	-.966	-1.160	-1.305	-1.478	-1.575	-1.602	-1.597	-1.571	-1.496	-1.363	-1.191	-.957	-.642
1.5	-.241	-.346	-.477	-.642	-.814	-.893	-.989	[illegible]	-1.078	-1.064	-1.030	-.978	-.924	-.826	-.684	-.511	-.260	.718
2.0	-.214	-.295	-.400	-.526	-.636	-.679	-.715	-.723	-.702	-.649	-.595	-.527	-.463	-.345	-.169	.089	.791	-.704
2.5	-.162	-.207	-.267	-.336	-.392	-.407	-.409	-.394	-.344	-.265	-.191	-.097	0.000	.202	.678	5.393	-.176	-2.210
3.0	-.105	-.120	-.135	-.143	-.133	-.118	-.080	-.031	.079	.245	.428	.753	1.342	-2.511	-.480	.517	-1.048	-1.961
3.5	-.052	-.049	-.035	-.004	.053	.093	.170	.254	.425	.548	.334	-.116	-.170	.105	.450	-.200	.232	-.042
4.0	-.008	.005	.030	.073	.136	.173	.232	.280	.297	.158	.059	.071	.146	.198	-.002	.170	.089	.137
4.5	.027	.044	.070	.111	.161	.185	.217	.236	.200	.112	.098	.134	.150	.071	.098	.033	.037	.047
5.0	.054	.071	.096	.129	.165	.180	.198	.205	.159	.104	.114	.130	.103	.065	.087	.068	.058	.019
5.5	.075	.090	.111	.137	.162	.172	.183	.184	.134	.103	.117	.106	.073	.083	.050	.060	.039	.046
6.0	.090	.103	.120	.139	.157	.163	.170	.167	.117	.104	.110	.081	.068	.081	.057	.039	.043	.032
6.5	.101	.111	.124	.139	.151	.155	.159	.154	.105	.103	.097	.068	.074	.060	.063	.054	.036	.022
7.0	.109	.116	.126	.137	.145	.147	.149	.142	.097	.100	.082	.067	.075	.050	.047	.040	.034	.034
7.5	.114	.119	.126	.134	.139	.140	.141	.131	.091	.093	.070	.070	.067	.054	.041	.036	.033	.023
8.0	.117	.121	.125	.130	.133	.133	.133	.122	.088	.085	.064	.070	.055	.057	.048	.042	.028	.021
8.5	.119	.121	.124	.126	.127	.127	.126	[illegible]	.085	.076	.063	.065	.049	.049	.043	.031	.031	.026
9.0	.119	.120	.121	.122	.122	.121	.120	.106	.083	.068	.063	.056	.049	.041	.034	.032	.023	.018
9.5	.119	.119	.119	.118	.117	.116	.114	.100	.081	.062	.063	.050	.051	.040	.036	.034	.028	.020
10.0	.118	.117	.116	.114	.112	.111	.109	.094	.078	.059	.061	.047	.050	.043	.038	.026	.021	.020

TABLE LIX REAL PART OF IRROTATIONAL ADMITTANCE COEFFICIENT $S\nu h$ = 5

ω \ M	.402	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-1.671	-1.715	-1.799	-1.943	-2.143	-2.258	-2.442	-2.588	-2.763	-2.841	-2.821	-2.724	-2.584	-2.274	-1.801	-1.293	-.784	-.379
1.0	-1.572	-1.605	-1.656	-1.731	-1.804	-1.827	-1.826	-1.788	-1.663	-1.476	-1.317	-1.141	-.994	-.779	-.554	-.369	-.214	-.103
1.5	-1.415	-1.441	-1.459	-1.465	-1.432	-1.392	-1.294	-1.191	-1.013	-.834	-.712	-.594	-.505	-.385	-.268	-.179	-.106	-.055
2.0	-1.213	-1.240	-1.240	-1.203	-1.114	-1.050	-.929	-.823	-.668	-.531	-.446	-.367	-.311	-.236	-.167	-.114	-.075	-.088
2.5	-.983	-1.011	-1.012	-.966	-.866	-.801	-.690	-.600	-.476	-.375	-.314	-.259	-.220	-.171	-.127	-.102	-.273	-.051
3.0	-.744	-.767	-.773	-.743	-.665	-.613	-.526	-.456	-.363	-.288	-.245	-.207	-.182	-.156	-.173	-8.852	-.074	-.056
3.5	-.512	-.524	-.526	-.508	-.462	-.430	-.376	-.333	-.274	-.228	-.205	-.194	-.212	-1.130	-.097	-.066	-.024	.008
4.0	-.296	-.298	-.288	-.261	-.215	-.187	-.138	-.091	.017	.306	1.054	1.335	.566	.285	1.000	.463	1.460	.585
4.5	-.101	-.095	-.074	-.030	.041	.086	.173	.277	.542	.772	.705	.597	.614	.904	.757	1.004	.919	1.000
5.0	.074	.082	.109	.163	.247	.300	.393	.493	.664	.713	.696	.733	.821	.853	.856	.871	.903	.995
5.5	.222	.235	.262	.318	.400	.448	.527	.607	.723	.746	.761	.823	.863	.844	.920	.920	.956	.945
6.0	.363	.367	.391	.442	.515	.556	.620	.686	.771	.785	.817	.367	.870	.884	.906	.943	.943	.966
6.5	.481	.480	.499	.543	.605	.639	.690	.746	.808	.821	.860	.885	.879	.920	.921	.933	.966	.983
7.0	.585	.578	.591	.625	.676	.704	.745	.794	.837	.854	.890	.894	.901	.929	.948	.960	.962	.971
7.5	.675	.662	.668	.694	.734	.756	.790	.831	.861	.883	.908	.905	.925	.929	.949	.959	.973	.980
8.0	.755	.735	.734	.751	.782	.799	.826	.862	.880	.907	.918	.921	.941	.940	.950	.961	.973	.988
8.5	.825	.799	.791	.800	.822	.835	.856	.887	.897	.925	.926	.938	.947	.957	.964	.975	.978	.981
9.0	.887	.855	.840	.841	.856	.865	.881	.907	.912	.938	.934	.952	.949	.964	.971	.971	.981	.988
9.5	.941	.903	.882	.877	.885	.891	.902	.925	.926	.946	.944	.958	.954	.963	.968	.977	.982	.990
10.0	.990	.946	.920	.908	.909	.913	.920	.939	.939	.952	.955	.961	.964	.966	.973	.983	.986	.986

TABLE LX IMAGINARY PART OF IRROTATIONAL ADMITTANCE COEFFICIENT $S\nu h$ = 5

ω \ M	.402	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.101	-.151	-.216	-.317	-.475	-.583	-.794	-1.010	-1.399	-1.831	-2.154	-2.484	-2.735	-3.049	-3.257	-3.252	-3.012	-2.537
1.0	-.177	-.267	-.380	-.544	-.771	-.907	-1.139	-1.335	-1.603	-1.801	-1.893	-1.945	-1.955	-1.917	-1.802	-1.625	-1.368	-1.026
1.5	-.212	-.326	-.464	-.644	-.857	-.966	-1.124	-1.231	-1.340	-1.379	-1.372	-1.339	-1.295	-1.203	-1.061	-.888	-.651	-.304
2.0	-.210	-.322	-.462	-.629	-.797	-.869	-.955	-.999	-1.019	-.992	-.950	-.892	-.833	-.727	-.575	-.387	-.086	.870
2.5	-.183	-.271	-.390	-.530	-.653	-.696	-.735	-.743	-.719	-.661	-.603	-.530	-.460	-.334	-.139	.169	1.625	.099
3.0	-.145	-.200	-.279	-.375	-.456	-.479	-.490	-.478	-.428	-.347	-.271	-.174	-.076	.130	.622	-3.544	.101	-.362
3.5	-.105	-.132	-.165	-.204	-.228	-.228	-.210	-.176	-.089	.044	.182	.402	.729	3.434	-.706	.479	-.698	-.565
4.0	-.068	-.074	-.076	-.064	-.028	.004	.072	.156	.359	.703	.860	-.418	-.461	.061	.838	-.264	-.587	-.364
4.5	-.035	-.029	-.014	.021	.083	.125	.200	.272	.332	.137	-.008	.041	.169	.219	-.037	.169	.207	-.097
5.0	-.006	.005	.027	.067	.123	.154	.198	.225	.187	.089	.090	.142	.147	.040	.120	.026	.012	.061
5.5	.017	.031	.055	.091	.134	.154	.179	.169	.143	.096	.113	.123	.085	.069	.062	.079	.063	.049
6.0	.036	.051	.072	.103	.135	.149	.165	.168	.122	.099	.113	.093	.064	.085	.046	.039	.029	.018
6.5	.052	.065	.084	.109	.134	.143	.154	.153	.107	.100	.103	.071	.068	.070	.063	.048	.047	.033
7.0	.064	.076	.092	.112	.131	.137	.145	.141	.097	.099	.087	.064	.074	.051	.054	.048	.027	.032
7.5	.073	.084	.097	.113	.127	.132	.137	.131	.091	.095	.073	.067	.071	.053	.039	.032	.039	.018
8.0	.080	.089	.100	.113	.123	.126	.130	.121	.086	.087	.065	.069	.059	.056	.045	.042	.025	.027
8.5	.086	.093	.102	.111	.119	.121	.123	.113	.084	.079	.062	.066	.050	.053	.046	.035	.033	.024
9.0	.090	.095	.102	.109	.115	.116	.117	.106	.082	.070	.062	.059	.048	.043	.036	.029	.022	.017
9.5	.092	.097	.102	.107	.110	.111	.112	.099	.080	.063	.062	.051	.050	.039	.034	.035	.029	.023
10.0	.094	.097	.101	.105	.107	.107	.107	.094	.077	.059	.061	.047	.051	.042	.038	.027	.021	.019

TABLE LXI REAL PART OF IRROTATIONAL ADMITTANCE COEFFICIENT $S_{\nu h}$ = 7

ω \ H	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-1.689	-1.736	-1.827	-1.985	-2.215	-2.355	-2.594	-2.807	-3.123	-3.383	-3.516	-3.587	-3.576	-3.411	-2.967	-2.308	-1.488	-.744
1.0	-1.643	-1.682	-1.754	-1.873	-2.025	-2.103	-2.209	-2.269	-2.284	-2.190	-2.060	-1.876	-1.697	-1.394	-1.030	-.703	-.412	-.196
1.5	-1.568	-1.599	-1.645	-1.711	-1.773	-1.788	-1.773	-1.724	-1.583	-1.387	-1.226	-1.053	-.913	-.709	-.500	-.332	-.192	-.092
2.0	-1.464	-1.492	-1.512	-1.528	-1.511	-1.480	-1.393	-1.295	-1.114	-.923	-.790	-.659	-.561	-.427	.291	0.000	-.114	-.057
2.5	-1.332	-1.365	-1.367	-1.342	-1.272	-1.214	-1.097	-.986	-.812	-.651	-.548	-.451	-.381	-.288	-.200	-.134	-.081	-.055
3.0	-1.177	-1.219	-1.217	-1.168	-1.067	-.999	-.875	-.769	-.616	-.485	-.405	-.332	-.279	-.212	-.149	-.104	-.080	-.071
3.5	-1.007	-1.054	-1.062	-1.008	-.898	-.828	-.711	-.616	-.485	-.379	-.315	-.259	-.219	-.169	-.126	-.112	-.321	-9.136
4.0	-.831	-.874	-.894	-.856	-.756	-.692	-.588	-.505	-.396	-.309	-.259	-.216	-.186	-.154	-.170	-.514	-.122	-.196
4.5	-.657	-.688	-.709	-.694	-.624	-.574	-.490	-.423	-.334	-.266	-.229	-.203	-.196	-.339	-.251	-.125	-.074	-.089
5.0	-.490	-.508	-.518	-.508	-.466	-.435	-.380	-.334	-.274	-.234	-.231	-.387	-3.142	-.104	-.109	-.039	-.001	.030
5.5	-.333	-.338	-.332	-.307	-.257	-.224	-.164	-.098	.109	1.015	.870	.315	.260	1.048	.399	1.514	1.439	.801
6.0	-.186	-.183	-.164	-.118	-.039	.015	.123	.257	.555	.597	.523	.604	.812	.722	.914	.778	.916	.855
6.5	-.050	-.042	-.015	.044	.139	.201	.313	.428	.586	.612	.654	.759	.783	.776	.843	.914	.894	.917
7.0	.074	.084	.116	.180	.277	.336	.432	.524	.636	.674	.732	.789	.787	.847	.856	.884	.941	.968
7.5	.187	.197	.230	.293	.385	.438	.520	.598	.685	.726	.783	.806	.817	.868	.899	.924	.930	.962
8.0	.290	.299	.330	.390	.473	.519	.590	.658	.725	.770	.815	.824	.852	.872	.906	.924	.953	.957
8.5	.384	.390	.417	.472	.546	.586	.647	.707	.759	.807	.836	.848	.879	.888	.909	.932	.948	.974
9.0	.469	.471	.494	.542	.608	.642	.695	.748	.788	.837	.851	.873	.891	.911	.930	.948	.963	.974
9.5	.547	.544	.562	.603	.660	.690	.735	.782	.814	.858	.866	.894	.898	.922	.938	.945	.960	.971
10.0	.617	.610	.623	.657	.705	.731	.770	.811	.837	.875	.882	.906	.908	.925	.938	.957	.970	.981

TABLE LXII IMAGINARY PART OF IRROTATIONAL ADMITTANCE COEFFICIENT $S_{\nu h}$ = 7

.5	-.069	-.105	-.152	-.226	-.343	-.425	-.591	-.767	-1.111	-1.537	-1.897	-2.316	-2.684	-3.253	-3.831	-4.163	-4.132	-3.650
1.0	-.127	-.198	-.286	-.415	-.617	-.745	-.986	-1.216	-1.588	-1.940	-2.161	-2.349	-2.462	-2.555	-2.535	-2.389	-2.107	-1.697
1.5	-.167	-.269	-.388	-.556	-.785	-.921	-1.149	-1.337	-1.588	-1.762	-1.837	-1.872	-1.870	-1.818	-1.692	-1.511	-1.254	-.910
2.0	-.185	-.309	-.449	-.631	-.853	-.970	-1.145	-1.270	-1.404	-1.462	-1.464	-1.437	-1.395	-1.304	-1.159	0.000	-.735	-.377
2.5	-.182	-.315	-.468	-.648	-.842	-.933	-1.052	-1.123	-1.176	-1.170	-1.138	-1.085	-1.029	-.923	-.771	-.584	-.307	.314
3.0	-.166	-.290	-.444	-.616	-.776	-.842	-.915	-.948	-.951	-.909	-.859	-.792	-.727	-.610	-.439	-.212	.271	-.647
3.5	-.144	-.242	-.383	-.541	-.672	-.718	-.758	-.765	-.736	-.671	-.608	-.529	-.453	-.312	-.084	.356	-1.809	-9.915
4.0	-.120	-.188	-.296	-.431	-.537	-.569	-.587	-.576	-.524	-.440	-.361	-.261	-.160	.052	.600	-1.999	.800	1.697
4.5	-.096	-.137	-.203	-.295	-.372	-.392	-.394	-.371	-.296	-.182	-.071	.089	.289	1.090	-1.009	.516	-.102	.907
5.0	-.074	-.094	-.124	-.161	-.185	-.184	-.159	-.113	.011	.227	.523	1.477	-5.748	-.366	.912	-.393	-.869	.872
5.5	-.053	-.061	-.065	-.059	-.025	.008	.086	.190	.469	.594	-.433	-.201	.144	.708	-.138	-.362	-.472	.578
6.0	-.035	-.034	-.025	.002	.058	.099	.175	.242	.213	.007	.050	.193	.178	-.037	.157	.139	.168	.120
6.5	-.019	-.012	.003	.035	.087	.118	.160	.178	.119	.078	.119	.112	.047	.092	.021	.026	.011	.009
7.0	-.004	.005	.023	.054	.096	.117	.142	.149	.103	.092	.107	.071	.058	.076	.062	.044	.048	.018
7.5	.008	.019	.037	.065	.099	.114	.131	.133	.092	.094	.090	.060	.070	.049	.056	.048	.026	.037
8.0	.019	.030	.047	.072	.099	.110	.123	.122	.086	.092	.073	.062	.071	.046	.036	.029	.036	.022
8.5	.028	.039	.055	.077	.099	.107	.117	.114	.081	.085	.063	.066	.059	.055	.044	.042	.025	.018
9.0	.036	.046	.061	.080	.097	.104	.112	.106	.079	.077	.059	.065	.048	.051	.045	.031	.031	.026
9.5	.043	.052	.065	.081	.095	.101	.107	.099	.077	.068	.059	.058	.046	.041	.033	.030	.022	.019
10.0	.048	.057	.068	.082	.093	.098	.102	.093	.075	.061	.060	.049	.049	.038	.034	.034	.027	.016

TABLE LXIII REAL PART OF IRROTATIONAL ADMITTANCE COEFFICIENT $S_{\nu h}$ = 9

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-1.696	-1.744	-1.837	-2.002	-2.245	-2.394	-2.659	-2.902	-3.289	-3.656	-3.894	-4.099	-4.215	-4.255	-3.998	-3.360	-2.326	-1.717
1.0	-1.670	-1.712	-1.794	-1.933	-2.126	-2.235	-2.405	-2.536	-2.677	-2.711	-2.658	-2.530	-2.371	-2.048	-1.509	-1.123	-.672	-.322
1.5	-1.627	-1.662	-1.726	-1.829	-1.954	-2.012	-2.077	-2.097	-2.046	-1.898	-1.743	-1.550	-1.376	-1.103	-.798	-.537	-.311	-.148
2.0	-1.568	-1.596	-1.639	-1.701	-1.756	-1.767	-1.745	-1.690	-1.541	-1.340	-1.180	-1.039	-.871	-.675	-.474	-.313	-.181	-.086
2.5	-1.491	-1.517	-1.539	-1.561	-1.554	-1.529	-1.450	-1.356	-1.174	-.977	-.838	-.700	-.596	-.454	-.315	-.207	-.120	-.059
3.0	-1.396	-1.427	-1.431	-1.418	-1.364	-1.314	-1.204	-1.094	-.912	-.737	-.622	-.513	-.433	-.327	-.226	-.149	-.088	-.051
3.5	-1.283	-1.325	-1.320	-1.280	-1.193	-1.128	-1.006	-.894	-.725	-.574	-.480	-.393	-.331	-.249	-.173	-.116	-.074	-.702
4.0	-1.156	-1.211	-1.207	-1.150	-1.043	-.972	-.847	-.741	-.589	-.461	-.383	-.313	-.263	-.199	-.140	-.100	-.157	-.084
4.5	-1.020	-1.082	-1.091	-1.029	-.914	-.842	-.721	-.623	-.489	-.380	-.315	-.258	-.218	-.167	-.125	-.139	-.079	-.051
5.0	-.881	-.939	-.966	-.916	-.802	-.733	-.621	-.533	-.415	-.322	-.268	-.221	-.190	-.154	-.175	-.174	-6.334	-.058
5.5	-.742	-.790	-.825	-.800	-.703	-.641	-.540	-.462	-.359	-.281	-.237	-.202	-.184	-.230	-.268	-.154	-.091	-.103
6.0	-.607	-.640	-.669	-.666	-.602	-.553	-.471	-.405	-.320	-.258	-.232	-.241	-.468	-.294	-.155	-.112	-.079	-.197
6.5	-.477	-.496	-.511	-.507	-.468	-.437	-.381	-.334	-.275	-.250	-.429	-.281	-.122	-.113	-.055	.253	.037	.149
7.0	-.353	-.361	-.359	-.336	-.284	-.247	-.174	-.085	.315	.834	.292	.263	.753	.511	1.333	.755	1.422	.925
7.5	-.235	-.235	-.219	-.175	-.092	-.033	.092	.250	.514	.469	.519	.747	.716	.717	.714	.784	.908	1.023
8.0	-.125	-.119	-.094	-.035	.065	.133	.257	.379	.518	.566	.660	.717	.708	.814	.844	.885	.890	.959
8.5	-.022	-.013	.019	.085	.189	.254	.362	.461	.571	.637	.709	.733	.764	.810	.863	.890	.930	.935
9.0	.074	.085	.120	.188	.290	.349	.444	.530	.621	.690	.743	.762	.806	.825	.862	.895	.922	.954
9.5	.163	.175	.210	.277	.374	.428	.511	.588	.663	.733	.766	.795	.829	.856	.887	.919	.943	.965
10.0	.247	.257	.291	.356	.445	.494	.568	.637	.699	.766	.787	.825	.840	.876	.903	.916	.938	.959

TABLE LXIV IMAGINARY PART OF IRROTATIONAL ADMITTANCE COEFFICIENT $S_{\nu h}$ = 9

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.052	-.081	-.117	-.175	-.267	-.332	-.465	-.610	-.901	-1.281	-1.621	-2.044	-2.446	-3.140	-4.000	-4.707	-5.031	-4.676
1.0	-.099	-.156	-.226	-.334	-.501	-.613	-.832	-1.053	-1.447	-1.872	-2.178	-2.480	-2.699	-2.955	-3.094	-3.040	-2.777	-2.311
1.5	-.137	-.220	-.319	-.465	-.678	-.813	-1.057	-1.280	-1.622	-1.917	-2.084	-2.209	-2.272	-2.297	-2.221	-2.051	-1.772	-1.388
2.0	-.163	-.270	-.392	-.562	-.792	-.928	-1.153	-1.337	-1.578	-1.740	-1.806	-1.833	-1.825	-1.765	-1.634	-1.451	-1.193	-.846
2.5	-.173	-.301	-.441	-.622	-.848	-.969	-1.154	-1.288	-1.437	-1.507	-1.515	-1.493	-1.454	-1.363	-1.217	-1.035	-.784	-.420
3.0	-.169	-.312	-.464	-.648	-.854	-.956	-1.096	-1.186	-1.264	-1.275	-1.252	-1.204	-1.150	-1.047	-.894	-.707	-.437	.087
3.5	-.156	-.300	-.462	-.642	-.823	-.904	-1.003	-1.057	-1.086	-1.061	-1.018	-.956	-.894	-.781	-.617	-.409	-.050	-3.544
4.0	-.138	-.270	-.435	-.608	-.763	-.826	-.892	-.919	-.913	-.863	-.807	-.735	-.665	-.539	-.351	-.074	1.051	.819
4.5	-.119	-.226	-.384	-.550	-.683	-.729	-.769	-.776	-.744	-.676	-.609	-.525	-.445	-.294	-.036	.627	-.258	.101
5.0	-.102	-.179	-.312	-.468	-.583	-.618	-.638	-.629	-.577	-.491	-.411	-.309	-.207	.010	.632	-.863	6.728	-.405
5.5	-.085	-.136	-.231	-.363	-.464	-.489	-.496	-.475	-.403	-.294	-.191	-.047	.121	.700	-1.059	.739	.477	-1.021
6.0	-.070	-.101	-.157	-.243	-.319	-.336	-.331	-.298	-.201	-.047	.124	.447	1.237	-.960	.508	-.387	-.268	-1.756
6.5	-.056	-.073	-.098	-.133	-.156	-.152	-.120	-.061	.109	.490	1.619	-1.650	-.341	.631	-.568	-6.239	-1.035	-1.574
7.0	-.044	-.051	-.057	-.054	-.022	.013	.101	.226	.544	-.287	-.190	.226	.609	-.325	.447	.549	-.358	-.501
7.5	-.032	-.033	-.028	-.007	.045	.085	.159	.213	.105	.029	.150	.133	-.031	.160	.012	-.033	.141	-.037
8.0	-.022	-.018	-.008	.019	.067	.096	.135	.145	.084	.092	.108	.047	.060	.053	.075	.066	.011	.047
8.5	-.012	-.006	.007	.035	.074	.094	.118	.122	.082	.091	.079	.054	.073	.039	.033	.023	.039	.027
9.0	-.003	.004	.019	.044	.077	.092	.109	.110	.078	.086	.063	.061	.063	.050	.039	.041	.022	.013
9.5	.004	.013	.027	.051	.078	.090	.103	.102	.074	.078	.056	.064	.048	.053	.046	.031	.030	.023
10.0	.011	.020	.034	.055	.078	.087	.098	.095	.073	.069	.055	.059	.043	.041	.033	.028	.022	.023

TABLE LXV REAL PART OF COMBINED ADMITTANCE COEFFICIENT $S_{\nu h} = 0$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	-.321	-.283	-.249	-.220	-.196	-.186	-.172	-.163	-.155	-.151	-.151	-.154	-.158	-.169	-.194	-.247	-.410	-1.477
1.0	-.667	-.607	-.557	-.516	-.486	-.475	-.461	-.459	-.478	-.529	-.589	-.682	-.793	-1.040	-1.408	-1.392	-.868	-.755
1.5	-.950	-.873	-.811	-.763	-.732	-.721	-.711	-.719	-.781	-.896	-1.007	-1.126	-1.196	-1.179	-.993	-.855	-1.023	-.958
2.0	-1.134	-1.046	-.975	-.919	-.882	-.870	-.857	-.869	-.943	-1.051	-1.111	-1.119	-1.077	-.973	-.921	-1.042	-.996	-1.071
2.5	-1.250	-1.154	-1.075	-1.012	-.968	-.953	-.936	-.947	-1.017	-1.084	-1.085	-1.038	-.988	-.953	-1.029	-1.025	-.990	-.957
3.0	-1.325	-1.222	-1.137	-1.068	-1.020	-1.002	-.981	-.991	-1.050	-1.076	-1.043	-.994	-.973	-1.008	-1.038	-.971	-1.019	-1.016
3.5	-1.375	-1.268	-1.178	-1.105	-1.052	-1.032	-1.009	-1.017	-1.064	-1.056	-1.014	-.987	-.999	-1.039	-.989	-1.023	-.985	-1.007
4.0	-1.410	-1.299	-1.206	-1.129	-1.073	-1.052	-1.027	-1.035	-1.067	-1.037	-1.002	-1.003	-1.027	-1.021	-.995	-1.008	-1.019	-.987
4.5	-1.435	-1.322	-1.226	-1.146	-1.088	-1.066	-1.040	-1.046	-1.066	-1.024	-1.005	-1.023	-1.034	-.998	-1.022	-.991	-.989	-1.015
5.0	-1.453	-1.338	-1.241	-1.159	-1.099	-1.076	-1.049	-1.054	-1.061	-1.018	-1.015	-1.034	-1.022	-1.001	-1.012	-1.016	-1.014	-.995
5.5	-1.467	-1.350	-1.252	-1.168	-1.107	-1.084	-1.056	-1.060	-1.056	-1.018	-1.027	-1.031	-1.009	-1.017	-.998	-1.005	-.994	-1.001
6.0	-1.478	-1.360	-1.260	-1.176	-1.113	-1.089	-1.061	-1.064	-1.051	-1.022	-1.034	-1.022	-1.007	-1.021	-1.007	-.998	-1.009	-1.007
6.5	-1.487	-1.368	-1.267	-1.181	-1.118	-1.094	-1.065	-1.067	-1.046	-1.028	-1.036	-1.014	-1.014	-1.012	-1.016	-1.012	-1.000	-.995
7.0	-1.493	-1.374	-1.272	-1.186	-1.122	-1.098	-1.069	-1.069	-1.043	-1.034	-1.032	-1.013	-1.022	-1.006	-1.007	-1.004	-1.004	-1.006
7.5	-1.499	-1.379	-1.276	-1.189	-1.125	-1.100	-1.071	-1.071	-1.041	-1.038	-1.027	-1.017	-1.023	-1.010	-1.003	-1.002	-1.004	-1.001
8.0	-1.503	-1.383	-1.280	-1.192	-1.127	-1.103	-1.074	-1.072	-1.040	-1.040	-1.023	-1.023	-1.018	-1.016	-1.011	-1.010	-1.001	-.999
8.5	-1.507	-1.386	-1.283	-1.195	-1.130	-1.105	-1.075	-1.072	-1.040	-1.039	-1.021	-1.026	-1.013	-1.016	-1.012	-1.003	-1.006	-1.005
9.0	-1.510	-1.389	-1.285	-1.197	-1.131	-1.106	-1.077	-1.073	-1.041	-1.037	-1.023	-1.025	-1.013	-1.011	-1.006	-1.003	-1.000	-.998
9.5	-1.513	-1.391	-1.287	-1.199	-1.133	-1.108	-1.078	-1.073	-1.043	-1.034	-1.026	-1.021	-1.017	-1.009	-1.006	-1.009	-1.006	-1.002
10.0	-1.516	-1.393	-1.289	-1.200	-1.134	-1.109	-1.079	-1.073	-1.044	-1.032	-1.029	-1.018	-1.020	-1.012	-1.011	-1.003	-1.000	-1.002

TABLE LXVI IMAGINARY PART OF COMBINED ADMITTANCE COEFFICIENT $S_{\nu h} = 0$

.5	-.474	-.442	-.417	-.397	-.383	-.379	-.374	-.377	-.397	-.429	-.458	-.494	-.529	-.594	-.699	-.654	-1.160	-1.811
1.0	-.601	-.638	-.604	-.579	-.565	-.561	-.559	-.572	-.620	-.687	-.743	-.803	-.845	-.857	-.615	-.047	.083	-.374
1.5	-.694	-.649	-.612	-.583	-.564	-.558	-.552	-.562	-.596	-.614	-.591	-.510	-.393	-.177	-.056	-.161	-.313	.013
2.0	-.638	-.593	-.556	-.524	-.500	-.491	-.480	-.482	-.484	-.431	-.344	-.229	-.149	-.112	-.190	-.222	-.038	-.131
2.5	-.569	-.527	-.490	-.458	-.432	-.422	-.409	-.406	-.382	-.292	-.205	-.140	-.127	-.168	-.107	-.067	-.142	-.088
3.0	-.505	-.466	-.432	-.401	-.375	-.365	-.351	-.346	-.304	-.206	-.146	-.130	-.149	-.172	-.092	-.102	-.050	-.034
3.5	-.451	-.415	-.383	-.354	-.329	-.319	-.307	-.300	-.246	-.157	-.128	-.140	-.156	-.121	-.078	-.101	-.084	-.076
4.0	-.405	-.373	-.343	-.315	-.293	-.283	-.272	-.263	-.202	-.131	-.127	-.143	-.134	-.080	-.101	-.053	-.051	-.037
4.5	-.367	-.337	-.309	-.284	-.263	-.254	-.244	-.233	-.169	-.120	-.128	-.130	-.100	-.078	-.083	-.070	-.055	-.039
5.0	-.335	-.307	-.282	-.258	-.238	-.231	-.221	-.209	-.145	-.115	-.125	-.106	-.078	-.088	-.056	-.064	-.049	-.046
5.5	-.308	-.282	-.258	-.236	-.218	-.211	-.202	-.189	-.127	-.114	-.115	-.085	-.074	-.083	-.059	-.042	-.039	-.025
6.0	-.284	-.260	-.238	-.218	-.201	-.194	-.185	-.172	-.114	-.111	-.101	-.073	-.078	-.064	-.066	-.053	-.045	-.036
6.5	-.264	-.242	-.221	-.202	-.186	-.179	-.172	-.157	-.105	-.106	-.086	-.071	-.078	-.053	-.052	-.047	-.030	-.029
7.0	-.246	-.225	-.206	-.188	-.173	-.167	-.160	-.144	-.098	-.099	-.074	-.072	-.071	-.055	-.042	-.035	-.040	-.023
7.5	-.231	-.211	-.193	-.176	-.162	-.156	-.149	-.133	-.094	-.090	-.068	-.072	-.059	-.059	-.047	-.042	-.026	-.030
8.0	-.217	-.199	-.181	-.165	-.152	-.147	-.140	-.124	-.090	-.081	-.065	-.068	-.052	-.053	-.047	-.037	-.034	-.020
8.5	-.205	-.187	-.171	-.156	-.143	-.138	-.132	-.115	-.088	-.072	-.065	-.060	-.050	-.044	-.037	-.030	-.025	-.023
9.0	-.194	-.177	-.162	-.147	-.135	-.131	-.125	-.108	-.085	-.066	-.065	-.053	-.052	-.040	-.035	-.035	-.028	-.022
9.5	-.184	-.168	-.153	-.140	-.128	-.124	-.118	-.101	-.082	-.062	-.063	-.048	-.052	-.043	-.039	-.030	-.024	-.017
10.0	-.175	-.160	-.146	-.133	-.122	-.118	-.113	-.095	-.079	-.059	-.060	-.048	-.048	-.043	-.036	-.026	-.023	-.021

TABLE LXVII REAL PART OF COMBINED ADMITTANCE COEFFICIENT $S_{\nu h}$ = 1

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	.164	.192	.215	.231	.235	.233	.224	.216	.205	.194	.185	.172	.159	.133	.100	.074	.056	.041
1.0	-.308	-.246	-.193	-.148	-.118	-.107	-.096	-.091	-.093	-.101	-.110	-.124	-.142	-.202	-.383	-1.080	-2.159	-.393
1.5	-.700	-.619	-.552	-.499	-.465	-.455	-.446	-.452	-.499	-.601	-.725	-.913	-1.099	-1.304	-1.103	-.773	.816	-1.105
2.0	-.960	-.868	-.795	-.740	-.709	-.702	-.700	-.721	-.812	-.956	-1.056	-1.106	-1.079	-.955	-.853	-.985	-1.023	-1.048
2.5	-1.125	-1.026	-.948	-.889	-.856	-.847	-.843	-.863	-.950	-1.041	-1.058	-1.017	-.963	-.917	-.997	-1.028	-.960	-.939
3.0	-1.232	-1.128	-1.045	-.981	-.942	-.930	-.920	-.936	-1.007	-1.047	-1.021	-.972	-.948	-.982	-1.031	-.956	-1.020	-1.021
3.5	-1.304	-1.196	-1.108	-1.040	-.995	-.981	-.965	-.978	-1.033	-1.034	-.996	-.968	-.979	-1.026	-.981	-1.012	-.975	-.997
4.0	-1.354	-1.243	-1.152	-1.080	-1.030	-1.013	-.994	-1.005	-1.045	-1.020	-.987	-.987	-1.013	-1.014	-.984	-1.005	-1.017	-.986
4.5	-1.390	-1.276	-1.183	-1.107	-1.054	-1.035	-1.014	-1.023	-1.048	-1.010	-.992	-1.011	-1.025	-.991	-1.015	-.985	-.984	-1.013
5.0	-1.416	-1.301	-1.205	-1.127	-1.072	-1.051	-1.028	-1.036	-1.047	-1.006	-1.004	-1.025	-1.016	-.993	-1.009	-1.011	-1.011	-.991
5.5	-1.436	-1.319	-1.222	-1.142	-1.084	-1.063	-1.039	-1.045	-1.044	-1.008	-1.018	-1.024	-1.003	-1.011	-.994	-1.003	-.992	-1.000
6.0	-1.452	-1.334	-1.235	-1.153	-1.094	-1.072	-1.047	-1.051	-1.041	-1.013	-1.027	-1.016	-1.001	-1.017	-1.003	-.995	-1.006	-1.005
6.5	-1.464	-1.345	-1.245	-1.162	-1.102	-1.079	-1.053	-1.056	-1.038	-1.021	-1.030	-1.009	-1.009	-1.009	-1.013	-1.010	-.999	-.994
7.0	-1.474	-1.354	-1.253	-1.169	-1.108	-1.085	-1.058	-1.060	-1.036	-1.028	-1.027	-1.008	-1.018	-1.002	-1.005	-1.002	-1.002	-1.005
7.5	-1.482	-1.362	-1.260	-1.175	-1.113	-1.090	-1.062	-1.063	-1.035	-1.033	-1.023	-1.013	-1.020	-1.006	-1.000	-1.000	-1.003	-.999
8.0	-1.488	-1.366	-1.265	-1.180	-1.117	-1.093	-1.065	-1.065	-1.034	-1.035	-1.019	-1.019	-1.016	-1.014	-1.008	-1.008	-.999	-.998
8.5	-1.494	-1.373	-1.270	-1.184	-1.120	-1.096	-1.068	-1.066	-1.035	-1.035	-1.018	-1.023	-1.011	-1.014	-1.010	-1.002	-1.005	-1.004
9.0	-1.498	-1.377	-1.274	-1.187	-1.123	-1.099	-1.070	-1.067	-1.037	-1.034	-1.019	-1.022	-1.011	-1.009	-1.004	-1.002	-.999	-.998
9.5	-1.503	-1.381	-1.277	-1.190	-1.125	-1.101	-1.072	-1.068	-1.038	-1.031	-1.023	-1.019	-1.014	-1.007	-1.004	-1.008	-1.005	-1.002
10.0	-1.506	-1.384	-1.280	-1.192	-1.127	-1.103	-1.074	-1.068	-1.041	-1.029	-1.026	-1.016	-1.018	-1.011	-1.010	-1.003	-.999	-1.002

TABLE LXVIII IMAGINARY PART OF COMBINED ADMITTANCE COEFFICIENT $S_{\nu h}$ = 1

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	.840	.879	.892	.874	.823	.789	.726	.678	.611	.544	.493	.437	.388	.310	.216	.116	-.042	-.306
1.0	-.200	-.148	-.115	-.105	-.116	-.128	-.152	-.179	-.237	-.308	-.371	-.452	-.536	-.711	-1.008	-1.538	.753	.144
1.5	-.473	-.425	-.394	-.382	-.391	-.401	-.423	-.456	-.533	-.625	-.684	-.700	-.635	-.338	.054	-.010	-.370	.133
2.0	-.523	-.479	-.449	-.434	-.433	-.438	-.447	-.467	-.500	-.478	-.299	-.264	-.152	-.075	-.162	-.264	-.013	-.107
2.5	-.503	-.462	-.433	-.413	-.403	-.401	-.399	-.405	-.393	-.309	-.215	-.137	-.116	-.158	-.202	-.068	-.144	-.073
3.0	-.464	-.427	-.398	-.375	-.360	-.354	-.347	-.346	-.310	-.212	-.147	-.126	-.144	-.176	-.097	-.096	-.052	-.037
3.5	-.424	-.390	-.362	-.338	-.320	-.313	-.304	-.299	-.249	-.159	-.127	-.138	-.156	-.127	-.074	-.105	-.081	-.078
4.0	-.387	-.355	-.328	-.305	-.287	-.279	-.270	-.263	-.204	-.132	-.125	-.142	-.137	-.081	-.100	-.053	-.054	-.034
4.5	-.354	-.325	-.299	-.277	-.259	-.252	-.242	-.233	-.170	-.119	-.127	-.131	-.103	-.077	-.086	-.069	-.052	-.042
5.0	-.325	-.298	-.274	-.253	-.235	-.228	-.220	-.209	-.145	-.115	-.125	-.108	-.079	-.087	-.056	-.066	-.051	-.045
5.5	-.300	-.275	-.253	-.232	-.216	-.209	-.201	-.189	-.127	-.113	-.116	-.086	-.073	-.084	-.058	-.042	-.038	-.025
6.0	-.278	-.255	-.234	-.215	-.199	-.193	-.185	-.172	-.114	-.111	-.102	-.073	-.077	-.065	-.066	-.053	-.046	-.037
6.5	-.259	-.237	-.218	-.199	-.184	-.179	-.171	-.157	-.105	-.106	-.087	-.070	-.078	-.053	-.053	-.048	-.030	-.028
7.0	-.243	-.222	-.203	-.186	-.172	-.166	-.159	-.144	-.098	-.099	-.075	-.072	-.071	-.055	-.042	-.035	-.040	-.023
7.5	-.228	-.208	-.191	-.174	-.161	-.156	-.149	-.133	-.094	-.090	-.068	-.072	-.060	-.058	-.047	-.042	-.027	-.030
8.0	-.215	-.196	-.179	-.164	-.151	-.146	-.140	-.124	-.090	-.081	-.065	-.068	-.052	-.054	-.047	-.037	-.033	-.019
8.5	-.203	-.185	-.169	-.155	-.143	-.138	-.132	-.115	-.087	-.073	-.065	-.060	-.050	-.044	-.030	-.030	-.025	-.023
9.0	-.192	-.176	-.160	-.146	-.135	-.130	-.125	-.108	-.085	-.066	-.065	-.053	-.052	-.040	-.035	-.035	-.028	-.022
9.5	-.183	-.167	-.152	-.139	-.128	-.124	-.118	-.101	-.082	-.062	-.063	-.048	-.052	-.042	-.038	-.030	-.025	-.017
10.0	-.174	-.159	-.145	-.132	-.122	-.118	-.112	-.095	-.079	-.059	-.060	-.048	-.048	-.043	-.036	-.027	-.023	-.021

TABLE LXIX REAL PART OF COMBINED ADMITTANCE COEFFICIENT $S_{\nu h} = 2$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	1.615	1.567	1.493	1.380	1.239	1.163	1.044	.975	.923	.887	.850	.790	.722	.583	.390	.223	.142	.070
1.0	.760	.798	.807	.776	.707	.662	.586	.529	.451	.375	.319	.261	.217	.166	.130	.097	.075	.095
1.5	.044	.128	.188	.221	.222	.213	.193	.173	.143	.116	.099	.084	.068	.035	-.037	-1.386	-.272	-.148
2.0	-.439	-.339	-.263	-.212	-.191	-.189	-.195	-.212	-.269	-.392	-.560	-.855	-1.187	-1.344	-.763	-.568	-1.323	-.638
2.5	-.750	-.645	-.566	-.516	-.500	-.504	-.523	-.564	-.695	-.888	-1.001	-1.013	-.937	-.805	-.840	-1.080	-.838	-.925
3.0	-.953	-.845	-.765	-.714	-.698	-.702	-.719	-.759	-.871	-.964	-.963	-.911	-.873	-.894	-1.011	-.916	-1.029	-1.036
3.5	-1.090	-.980	-.897	-.843	-.821	-.820	-.827	-.857	-.940	-.971	-.941	-.909	-.916	-.983	-.961	-.973	-.947	-.965
4.0	-1.185	-1.073	-.988	-.928	-.899	-.893	-.891	-.915	-.975	-.970	-.940	-.937	-.968	-.992	-.952	-.998	-1.006	-.987
4.5	-1.254	-1.140	-1.052	-.988	-.952	-.942	-.934	-.953	-.994	-.969	-.951	-.972	-.996	-.971	-.992	-.968	-.974	-1.005
5.0	-1.304	-1.189	-1.098	-1.030	-.989	-.976	-.964	-.980	-1.004	-.971	-.970	-.996	-.996	-.971	-.998	-.996	-1.001	-.982
5.5	-1.343	-1.226	-1.133	-1.062	-1.015	-1.002	-.986	-.999	-1.009	-.978	-.989	-1.004	-.986	-.991	-.982	-.997	-.988	-1.000
6.0	-1.372	-1.255	-1.160	-1.086	-1.037	-1.021	-1.003	-1.013	-1.012	-.987	-1.004	-1.000	-.984	-1.004	-.989	-.985	-.997	-.998
6.5	-1.396	-1.278	-1.181	-1.105	-1.053	-1.035	-1.016	-1.024	-1.013	-.998	-1.011	-.994	-.993	-.999	-1.004	-1.001	-.996	-.991
7.0	-1.415	-1.296	-1.198	-1.120	-1.066	-1.047	-1.026	-1.032	-1.014	-1.008	-1.012	-.994	-1.004	-.993	-.999	-.998	-.995	-1.003
7.5	-1.430	-1.311	-1.212	-1.132	-1.077	-1.057	-1.034	-1.038	-1.016	-1.016	-1.010	-1.000	-1.010	-.996	-.993	-.993	-1.001	-.995
8.0	-1.443	-1.323	-1.223	-1.142	-1.085	-1.064	-1.041	-1.043	-1.018	-1.021	-1.007	-1.008	-1.007	-1.005	-1.001	-1.003	-.995	-.998
8.5	-1.453	-1.333	-1.232	-1.150	-1.092	-1.071	-1.046	-1.047	-1.020	-1.023	-1.007	-1.013	-1.003	-1.008	-1.005	-.999	-1.003	-1.002
9.0	-1.463	-1.341	-1.240	-1.157	-1.098	-1.076	-1.051	-1.050	-1.023	-1.023	-1.010	-1.014	-1.003	-1.004	-1.000	-.998	-.996	-.996
9.5	-1.470	-1.348	-1.247	-1.163	-1.103	-1.081	-1.055	-1.053	-1.026	-1.022	-1.014	-1.012	-1.007	-1.001	-1.000	-1.004	-1.003	-1.001
10.0	-1.477	-1.355	-1.253	-1.168	-1.107	-1.085	-1.058	-1.054	-1.029	-1.020	-1.018	-1.010	-1.012	-1.005	-1.006	-1.000	-.997	-1.000

TABLE LXX IMAGINARY PART OF COMBINED ADMITTANCE COEFFICIENT $S_{\nu h} = 2$

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	4.767	4.771	4.655	4.386	3.990	3.766	3.407	3.153	2.829	2.523	2.297	2.058	1.864	1.587	1.320	1.132	.920	.635
1.0	1.245	1.318	1.335	1.287	1.181	1.113	1.000	.910	.782	.662	.582	.505	.446	.358	.230	.063	-.212	-.968
1.5	.193	.258	.286	.276	.234	.205	.154	.107	.028	-.060	-.135	-.235	-.335	-.544	-1.050	-3.544	.956	-.632
2.0	-.176	-.126	-.107	-.119	-.161	-.190	-.242	-.299	-.420	-.575	-.692	-.754	-.643	.011	.202	-.192	-.417	-.185
2.5	-.304	-.265	-.249	-.258	-.291	-.313	-.351	-.392	-.449	-.420	-.305	-.142	-.053	-.079	-.242	-.131	-.095	.034
3.0	-.342	-.308	-.291	-.292	-.308	-.318	-.334	-.349	-.337	-.239	-.150	-.105	-.120	-.184	-.129	-.062	-.079	-.069
3.5	-.344	-.313	-.294	-.289	-.292	-.294	-.297	-.300	-.261	-.166	-.122	-.127	-.153	-.145	-.064	-.118	-.065	-.076
4.0	-.332	-.303	-.284	-.273	-.269	-.267	-.265	-.262	-.211	-.133	-.120	-.140	-.144	-.087	-.094	-.057	-.067	-.026
4.5	-.315	-.288	-.268	-.255	-.246	-.243	-.239	-.233	-.174	-.118	-.124	-.134	-.110	-.073	-.092	-.062	-.045	-.051
5.0	-.296	-.271	-.252	-.237	-.227	-.222	-.217	-.209	-.148	-.113	-.124	-.113	-.081	-.085	-.059	-.070	-.057	-.040
5.5	-.278	-.254	-.236	-.220	-.209	-.204	-.199	-.189	-.129	-.111	-.117	-.089	-.072	-.085	-.055	-.043	-.034	-.025
6.0	-.261	-.239	-.221	-.206	-.194	-.189	-.183	-.171	-.115	-.110	-.104	-.074	-.075	-.068	-.066	-.050	-.048	-.039
6.5	-.246	-.225	-.207	-.192	-.180	-.176	-.170	-.157	-.105	-.106	-.088	-.069	-.078	-.054	-.055	-.050	-.029	-.025
7.0	-.232	-.212	-.195	-.180	-.169	-.164	-.158	-.144	-.098	-.099	-.076	-.071	-.072	-.054	-.042	-.035	-.040	-.025
7.5	-.219	-.200	-.184	-.170	-.158	-.154	-.148	-.133	-.093	-.091	-.068	-.072	-.061	-.058	-.046	-.041	-.027	-.029
8.0	-.207	-.189	-.174	-.160	-.149	-.145	-.139	-.124	-.090	-.082	-.065	-.069	-.052	-.055	-.048	-.038	-.033	-.019
8.5	-.197	-.180	-.165	-.152	-.141	-.137	-.131	-.115	-.087	-.073	-.065	-.061	-.050	-.045	-.039	-.030	-.026	-.024
9.0	-.187	-.171	-.157	-.144	-.133	-.129	-.124	-.108	-.085	-.066	-.065	-.054	-.052	-.040	-.034	-.035	-.027	-.021
9.5	-.178	-.163	-.149	-.137	-.127	-.123	-.118	-.101	-.082	-.062	-.063	-.049	-.052	-.042	-.038	-.031	-.025	-.017
10.0	-.170	-.155	-.142	-.130	-.121	-.117	-.112	-.095	-.079	-.059	-.060	-.047	-.049	-.043	-.036	-.026	-.022	-.021

TABLE LXXI REAL PART OF COMBINED ADMITTANCE COEFFICIENT $S_{\nu h}$ = 3

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	4.017	3.765	3.462	3.098	2.713	2.524	2.243	2.115	2.067	2.032	1.959	1.818	1.653	1.331	.903	.526	.297	.140
1.0	2.513	2.444	2.283	2.034	1.735	1.583	1.354	1.206	1.026	.947	.712	.571	.462	.323	.224	.150	.092	.052
1.5	1.269	1.318	1.293	1.186	1.017	.923	.777	.671	.530	.406	.328	.262	.222	.177	.130	.100	.088	18.227
2.0	.424	.523	.574	.570	.516	.477	.411	.358	.287	.231	.201	.176	.158	.143	.156	1.377	.038	.063
2.5	-.126	-.013	.062	.098	.097	.086	.063	.038	-.010	-.093	-.214	-.506	-1.187	-1.681	-.441	-.381	-1.540	-1.244
3.0	-.488	-.373	-.295	-.254	-.255	-.270	-.307	-.363	-.530	-.804	-.975	-.949	-.808	-.669	-.868	-1.000	-.907	-.882
3.5	-.733	-.618	-.540	-.500	-.505	-.522	-.564	-.626	-.774	-.883	-.869	-.813	-.794	-.883	-.960	-.875	-.929	-.912
4.0	-.904	-.789	-.710	-.668	-.667	-.679	-.708	-.756	-.857	-.889	-.862	-.851	-.885	-.955	-.902	-.983	-.971	-1.001
4.5	-1.027	-.912	-.830	-.783	-.773	-.779	-.795	-.832	-.903	-.901	-.883	-.904	-.943	-.941	-.947	-.947	-.967	-.980
5.0	-1.118	-1.002	-.918	-.866	-.847	-.847	-.854	-.883	-.932	-.913	-.911	-.946	-.961	-.935	-.978	-.966	-.977	-.973
5.5	-1.187	-1.071	-.984	-.927	-.901	-.896	-.896	-.920	-.950	-.928	-.941	-.967	-.958	-.957	-.965	-.986	-.983	-.999
6.0	-1.241	-1.123	-1.035	-.973	-.941	-.933	-.928	-.947	-.962	-.944	-.965	-.972	-.956	-.980	-.967	-.971	-.982	-.985
6.5	-1.283	-1.165	-1.074	-1.009	-.972	-.961	-.952	-.968	-.971	-.960	-.980	-.970	-.966	-.983	-.987	-.986	-.992	-.989
7.0	-1.317	-1.198	-1.106	-1.037	-.996	-.983	-.972	-.984	-.978	-.975	-.987	-.971	-.982	-.977	-.988	-.991	-.985	-.999
7.5	-1.345	-1.226	-1.131	-1.060	-1.015	-1.001	-.987	-.997	-.984	-.988	-.988	-.979	-.992	-.980	-.982	-.983	-.996	-.989
8.0	-1.367	-1.248	-1.152	-1.079	-1.031	-1.016	-1.000	-1.007	-.989	-.997	-.988	-.989	-.993	-.991	-.989	-.995	-.988	-.996
8.5	-1.386	-1.266	-1.170	-1.094	-1.045	-1.028	-1.010	-1.015	-.995	-1.002	-.989	-.997	-.990	-.997	-.997	-.994	-.998	-.997
9.0	-1.403	-1.282	-1.184	-1.107	-1.056	-1.038	-1.019	-1.022	-1.000	-1.005	-.993	-1.001	-.990	-.995	-.993	-.991	-.991	-.993
9.5	-1.416	-1.295	-1.197	-1.118	-1.065	-1.046	-1.026	-1.027	-1.005	-1.006	-.999	-1.000	-.995	-.992	-.992	-.998	-.999	-1.000
10.0	-1.428	-1.306	-1.207	-1.128	-1.073	-1.054	-1.032	-1.032	-1.011	-1.006	-1.004	-.999	-1.001	-.996	-.998	-.996	-.994	-.996

TABLE LXXII IMAGINARY PART OF COMBINED ADMITTANCE COEFFICIENT $S_{\nu h}$ = 3

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	11.272	11.073	10.552	9.663	8.541	7.950	7.053	6.449	5.685	4.955	4.426	3.884	3.463	2.892	2.371	2.027	1.721	1.330
1.0	3.649	3.720	3.642	3.390	3.012	2.802	2.475	2.732	1.897	1.607	1.428	1.271	1.165	1.034	.889	.711	.487	.158
1.5	1.309	1.411	1.442	1.393	1.270	1.192	1.062	.959	.813	.689	.612	.539	.477	.364	.203	-.011	-.438	-5.561
2.0	.408	.484	.517	.508	.460	.427	.368	.316	.232	.142	.067	-.030	-.129	-.334	-.816	-4.861	.496	-2.571
2.5	.031	.081	.096	.076	.023	-.012	-.075	-.140	-.276	-.473	-.675	-.971	-1.164	.538	.311	-.399	.698	-.919
3.0	-.135	-.099	-.092	-.118	-.173	-.209	-.272	-.337	-.441	-.436	-.270	-.036	.033	-.096	-.318	.072	-.266	-.224
3.5	-.209	-.179	-.173	-.192	-.231	-.253	-.287	-.315	-.309	-.194	-.106	-.088	-.131	-.193	-.052	-.122	-.009	-.027
4.0	-.239	-.213	-.204	-.214	-.235	-.246	-.260	-.268	-.228	-.137	-.109	-.131	-.154	-.106	-.075	-.079	-.093	-.030
4.5	-.249	-.224	-.213	-.216	-.225	-.229	-.234	-.234	-.183	-.117	-.117	-.137	-.124	-.068	-.101	-.049	-.033	-.062
5.0	-.248	-.224	-.212	-.209	-.211	-.212	-.213	-.209	-.153	-.110	-.122	-.120	-.088	-.078	-.068	-.075	-.064	-.029
5.5	-.241	-.220	-.206	-.200	-.198	-.197	-.195	-.189	-.131	-.109	-.118	-.095	-.071	-.087	-.051	-.047	-.032	-.032
6.0	-.233	-.212	-.198	-.190	-.185	-.183	-.181	-.172	-.116	-.108	-.107	-.076	-.072	-.074	-.064	-.045	-.049	-.039
6.5	-.223	-.204	-.190	-.180	-.174	-.171	-.168	-.157	-.105	-.105	-.092	-.062	-.077	-.055	-.059	-.053	-.030	-.022
7.0	-.214	-.195	-.181	-.171	-.163	-.160	-.157	-.144	-.098	-.100	-.078	-.069	-.074	-.052	-.043	-.036	-.039	-.029
7.5	-.204	-.186	-.173	-.162	-.154	-.151	-.147	-.133	-.093	-.092	-.069	-.071	-.064	-.057	-.044	-.039	-.029	-.027
8.0	-.195	-.178	-.165	-.154	-.146	-.142	-.138	-.124	-.089	-.083	-.065	-.069	-.053	-.056	-.048	-.040	-.031	-.019
8.5	-.186	-.170	-.157	-.146	-.138	-.135	-.130	-.115	-.086	-.075	-.064	-.063	-.049	-.047	-.040	-.030	-.028	-.026
9.0	-.178	-.163	-.150	-.139	-.131	-.128	-.123	-.108	-.084	-.067	-.064	-.055	-.051	-.040	-.034	-.034	-.025	-.020
9.5	-.171	-.156	-.144	-.133	-.125	-.121	-.117	-.101	-.082	-.062	-.063	-.049	-.052	-.041	-.037	-.032	-.027	-.018
10.0	-.164	-.150	-.138	-.127	-.119	-.116	-.111	-.095	-.079	-.059	-.060	-.047	-.049	-.043	-.037	-.026	-.022	-.021

TABLE LXXIII REAL PART OF COMBINED ADMITTANCE COEFFICIENT $S_{\nu h}$ = 4

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	7.348	6.761	6.138	5.447	4.748	4.409	3.912	3.728	3.723	3.688	3.545	3.269	2.955	2.373	1.636	.986	.531	.247
1.0	4.923	4.631	4.193	3.641	3.056	2.771	2.356	2.107	1.811	1.502	1.266	1.020	.829	.579	.381	.249	.144	.071
1.5	2.955	2.885	2.657	2.297	1.885	1.683	1.389	1.192	.938	.712	.568	.441	.359	.270	.187	.127	.079	.053
2.0	1.619	1.677	1.617	1.436	1.185	1.053	.861	.724	.547	.405	.327	.266	.227	.175	.130	.101	.121	.075
2.5	.743	.853	.889	.842	.725	.655	.545	.462	.356	.278	.237	.202	.179	.157	.176	4.595	.070	.252
3.0	.162	.285	.356	.373	.342	.316	.270	.234	.184	.140	.104	.047	-.111	-5.560	-.127	-.111	-.381	-1.184
3.5	-.233	-.110	-.033	-.002	-.017	-.038	-.083	-.140	-.307	-.682	-1.049	-.974	-.688	-.508	-.948	-.771	-1.264	-1.313
4.0	-.510	-.389	-.313	-.284	-.305	-.333	-.396	-.479	-.677	-.820	-.786	-.715	-.716	-.883	-.873	-.907	-.847	-.988
4.5	-.709	-.590	-.514	-.484	-.504	-.528	-.580	-.648	-.774	-.809	-.786	-.797	-.855	-.910	-.869	-.940	-.978	-.929
5.0	-.856	-.739	-.662	-.627	-.637	-.655	-.691	-.742	-.827	-.831	-.828	-.868	-.909	-.891	-.941	-.920	-.938	-.979
5.5	-.969	-.851	-.772	-.732	-.732	-.742	-.766	-.806	-.865	-.857	-.870	-.912	-.919	-.909	-.945	-.966	-.978	-.985
6.0	-1.056	-.938	-.857	-.817	-.801	-.806	-.820	-.853	-.891	-.882	-.907	-.931	-.920	-.943	-.938	-.956	-.960	-.970
6.5	-1.125	-1.007	-.923	-.872	-.854	-.854	-.862	-.889	-.911	-.906	-.934	-.936	-.930	-.959	-.960	-.963	-.985	-.989
7.0	-1.180	-1.061	-.975	-.919	-.896	-.892	-.894	-.917	-.926	-.929	-.950	-.940	-.949	-.957	-.973	-.980	-.972	-.988
7.5	-1.224	-1.106	-1.017	-.958	-.929	-.922	-.920	-.939	-.938	-.948	-.957	-.949	-.966	-.958	-.967	-.971	-.989	-.983
8.0	-1.261	-1.142	-1.052	-.989	-.955	-.947	-.941	-.956	-.949	-.963	-.961	-.962	-.973	-.970	.972	-.981	-.980	-.994
8.5	-1.292	-1.173	-1.081	-1.015	-.978	-.967	-.958	-.970	-.959	-.973	-.964	-.974	-.973	-.981	-.984	-.987	-.991	-.990
9.0	-1.319	-1.198	-1.105	-1.037	-.996	-.984	-.973	-.982	-.968	-.979	-.970	-.981	-.973	-.982	-.984	-.981	-.986	-.990
9.5	-1.341	-1.220	-1.126	-1.055	-1.012	-.998	-.985	-.992	-.976	-.983	-.977	-.984	-.978	-.980	-.981	-.989	-.992	-.996
10.0	-1.359	-1.239	-1.143	-1.071	-1.025	-1.010	-.995	-1.000	-.984	-.986	-.985	-.984	-.986	-.983	-.988	-.991	-.991	-.991

TABLE LXXIV IMAGINARY PART OF COMBINED ADMITTANCE COEFFICIENT . $S_{\nu h}$ = 4

ω	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	20.303	19.633	18.380	16.506	14.317	13.210	11.571	10.502	9.138	7.817	6.873	5.931	5.220	4.287	3.459	2.912	2.480	1.972
1.0	6.997	6.978	6.652	6.013	5.199	4.779	4.156	3.703	3.086	2.566	2.259	1.999	1.828	1.627	1.436	1.221	.967	.644
1.5	2.878	3.016	2.994	2.792	2.464	2.282	2.005	1.794	1.510	1.288	1.164	1.059	.984	.862	.699	.517	.261	-.218
2.0	1.236	1.367	1.429	1.399	1.283	1.205	1.077	.972	.828	.710	.635	.555	.481	.353	.171	-.089	-.794	.705
2.5	.506	.595	.645	.654	.617	.585	.523	.467	.376	.281	.201	.100	0.000	-.206	-.684	-5.389	.177	2.213
3.0	.158	.213	.235	.223	.181	.150	.093	.035	-.085	-.257	-.441	-.769	-1.367	2.589	.482	-.518	1.049	1.960
3.5	-.017	.019	.025	-.005	-.069	-.111	-.190	-.275	-.447	-.564	-.342	.118	.172	-.106	-.452	.208	-.232	.042
4.0	-.108	-.080	-.079	-.109	-.165	-.199	-.254	-.299	-.309	-.162	-.060	-.072	-.147	-.199	.002	-.170	-.089	-.132
4.5	-.155	-.131	-.130	-.152	-.189	-.208	-.234	-.248	-.207	-.114	-.100	-.136	-.151	-.071	-.098	-.033	-.037	-.047
5.0	-.179	-.157	-.154	-.167	-.188	-.199	-.211	-.214	-.163	-.106	-.115	-.131	-.104	-.065	-.087	-.069	-.058	-.019
5.5	-.189	-.169	-.163	-.170	-.182	-.187	-.192	-.191	-.137	-.104	-.118	-.106	-.073	-.084	-.050	-.060	-.039	-.046
6.0	-.192	-.174	-.166	-.167	-.173	-.175	-.178	-.173	-.119	-.105	-.111	-.082	-.068	-.081	-.057	-.039	-.043	-.032
6.5	-.191	-.173	-.165	-.163	-.164	-.165	-.165	-.158	-.106	-.104	-.097	-.069	-.074	-.060	-.063	-.054	-.036	-.022
7.0	-.188	-.171	-.161	-.157	-.156	-.155	-.154	-.145	-.098	-.101	-.082	-.067	-.076	-.050	-.047	-.040	-.034	-.034
7.5	-.183	-.167	-.157	-.151	-.148	-.147	-.145	-.134	-.092	-.094	-.070	-.070	-.067	-.054	-.041	-.036	-.033	-.023
8.0	-.178	-.162	-.151	-.145	-.141	-.139	-.136	-.124	-.089	-.085	-.064	-.070	-.055	-.057	-.048	-.042	-.028	-.021
8.5	-.172	-.157	-.146	-.139	-.134	-.132	-.129	-.115	-.086	-.076	-.063	-.065	-.049	-.049	-.043	-.031	-.031	-.026
9.0	-.166	-.152	-.141	-.133	-.127	-.125	-.122	-.108	-.083	-.069	-.063	-.057	-.050	-.041	-.034	-.032	-.023	-.018
9.5	-.161	-.147	-.136	-.128	-.122	-.119	-.116	-.101	-.081	-.063	-.063	-.050	-.052	-.040	-.036	-.034	-.028	-.020
10.0	-.155	-.141	-.131	-.123	-.116	-.114	-.111	-.095	-.079	-.059	-.061	-.047	-.050	-.043	-.038	-.026	-.021	-.020

TABLE LXXV REAL PART OF COMBINED ADMITTANCE COEFFICIENT $S_{\nu h}$ = 5

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	11.587	10.568	9.562	8.471	7.375	6.845	6.069	5.833	5.906	5.856	5.600	5.129	4.616	3.699	2.576	1.592	.845	.389
1.0	7.964	7.355	6.577	5.668	4.735	4.285	3.635	3.766	2.824	2.344	1.977	1.601	1.311	.927	.600	.386	.218	.104
1.5	5.078	4.789	4.278	3.622	2.938	2.612	2.149	1.849	1.463	1.115	.892	.692	.559	.408	.277	.181	.106	.055
2.0	3.132	3.072	2.800	2.369	1.8[illegible]4	1.667	1.346	1.129	.850	.623	.494	.391	.326	.243	.169	.115	.075	.088
2.5	1.852	1.919	1.830	1.588	1.275	1.119	.898	.743	.549	.404	.329	.269	.226	.174	.128	.103	.274	.051
3.0	.997	1.116	1.137	1.044	.868	.771	.626	.521	.391	.300	.252	.211	.185	.157	.174	8.857	.074	.056
3.5	.410	.543	.609	.604	.536	.489	.413	.356	.284	.233	.208	.197	.213	1.133	.098	.066	.024	-.008
4.0	-.002	.129	.205	.227	.199	.174	.128	.081	-.028	-.323	-1.087	-1.363	-.576	-.289	-1.006	-.464	-1.461	-.586
4.5	-.299	-.173	-.098	-.076	-.109	-.143	-.217	-.315	-.576	-.800	-.723	-.608	-.622	-.911	-.761	-1.006	-.920	-1.000
5.0	-.520	-.397	-.324	-.303	-.339	-.375	-.449	-.538	-.697	-.734	-.710	-.744	-.829	-.858	-.859	-.872	-.904	-.996
5.5	-.687	-.567	-.494	-.470	-.499	-.528	-.584	-.651	-.752	-.764	-.774	-.832	-.871	-.849	-.922	-.922	-.956	-.945
6.0	-.818	-.698	-.623	-.595	-.613	-.634	-.674	-.727	-.797	-.801	-.828	-.875	-.876	-.888	-.908	-.944	-.944	-.966
6.5	-.921	-.802	-.724	-.690	-.697	-.711	-.740	-.783	-.832	-.835	-.871	-.893	-.885	-.924	-.922	-.934	-.966	-.984
7.0	-1.003	-.884	-.805	-.764	-.762	-.771	-.791	-.827	-.858	-.867	-.900	-.900	-.905	-.932	-.950	-.961	-.962	-.971
7.5	-1.070	-.951	-.869	-.824	-.814	-.8[illegible]8	-.831	-.861	-.879	-.895	-.916	-.911	-.930	-.932	-.951	-.960	-.973	-.980
8.0	-1.125	-1.006	-.922	-.872	-.856	-.856	-.864	-.889	-.897	-.917	-.926	-.926	-.945	-.943	-.951	-.962	-.973	-.988
8.5	-1.171	-1.052	-.966	-.912	-.890	-.887	-.890	-.911	-.912	-.934	-.9[illegible]2	-.943	-.950	-.959	-.965	-.976	-.978	-.981
9.0	-1.210	-1.091	-1.003	-.945	-.918	-.913	-.913	-.930	-.926	-.946	-.940	-.956	-.952	-.966	-.972	-.971	-.981	-.988
9.5	-1.243	-1.123	-1.034	-.973	-.942	-.935	-.931	-.945	-.938	-.954	-.949	-.962	-.957	-.965	-.969	-.977	-.982	-.990
10.0	-1.271	-1.151	-1.060	-.997	-.962	-.953	-.947	-.958	-.950	-.959	-.960	-.965	-.966	-.967	-.974	-.983	-.987	-.987

TABLE LXXVI IMAGINARY PART OF COMBINED ADMITTANCE COEFFICIENT $S_{\nu h}$ = 5

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	31.804	30.365	28.069	24.876	21.301	19.534	16.955	15.310	13.182	11.098	9.633	8.202	7.145	5.786	4.598	3.812	3.223	2.592
1.0	11.259	11.020	10.291	9.115	7.739	7.056	6.066	5.352	4.381	3.581	3.123	2.742	2.494	2.205	1.948	1.691	1.392	1.032
1.5	4.892	5.018	4.839	4.376	3.765	3.453	2.995	2.654	2.201	1.861	1.678	1.529	1.429	1.284	1.098	.904	.656	.305
2.0	2.311	2.511	2.551	2.404	2.131	1.978	1.744	1.565	1.329	1.153	1.055	.963	.884	.753	.587	.390	.086	-.871
2.5	1.129	1.290	1.388	1.385	1.280	1.206	1.081	.979	.840	.725	.647	.558	.477	.342	.141	-.171	-1.630	-.099
3.0	.543	.646	.723	.757	.732	.699	.634	.572	.474	.371	.285	.180	.078	-.132	-.628	3.553	-.101	.362
3.5	.234	.296	.333	.343	.321	.299	.251	.199	.096	-.046	-.188	-.412	-.743	-3.481	.711	-.480	.699	.565
4.0	.064	.103	.113	.091	.035	-.005	-.083	-.172	-.381	-.729	-.885	.426	.469	-.061	-.842	.265	.588	.364
4.5	-.033	-.005	-.005	-.039	-.105	-.147	-.222	-.294	-.348	-.141	.008	-.042	-.170	-.220	.037	-.170	-.207	.097
5.0	-.089	-.067	-.069	-.099	-.149	-.178	-.217	-.240	-.194	-.091	-.091	-.144	-.148	-.041	-.121	-.026	-.012	-.061
5.5	-.121	-.102	-.103	-.125	-.158	-.174	-.194	-.200	-.148	-.097	-.115	-.124	-.085	-.070	-.062	-.079	-.064	-.049
6.0	-.140	-.122	-.121	-.135	-.157	-.166	-.176	-.176	-.125	-.100	-.114	-.094	-.065	-.086	-.046	-.039	-.029	-.018
6.5	-.150	-.133	-.130	-.139	-.152	-.157	-.163	-.160	-.109	-.102	-.104	-.072	-.068	-.070	-.064	-.048	-.047	-.033
7.0	-.155	-.139	-.134	-.138	-.146	-.149	-.152	-.146	-.099	-.100	-.088	-.065	-.075	-.051	-.054	-.048	-.027	-.032
7.5	-.157	-.141	-.135	-.136	-.140	-.142	-.143	-.135	-.092	-.096	-.074	-.067	-.071	-.050	-.039	-.032	-.039	-.018
8.0	-.155	-.141	-.134	-.133	-.134	-.135	-.135	-.125	-.088	-.088	-.065	-.069	-.059	-.056	-.045	-.042	-.025	-.027
8.5	-.153	-.139	-.132	-.129	-.128	-.128	-.127	-.116	-.085	-.079	-.062	-.067	-.050	-.053	-.046	-.035	-.033	-.024
9.0	-.150	-.137	-.129	-.125	-.123	-.122	-.121	-.108	-.082	-.071	-.062	-.059	-.048	-.043	-.036	-.029	-.022	-.017
9.5	-.147	-.134	-.126	-.121	-.118	-.117	-.115	-.101	-.080	-.064	-.063	-.051	-.050	-.039	-.034	-.035	-.029	-.023
10.0	-.144	-.131	-.122	-.117	-.113	-.112	-.110	-.095	-.078	-.059	-.061	-.047	-.051	-.042	-.038	-.027	-.021	-.018

TABLE LXXVII REAL PART OF COMBINED ADMITTANCE COEFFICIENT $S_{\nu h}$ = 7

ω \ M	.902	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198	.179	.152	.124	.099	.074	.050
.5	22.746	20.697	18.738	16.597	14.430	13.380	11.847	11.547	11.880	11.717	11.095	10.056	8.992	7.183	5.059	3.220	1.721	.779
1.0	15.917	14.505	12.895	11.066	9.205	8.314	7.032	6.361	5.526	4.572	3.857	3.140	2.597	1.872	1.214	.766	.427	.198
1.5	10.575	9.642	8.459	7.090	5.714	5.068	4.159	3.596	2.862	2.194	1.769	1.387	1.123	.805	.537	.344	.194	.092
2.0	7.047	6.504	5.680	4.695	3.710	3.254	2.620	2.206	1.674	1.234	.977	.764	.624	.457	.307	0.000	.115	.057
2.5	4.748	4.505	3.961	3.254	2.541	2.212	1.760	1.457	1.074	.780	.620	.492	.406	.299	.204	.135	.081	.055
3.0	3.199	3.168	2.849	2.349	1.823	1.580	1.247	1.022	.744	.542	.437	.350	.290	.217	.151	.105	.080	.071
3.5	2.122	2.212	2.076	1.745	1.357	1.175	.925	.755	.549	.407	.332	.268	.225	.171	.127	.113	.322	9.144
4.0	1.355	1.495	1.488	1.307	1.036	.901	.713	.583	.430	.325	.268	.220	.189	.156	.171	.516	.122	.196
4.5	.797	.947	1.005	.944	.786	.695	.562	.466	.353	.275	.235	.206	.198	.342	.252	.125	.074	.089
5.0	.381	.527	.605	.606	.539	.491	.414	.355	.284	.239	.235	.394	3.206	.105	.109	.039	0.000	-.030
5.5	.065	.204	.281	.295	.255	.221	.157	.088	-.126	-1.061	-.899	-.323	-.266	-1.059	-.402	-1.518	-1.441	-.802
6.0	-.181	-.048	.025	.035	-.016	-.063	-.165	-.297	-.594	-.622	-.539	-.616	-.824	-.729	-.918	-.779	-.916	-.856
6.5	-.375	-.247	-.177	-.168	-.224	-.273	-.369	-.475	-.620	-.633	-.670	-.771	-.793	-.782	-.847	-.916	-.894	-.918
7.0	-.530	-.406	-.336	-.326	-.375	-.416	-.492	-.571	-.668	-.694	-.748	-.800	-.795	-.852	-.859	-.885	-.942	-.968
7.5	-.656	-.534	-.464	-.449	-.487	-.520	-.579	-.643	-.715	-.745	-.797	-.816	-.824	-.873	-.902	-.925	-.931	-.962
8.0	-.760	-.640	-.568	-.547	-.575	-.601	-.647	-.701	-.753	-.788	-.828	-.833	-.859	-.876	-.908	-.925	-.954	-.957
8.5	-.848	-.727	-.654	-.627	-.645	-.665	-.701	-.747	-.784	-.823	-.847	-.856	-.885	-.892	-.911	-.933	-.948	-.974
9.0	-.921	-.801	-.725	-.693	-.703	-.717	-.746	-.786	-.812	-.851	-.862	-.881	-.897	-.914	-.932	-.949	-.963	-.975
9.5	-.983	-.863	-.785	-.749	-.751	-.761	-.783	-.817	-.835	-.872	-.876	-.901	-.903	-.926	-.940	-.946	-.960	-.971
10.0	-1.036	-.917	-.837	-.796	-.791	-.798	-.814	-.843	-.857	-.887	-.891	-.912	-.913	-.928	-.940	-.958	-.970	-.981

TABLE LXXVIII IMAGINARY PART OF COMBINED ADMITTANCE COEFFICIENT $S_{\nu h}$ = 7

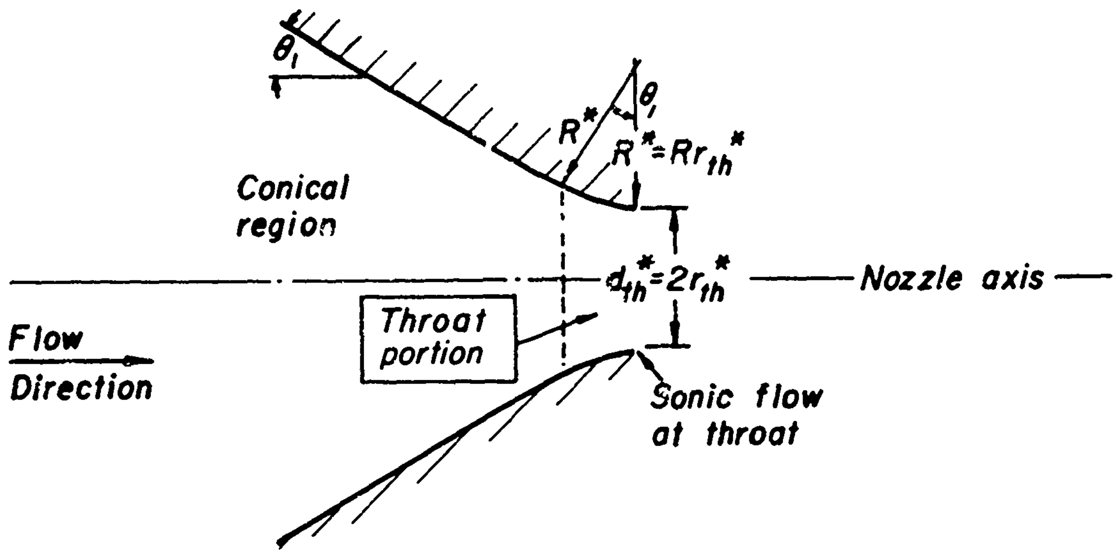
.5	62.021	58.222	52.971	46.170	38.878	35.356	30.303	27.175	23.009	18.890	16.077	13.415	11.503	9.103	7.049	5.687	4.702	3.805
1.0	22.397	21.323	19.412	16.776	13.915	12.546	10.607	9.215	7.315	5.813	4.986	4.314	3.879	3.375	2.946	2.584	2.178	1.715
1.5	10.175	10.035	9.292	8.105	6.768	6.126	5.219	4.546	3.664	3.033	2.706	2.446	2.275	2.056	1.811	1.565	1.273	.915
2.0	5.184	5.404	5.177	4.626	3.943	3.606	3.123	2.760	2.297	1.975	1.808	1.667	1.561	1.399	1.205	0.000	.741	.378
2.5	2.831	3.149	3.173	2.937	2.570	2.378	2.095	1.879	1.605	1.412	1.303	1.196	1.106	.966	.790	.592	.309	-.314
3.0	1.611	1.898	2.040	1.979	1.785	1.671	1.493	1.354	1.174	1.036	.945	.847	.764	.629	.447	.213	-.272	.648
3.5	.935	1.143	1.315	1.355	1.269	1.201	1.035	.990	.857	.740	.652	.555	.470	.320	.085	-.358	1.814	9.926
4.0	.542	.675	.813	.898	.881	.845	.770	.699	.590	.474	.381	.271	.165	-.052	-.606	2.008	-.802	-1.699
4.5	.305	.386	.467	.536	.551	.535	.486	.431	.325	.193	.074	-.092	-.295	-1.105	1.017	-.518	.102	-.907
5.0	.158	.207	.242	.262	.254	.236	.187	.127	-.012	-.238	-.542	-1.512	5.870	.370	-.917	.394	.871	-.872
5.5	.064	.096	.105	.087	.033	-.009	-.099	-.211	-.500	-.619	.446	.205	-.146	-.714	.139	.363	.472	-.579
6.0	.003	.026	.024	-.008	-.074	-.119	-.197	-.264	-.225	-.007	-.051	-.196	-.180	.038	-.158	-.139	-.168	-.120
6.5	-.038	-.020	-.024	-.055	-.108	-.138	-.177	-.192	-.125	-.080	-.121	-.114	-.048	-.093	-.021	-.026	-.012	-.009
7.0	-.065	-.050	-.054	-.079	-.117	-.135	-.155	-.158	-.107	-.095	-.108	-.072	-.058	-.077	-.062	-.044	-.048	-.018
7.5	-.084	-.069	-.072	-.091	-.118	-.129	-.142	-.141	-.096	-.096	-.091	-.060	-.071	-.050	-.056	-.048	-.026	-.037
8.0	-.095	-.083	-.084	-.097	-.116	-.124	-.132	-.129	-.088	-.093	-.074	-.062	-.071	-.047	-.036	-.030	-.036	-.022
8.5	-.103	-.091	-.091	-.100	-.114	-.119	-.125	-.119	-.083	-.087	-.063	-.067	-.059	-.055	-.044	-.042	-.025	-.018
9.0	-.108	-.097	-.095	-.101	-.111	-.114	-.118	-.110	-.081	-.078	-.059	-.065	-.048	-.052	-.045	-.031	-.031	-.026
9.5	-.112	-.100	-.097	-.101	-.107	-.110	-.113	-.103	-.079	-.069	-.059	-.058	-.046	-.041	-.033	-.030	-.022	-.019
10.0	-.113	-.102	-.098	-.100	-.104	-.106	-.107	-.096	-.077	-.062	-.060	-.050	-.049	-.038	-.034	-.034	-.027	-.016

TABLE LXXIX REAL PART OF COMBINED ADMITTANCE COEFFICIENT $S_{\nu h}$ = 9

ω \ M	.90	.805	.705	.599	.501	.457	.395	.351	.294	.250	.223	.198		.179	.152		.124	.09
.5	37.497	34.260	31.060	27.508	23.892	22.136	19.581	19.299	20.019	19.603	18.417	16.578	14.762	11.765	8.328	5.381	2.917	1.311
1.0	26.426	24.027	21.339	18.279	15.166	13.678	11.546	10.493	9.116	7.510	6.33[illegible]	5.171	4.298	3.135	2.053	1.286	.712	.328
1.5	17.749	16.018	14.003	11.702	9.401	8.327	6.821	5.915	4.714	3.622	2.937	2.321	1.889	1.355	.895	.569	.319	.149
2.0	12.091	10.891	9.431	7.761	6.113	5.356	4.309	3.639	2.771	2.056	1.639	1.282	1.043	.756	.504	.323	.183	.087
2.5	8.466	7.681	6.624	5.400	4.200	3.654	2.906	2.414	1.788	1.304	1.035	.813	.666	.486	.326	.211	.121	.059
3.0	6.050	5.605	4.840	3.924	3.027	2.620	2.067	1.698	1.237	.897	.716	.567	.466	.342	.231	.151	.089	.051
3.5	4.365	4.191	3.652	2.956	2.267	1.956	1.534	1.251	.904	.659	.530	.421	.347	.257	.176	.117	.075	.703
4.0	3.151	3.167	2.823	2.292	1.753	1.510	1.180	.959	.692	.509	.411	.328	.272	.203	.142	.101	.157	.084
4.5	2.258	2.380	2.207	1.817	1.391	1.197	.935	.759	.551	.409	.332	.267	.223	.170	.126	.139	.079	.051
5.0	1.588	1.752	1.715	1.458	1.127	.971	.761	.619	.453	.339	.278	.227	.193	.156	.176	.175	6.346	.059
5.5	1.075	1.245	1.292	1.162	.923	.801	.632	.518	.384	.292	.244	.206	.187	.233	.269	.155	.091	.103
6.0	.675	.838	.918	.884	.743	.657	.531	.441	.336	.266	.235	.244	.476	.297	.156	.113	.078	.197
6.5	.358	.511	.595	.604	.539	.492	.413	.354	.284	.256	.439	.287	.123	.114	.054	-.253	-.037	-.149
7.0	.105	.248	.326	.338	.289	.248	.169	.074	-.346	-.874	-.304	-.271	-.766	-.517	-1.340	-.757	-1.424	-.926
7.5	-.202	.034	.106	.111	.047	-.009	-.133	-.292	-.553	-.491	-.536	-.763	-.727	-.724	-.718	-.786	-.909	-1.023
8.0	-.272	-.142	-.073	-.072	-.141	-.199	-.312	-.427	-.553	-.588	-.678	-.730	-.718	-.821	-.848	-.887	-.890	-.959
8.5	-.415	-.287	-.220	-.218	-.282	-.332	-.422	-.510	-.604	-.659	-.726	-.745	-.773	-.816	-.866	-.891	-.931	-.936
9.0	-.534	-.409	-.342	-.336	-.390	-.432	-.505	-.578	-.652	-.711	-.758	-.773	-.814	-.831	-.865	-.896	-.922	-.954
9.5	-.636	-.512	-.444	-.434	-.477	-.512	-.572	-.635	-.693	-.752	-.781	-.805	-.836	-.861	-.890	-.920	-.943	-.965
10.0	-.722	-.600	-.531	-.515	-.549	-.577	-.627	-.682	-.728	-.784	-.801	-.835	-.847	-.880	-.905	-.917	-.939	-.959

TABLE LXXX IMAGINARY PART OF COMBINED ADMITTANCE COEFFICIENT $S_{\nu h}$ = 9

.5	101.66	94.557	85.221	73.518	61.239	55.384	47.064	42.022	35.116	28.293	23.738	19.517	16.533	12.845	9.728	7.672	6.203	5.003
1.0	36.885	34.557	31.008	26.400	21.566	19.294	16.118	13.829	10.710	8.324	7.045	6.018	5.358	4.593	3.951	3.454	2.931	2.351
1.5	17.009	16.321	14.783	12.636	10.353	9.286	7.806	6.698	5.268	4.280	3.778	3.381	3.123	2.805	2.482	2.172	1.816	1.399
2.0	8.939	8.938	8.278	7.202	6.005	5.438	4.648	4.051	3.300	2.799	2.544	2.335	2.187	1.980	1.741	1.499	1.210	.850
2.5	5.123	5.417	5.185	4.626	3.946	3.614	3.144	2.787	2.344	2.048	1.890	1.747	1.635	1.468	1.268	1.057	.791	.422
3.0	3.099	3.501	3.501	3.217	2.811	2.602	2.300	2.068	1.780	1.580	1.462	1.345	1.249	1.102	.919	.717	.440	-.087
3.5	1.933	2.336	2.473	2.353	2.105	1.968	1.761	1.600	1.397	1.243	1.142	1.037	.950	.811	.630	.414	[illegible]	3.549
4.0	1.226	1.566	1.779	1.769	1.622	1.529	1.381	1.263	1.107	.975	.882	.782	.697	.555	.356	.075	-1.055	-.820
4.5	.785	1.036	1.268	1.336	1.261	1.198	1.089	.997	.868	.745	.653	.552	.462	.301	.036	-.631	.259	-.101
5.0	.505	.672	.871	.987	.968	.927	.846	.771	.654	.532	.435	.322	.213	-.010	-.638	.867	-6.742	.405
5.5	.323	.427	.563	.684	.708	.686	.625	.561	.447	.314	.200	.049	-.124	-.711	1.068	-.742	-.477	1.022
6.0	.201	.265	.339	.420	.455	.446	.401	.343	.219	.049	-.129	-.460	-1.263	.973	-.512	.388	.269	1.757
6.5	.117	.158	.189	.213	.211	.194	.141	.068	-.118	-.513	-1.673	1.692	.347	-.638	.571	6.256	1.037	1.575
7.0	.058	.086	.095	.080	.028	-.016	-.117	-.250	-.579	.300	.196	-.231	-.419	.328	-.449	-.550	.359	.501
7.5	.016	.037	.036	.007	-.057	-.102	-.181	-.233	-.111	-.029	-.153	-.135	.031	-.161	-.012	.034	-.141	.037
8.0	-.013	.002	-.002	-.031	-.084	-.114	-.151	-.157	-.088	-.094	-.111	-.048	-.061	-.053	-.075	-.066	-.011	-.047
8.5	-.035	-.022	-.027	-.052	-.092	-.110	-.131	-.131	-.065	-.093	-.080	-.054	-.073	-.039	-.034	-.023	-.039	-.027
9.0	-.051	-.039	-.044	-.064	-.093	-.106	-.119	-.118	-.081	-.088	-.064	-.062	-.064	-.051	-.039	-.041	-.072	-.013
9.5	-.063	-.052	-.055	-.071	-.093	-.102	-.112	-.108	-.077	-.079	-.057	-.065	-.049	-.053	-.046	-.031	-.030	-.023
10.0	-.072	-.061	-.063	-.076	-.092	-.099	-.106	-.100	-.075	-.070	-.055	-.059	-.043	-.041	-.033	-.028	-.022	-.023



1 Geometry of convergent portion of nozzle

$$A(z,\omega,s_{\nu h}) = A_{ref}\left(\beta z, \frac{\omega}{\beta}, \frac{s_{\nu h}}{\beta}\right)$$

$$B(z,\omega,s_{\nu h}) = \beta B_{ref}\left(\beta z, \frac{\omega}{\beta}, \frac{s_{\nu h}}{\beta}\right)$$

$$C(z,\omega,s_{\nu h}) = C_{ref}\left(\beta z, \frac{\omega}{\beta}, \frac{s_{\nu h}}{\beta}\right)$$

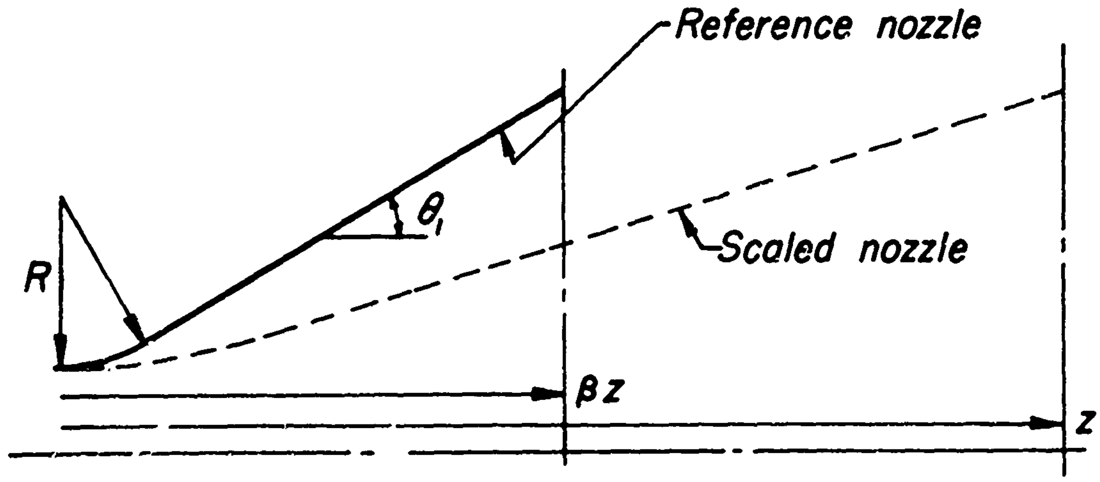


Fig.2 Scaling of admittance coefficients

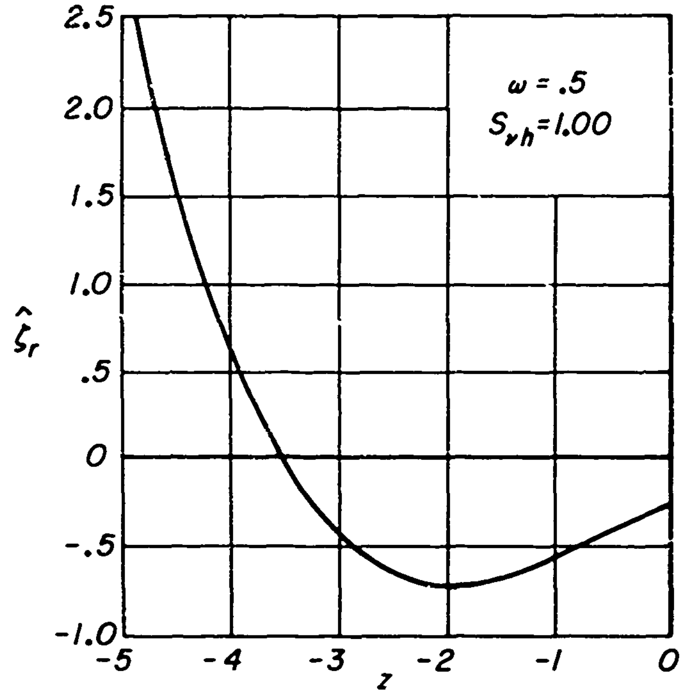


Fig. 3 Real part of $\hat{\zeta}$ versus axial distance

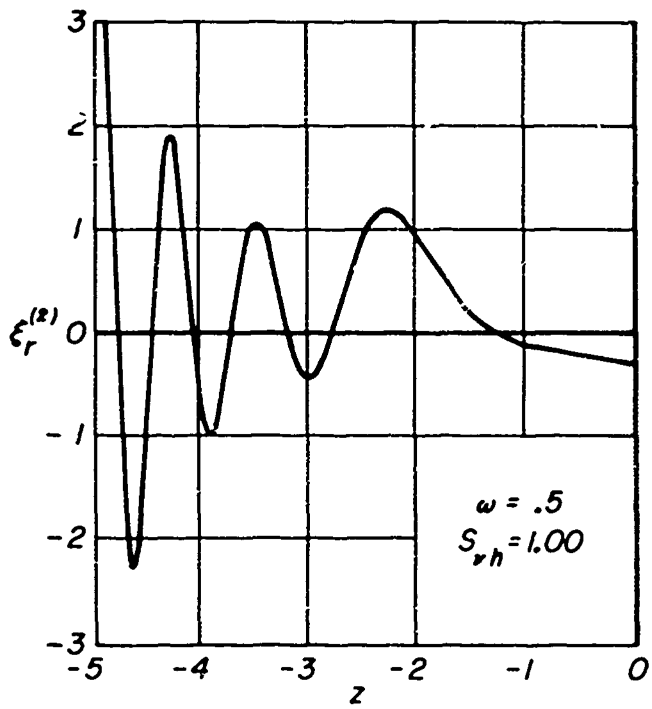


Fig. 4 Real part of $\xi^{(2)}$ versus axial distance

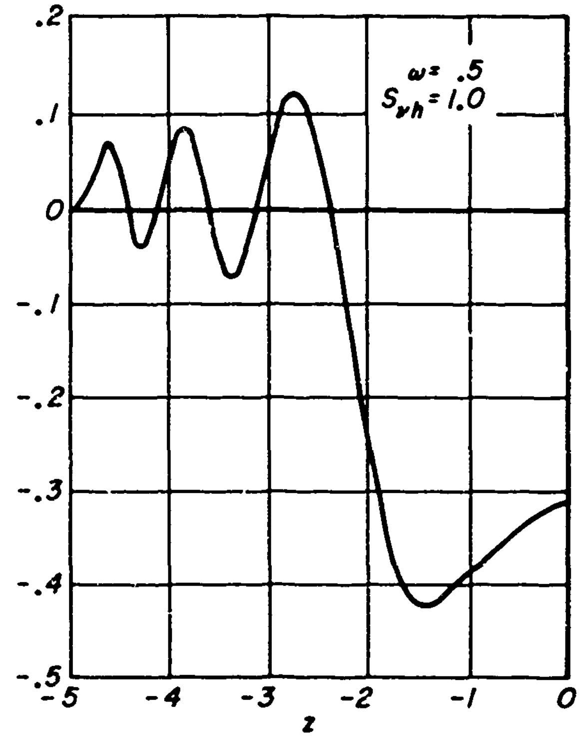


Fig. 5 Real part of pressure admittance coefficient versus axial distance

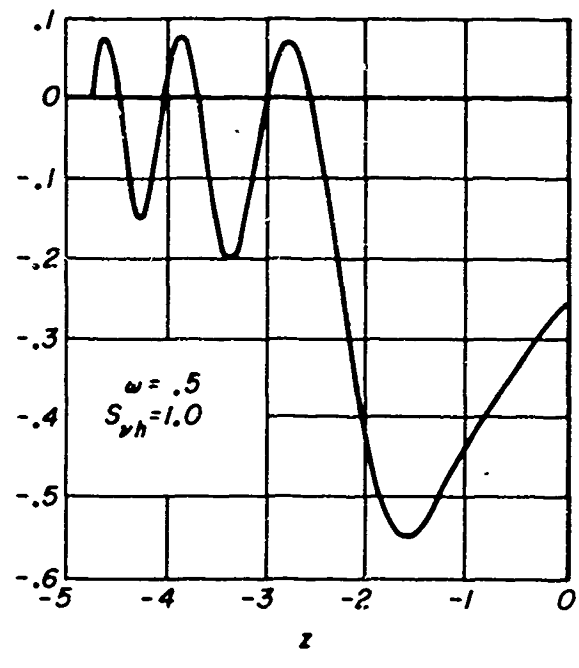


Fig. 6 Imaginary part of radial velocity admittance coefficient versus axial distance

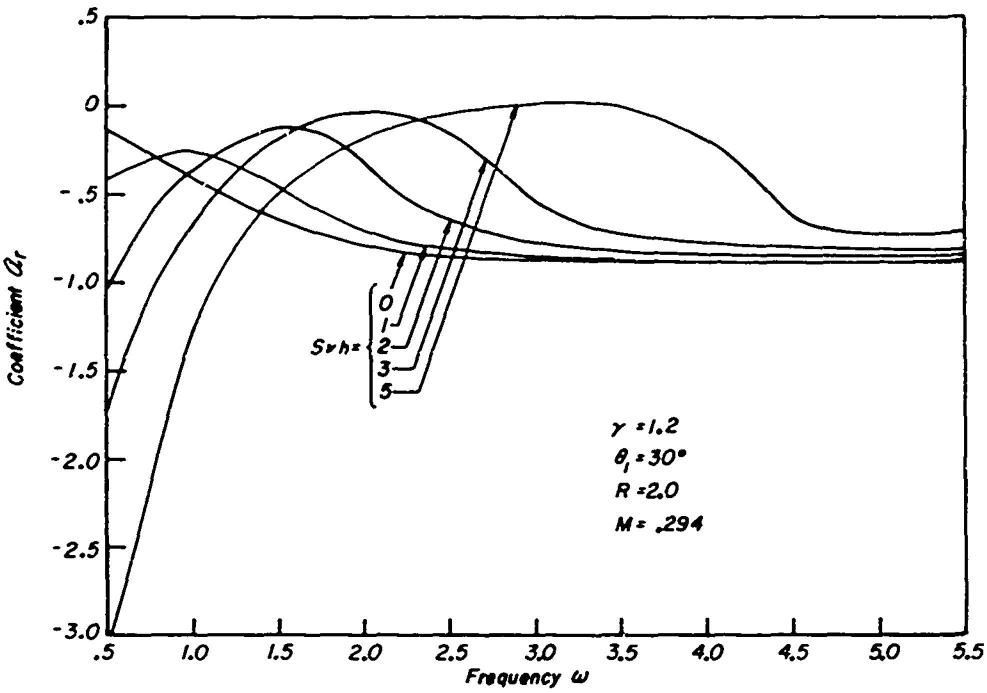


Fig. 7(a) Real part of pressure admittance coefficient versus frequency

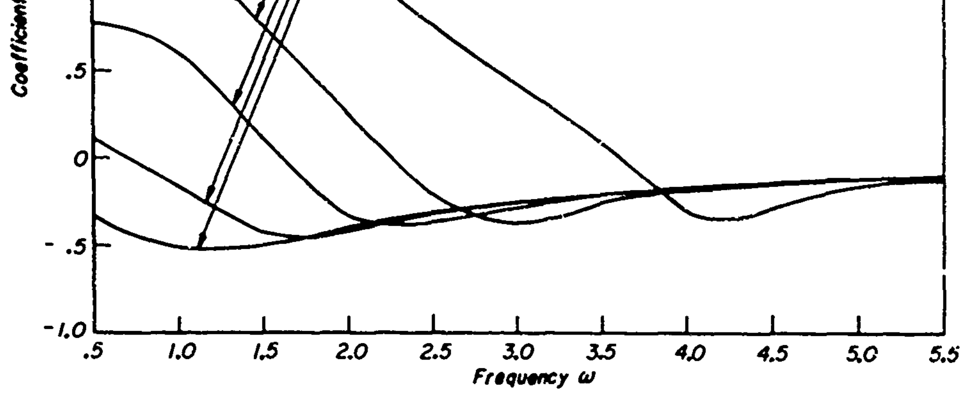


Fig. 7(b) Imaginary part of pressure admittance coefficient versus frequency

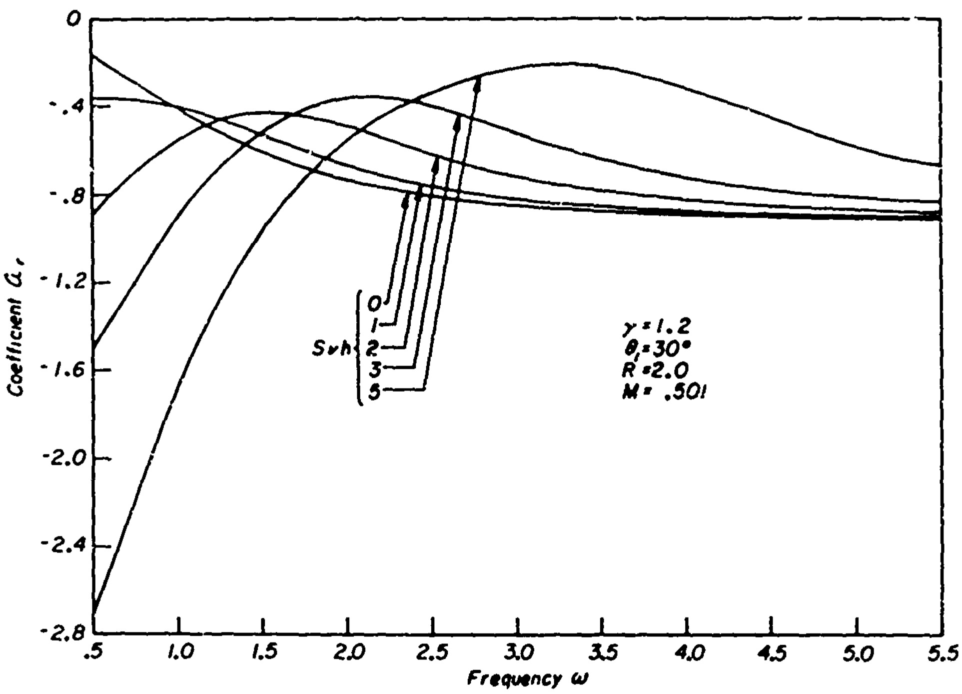


Fig.7(c) Real part of pressure admittance coefficient versus frequency

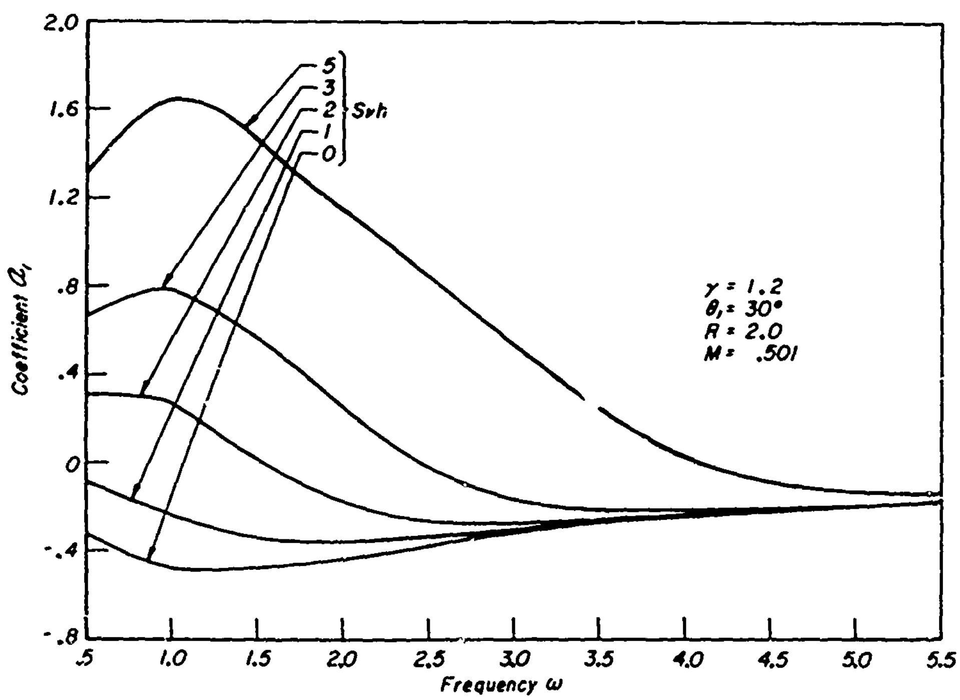


Fig.7(d) Imaginary part of pressure admittance coefficient versus frequency

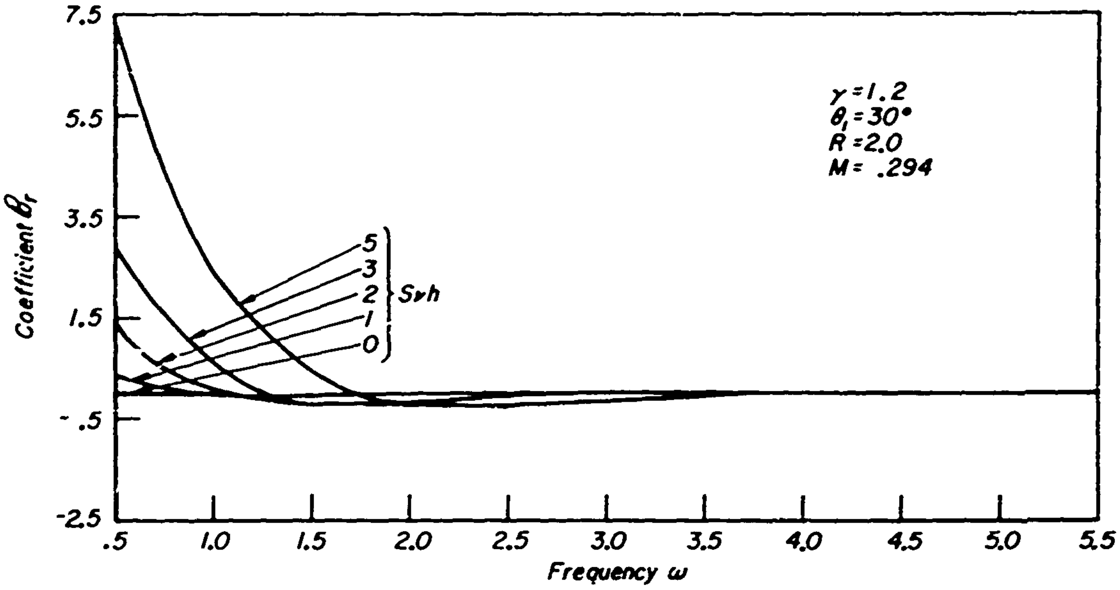


Fig. 8(a) Real part of radial velocity admittance coefficient versus frequency

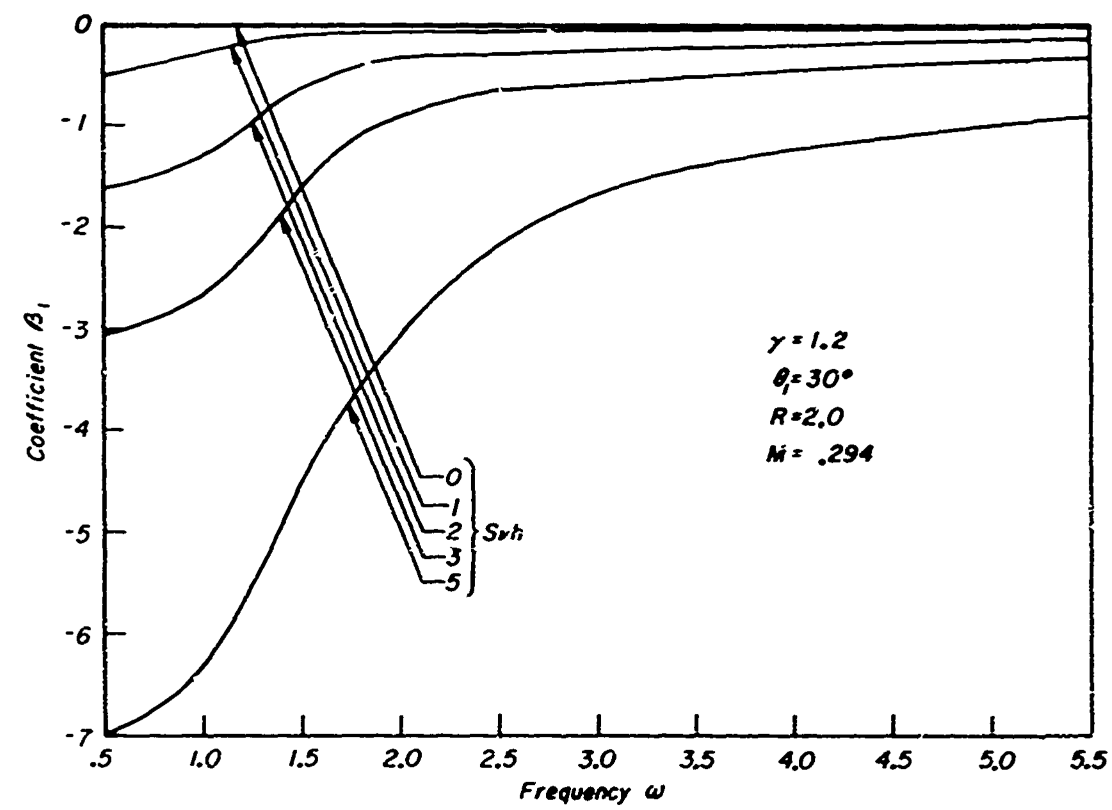


Fig. 8(b) Imaginary part of radial velocity admittance coefficient versus frequency

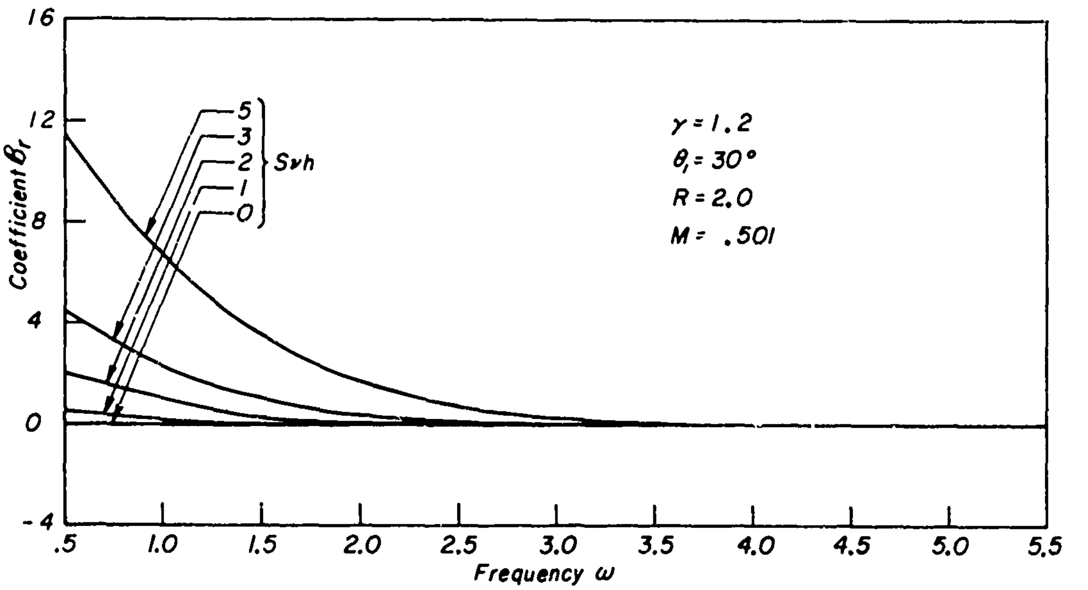


Fig. 8(c) Real part of radial velocity admittance coefficient versus frequency

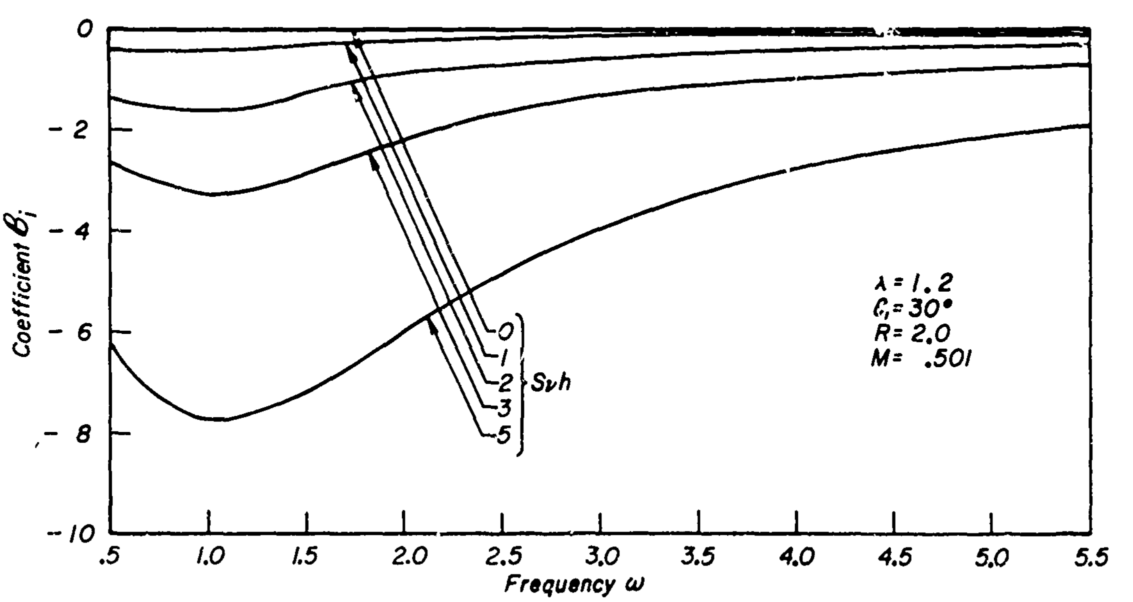


Fig. 8(d) Imaginary part of radial velocity admittance coefficient versus frequency

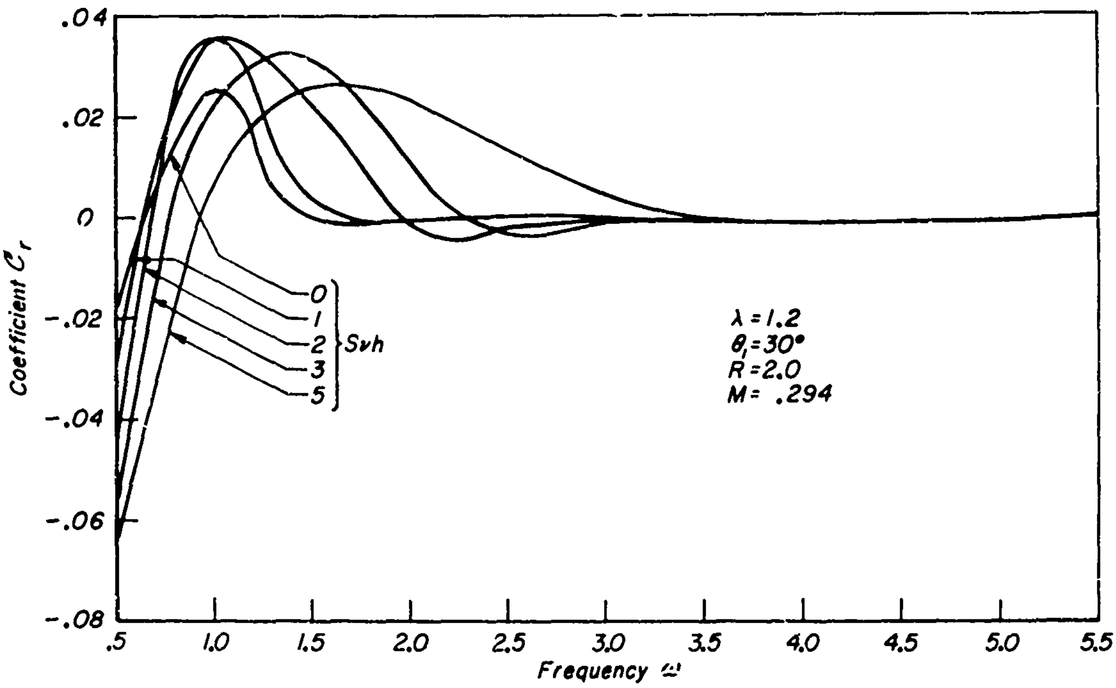


Fig. 9(a) Real part of entropy admittance coefficient versus frequency

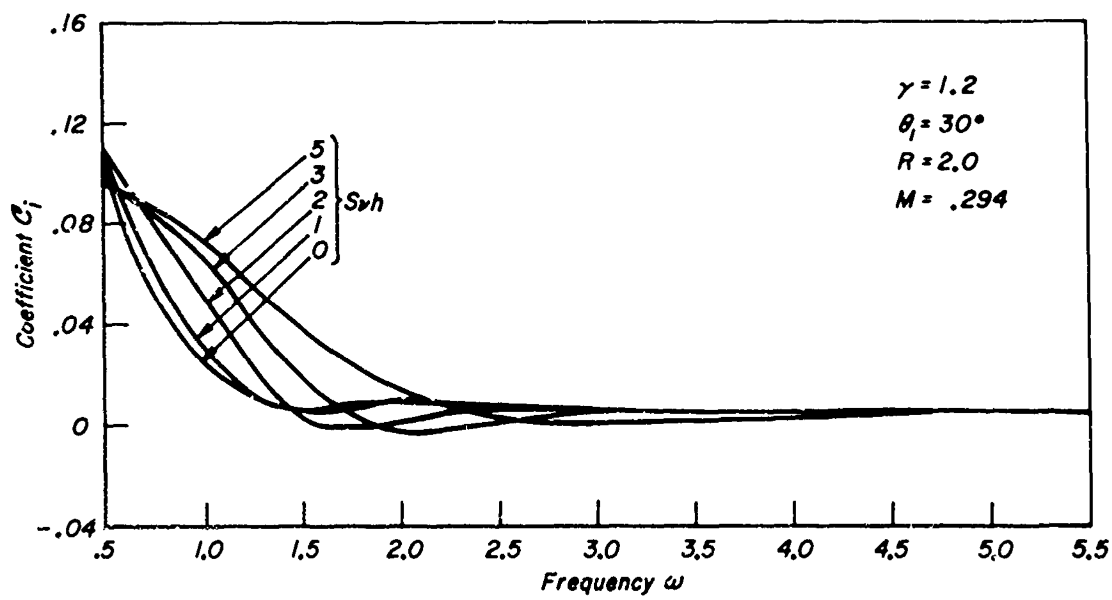


Fig. 9(b) Imaginary part of entropy admittance coefficient versus frequency

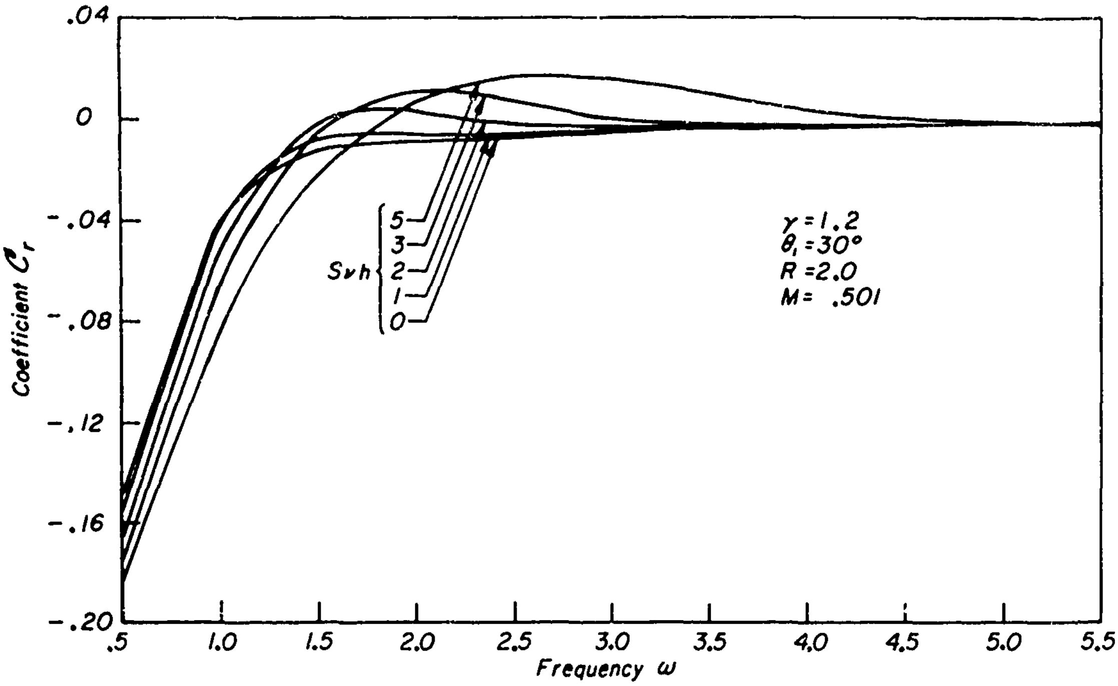


Fig.9(c) Real part of entropy admittance coefficient versus frequency

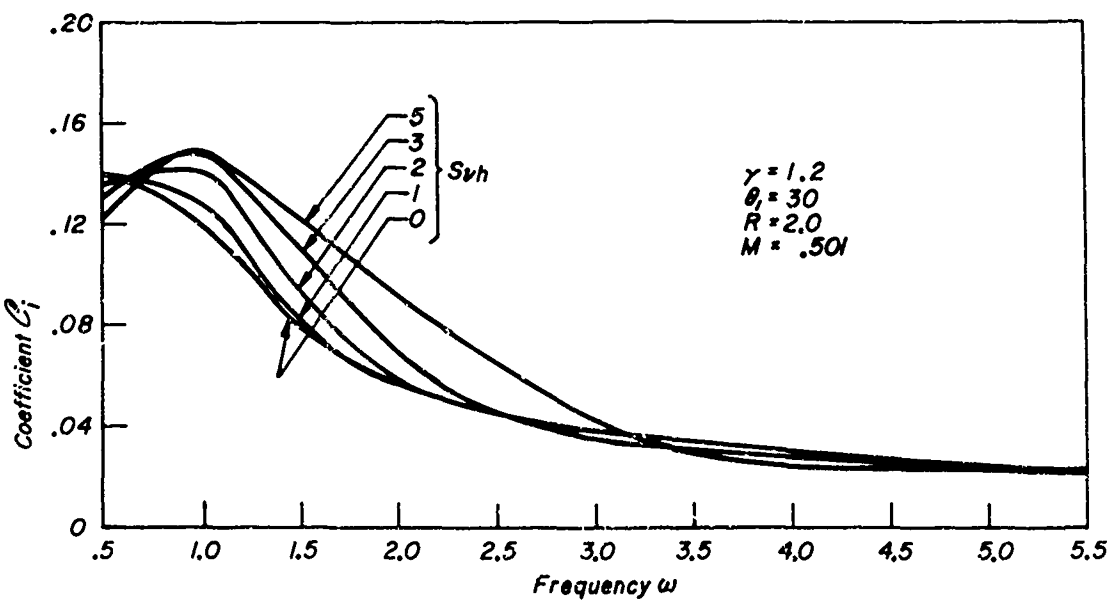


Fig.9(d) Imaginary part of entropy admittance coefficient versus frequency

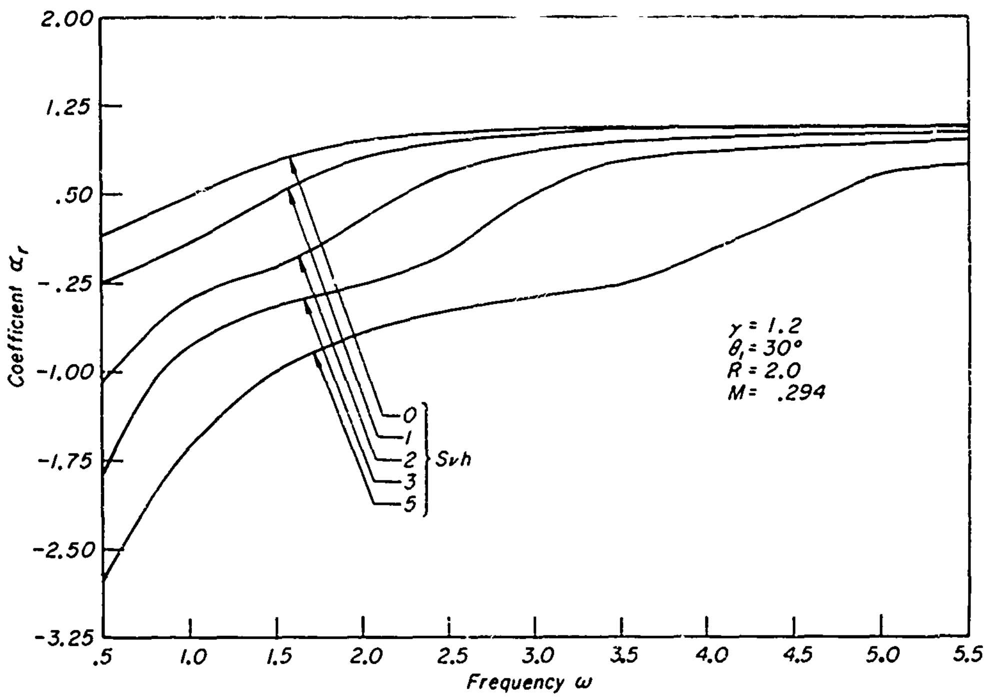


Fig. 10(a) Real part of irrotational admittance coefficient versus frequency

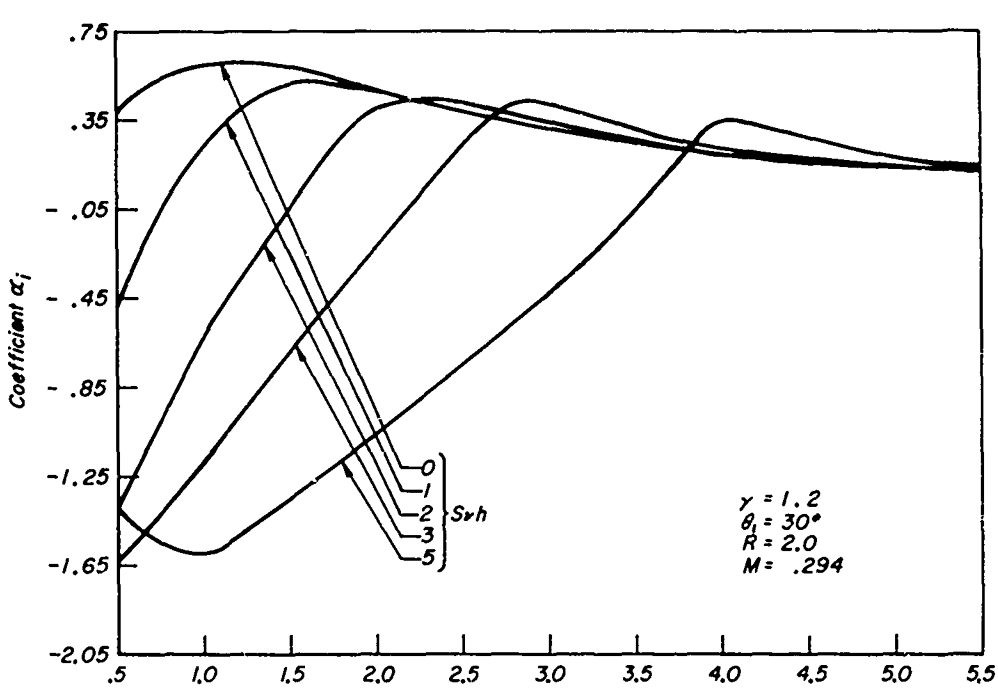


Fig. 10(b) Imaginary part of irrotational admittance coefficient versus frequency

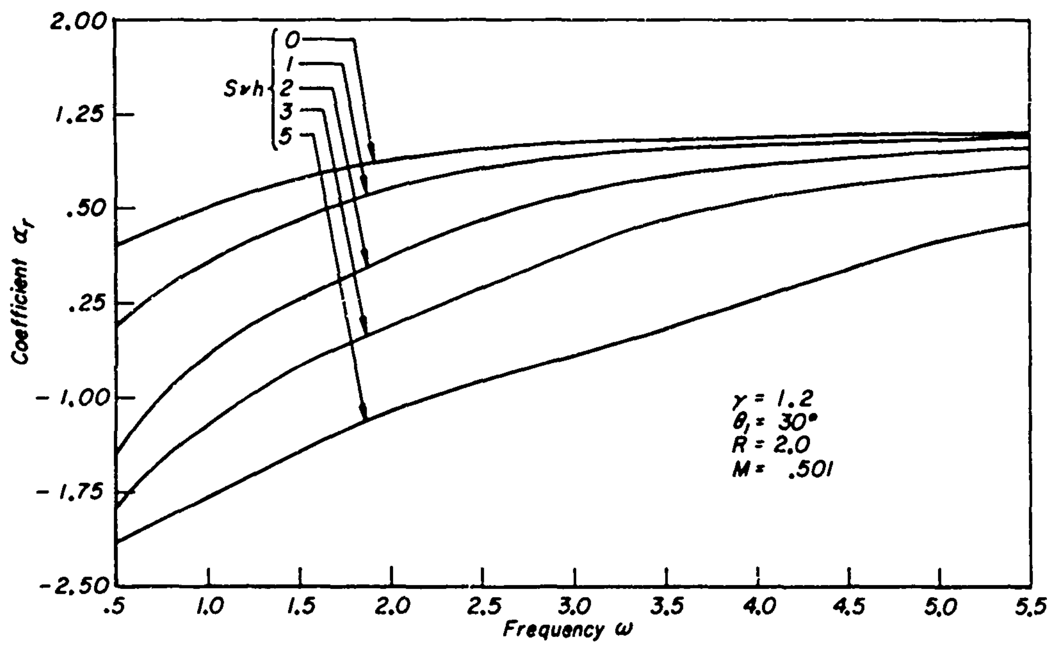


Fig. 10(c) Real part of irrotational admittance coefficient versus frequency

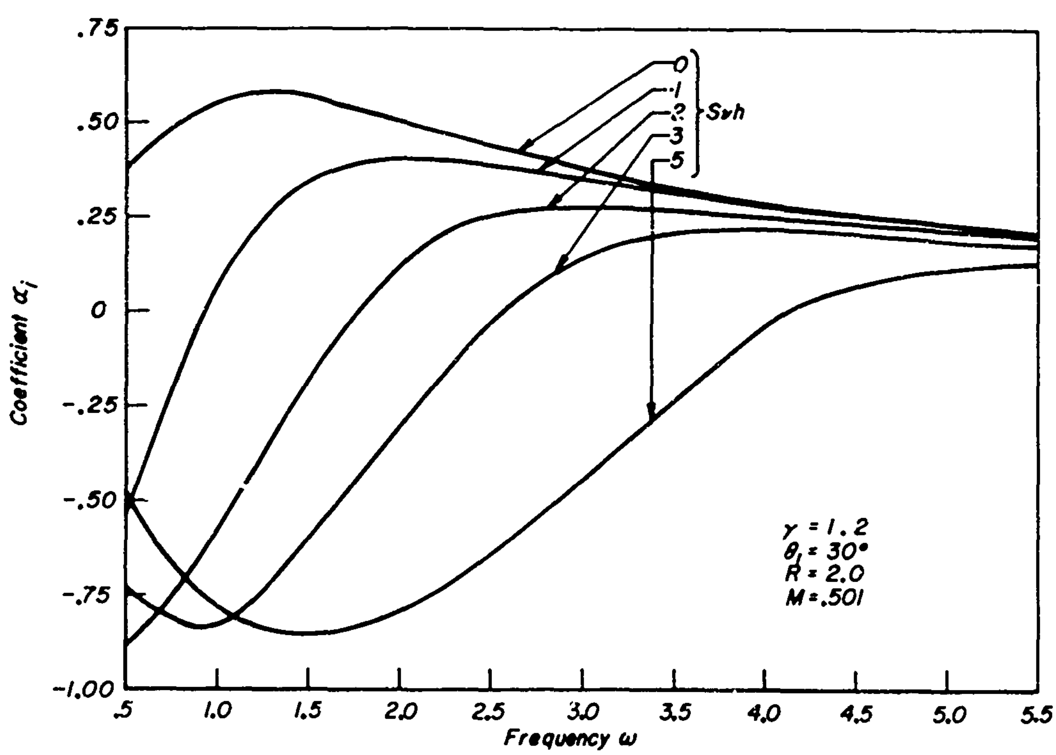


Fig. 10(d) Imaginary part of irrotational admittance coefficient versus frequency

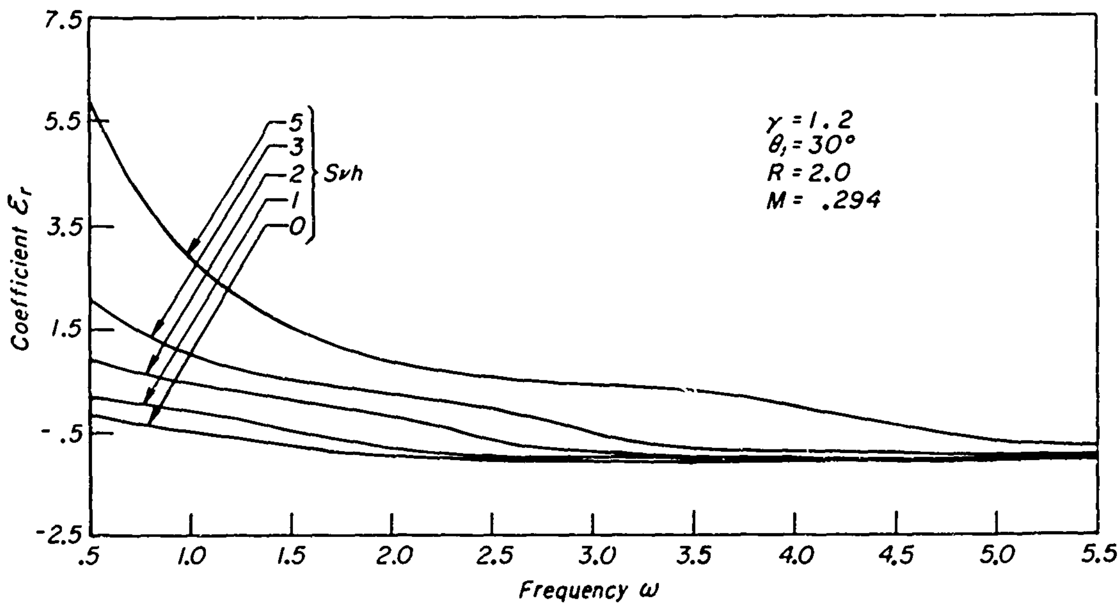


Fig. 11(a) Real part of combined admittance coefficient versus frequency

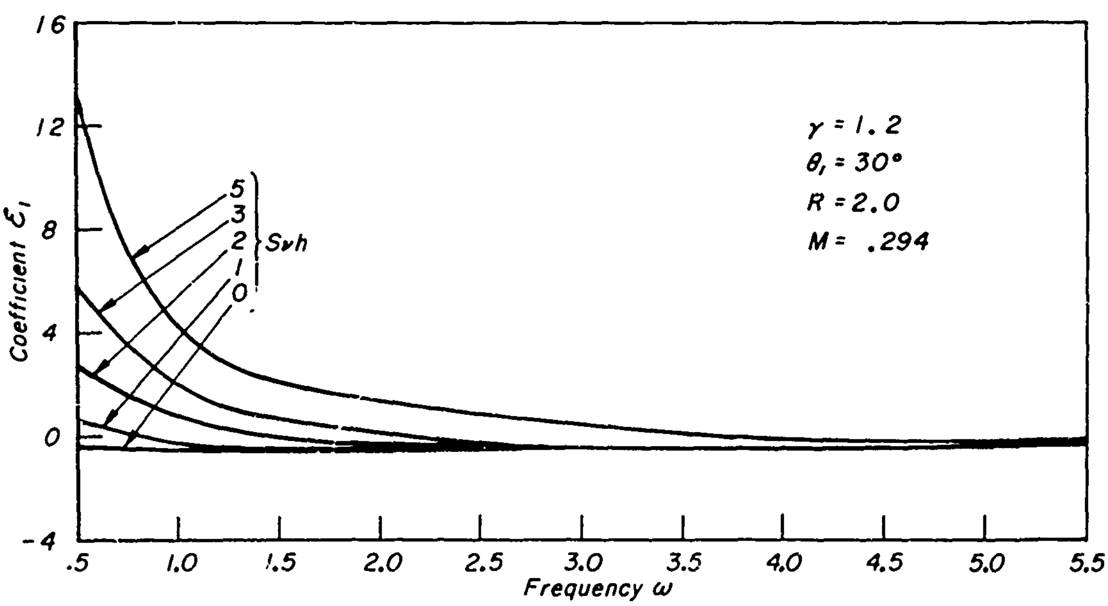


Fig. 11(b) Imaginary part of combined admittance coefficient versus frequency

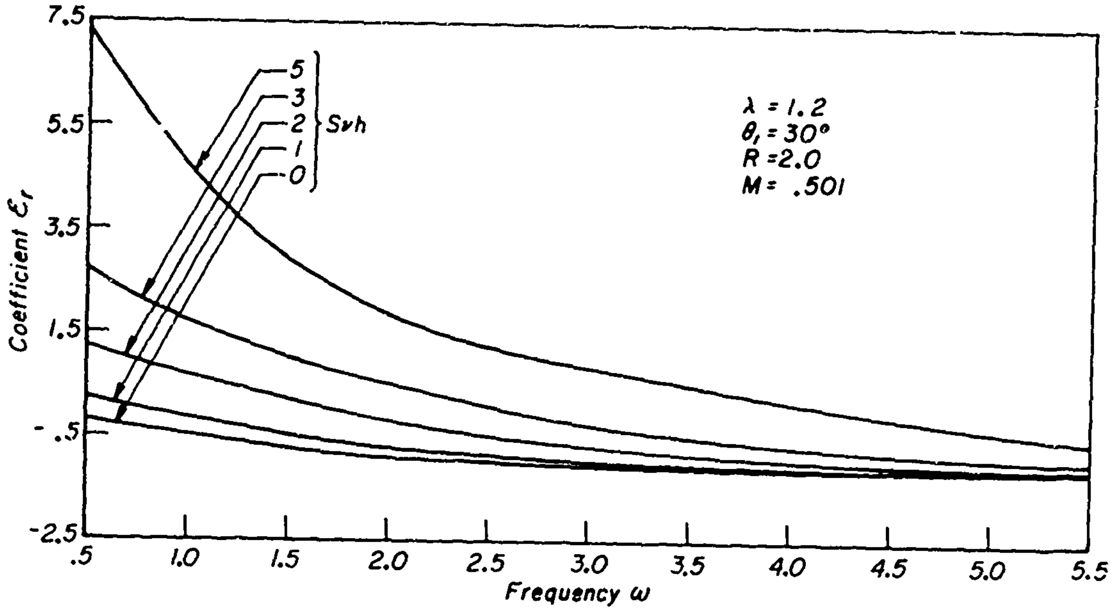


Fig. 11(c) Real part of combined admittance coefficient versus frequency

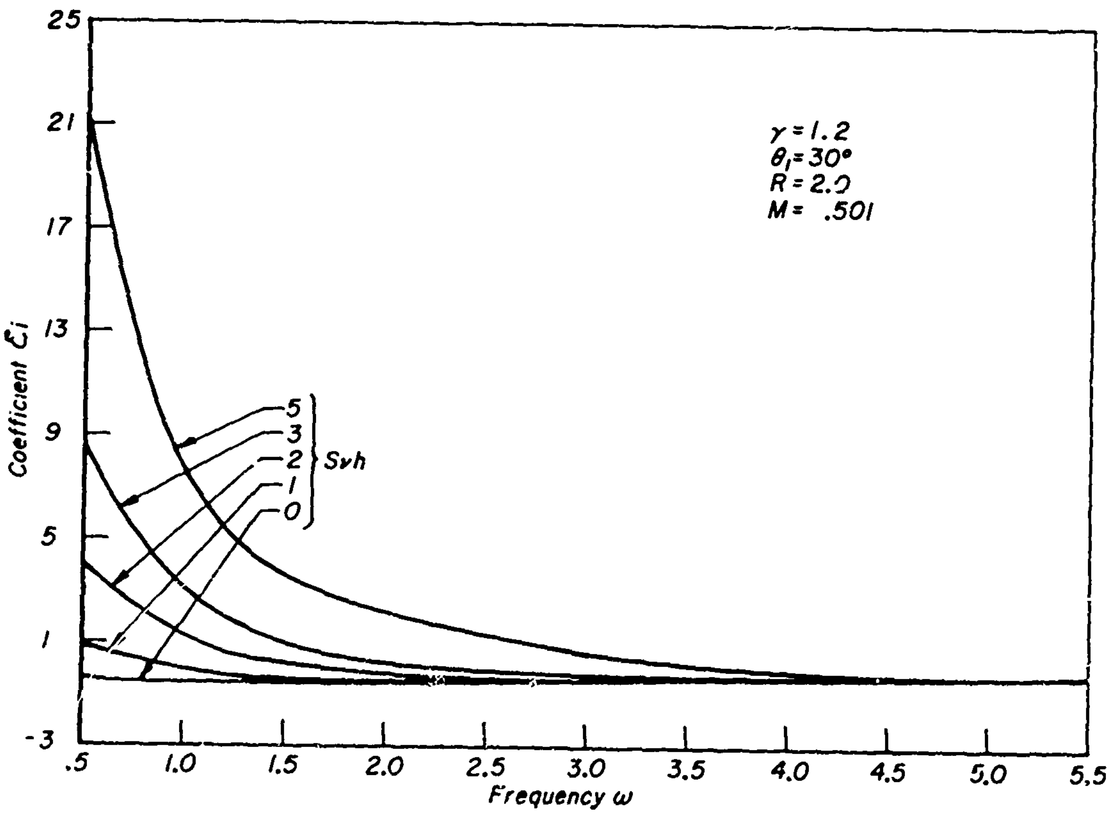


Fig. 11(d) Imaginary part of combined admittance coefficient versus frequency

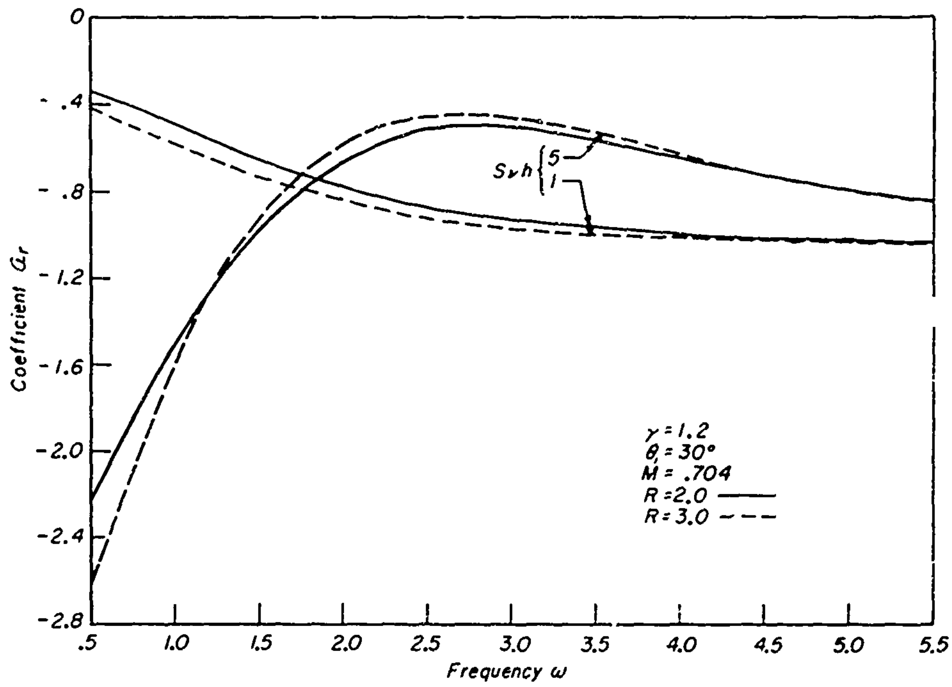


Fig. 12(a) Real part of pressure admittance coefficient versus frequency: Effect of throat wall curvature

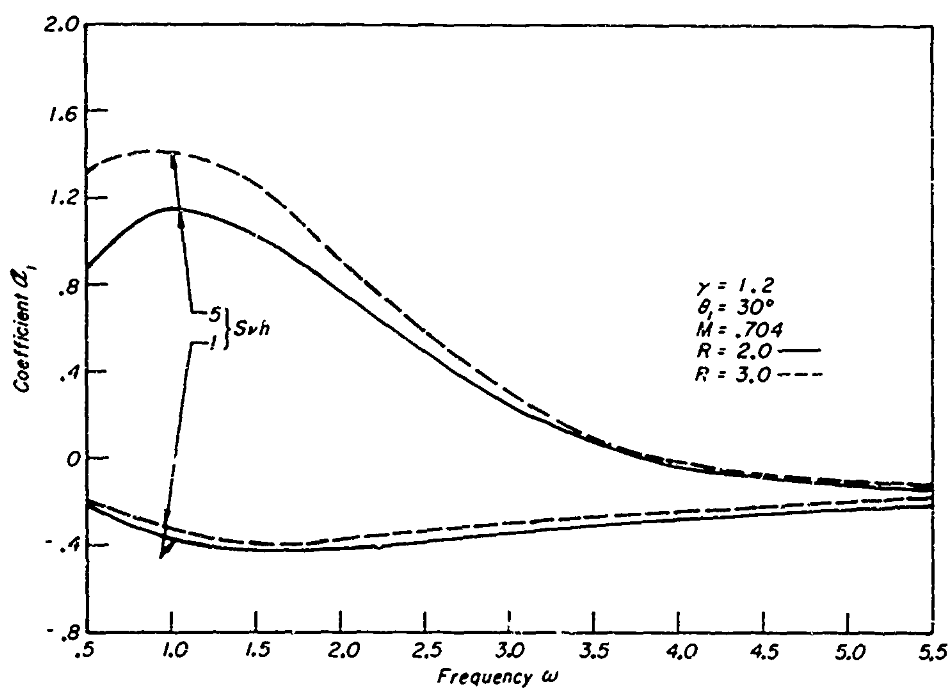


Fig. 12(b) Imaginary part of pressure admittance coefficient versus frequency: Effect of throat wall curvature

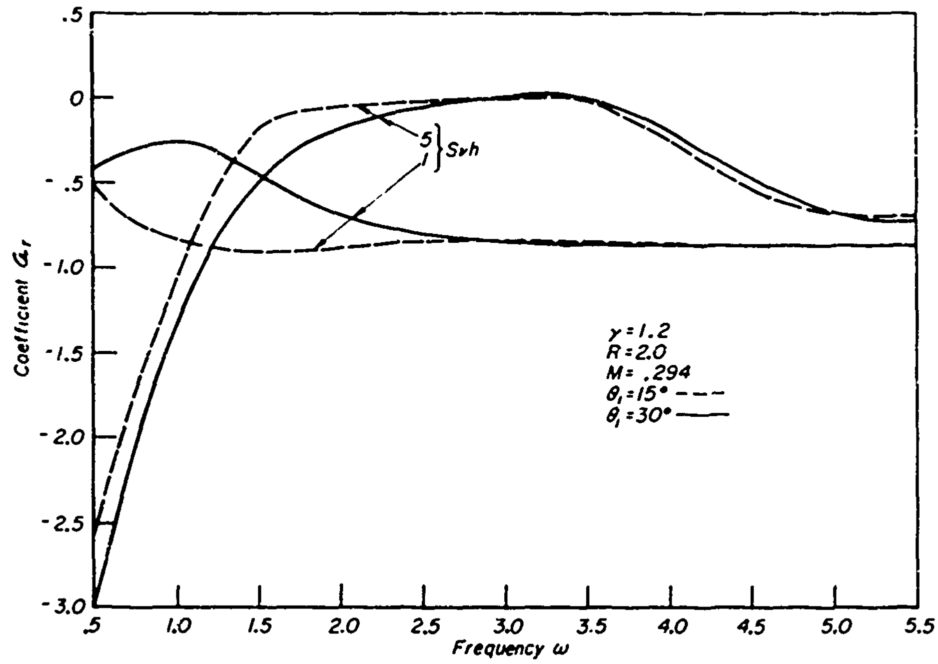


Fig. 12(c) Real part of pressure admittance coefficient versus frequency: Effect of cone angle

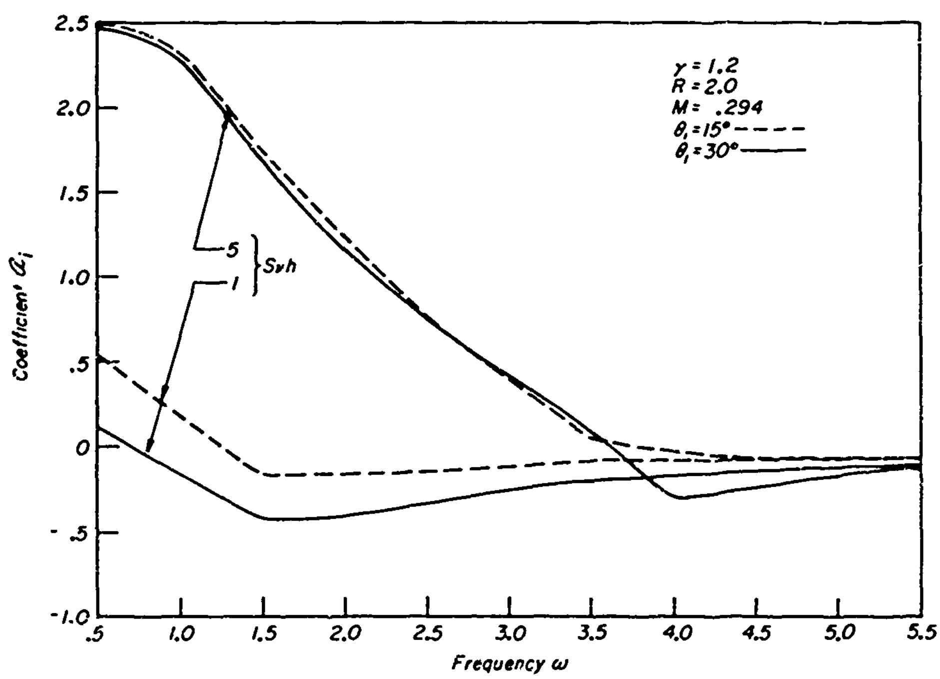


Fig. 12(d) Imaginary part of pressure admittance coefficient versus frequency: Effect of cone angle

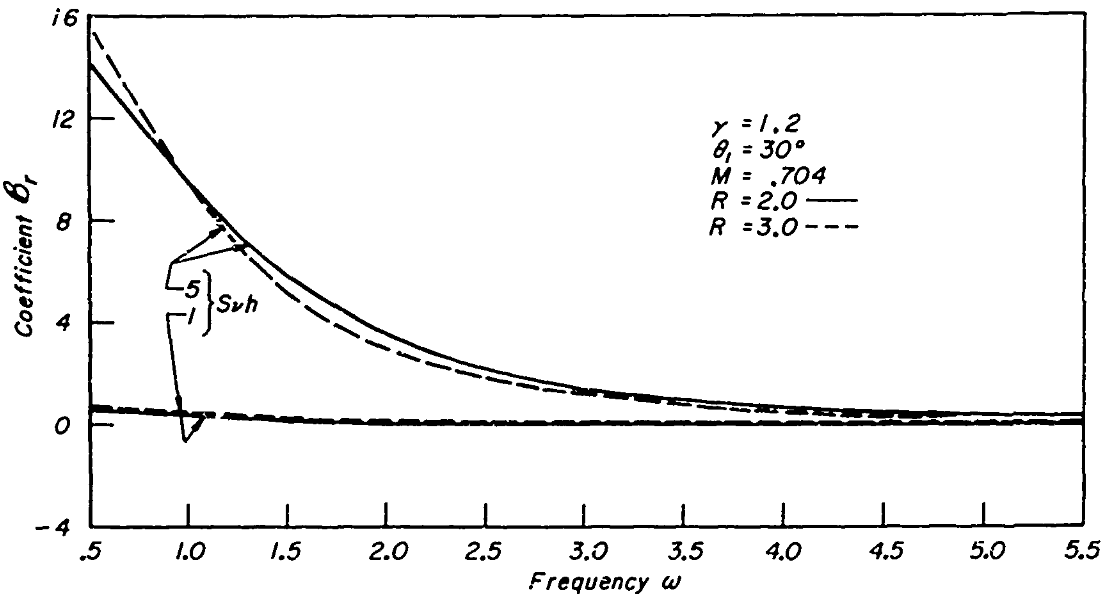


Fig. 13(a) Real part of radial velocity admittance coefficient versus frequency: Effect of throat wall curvature

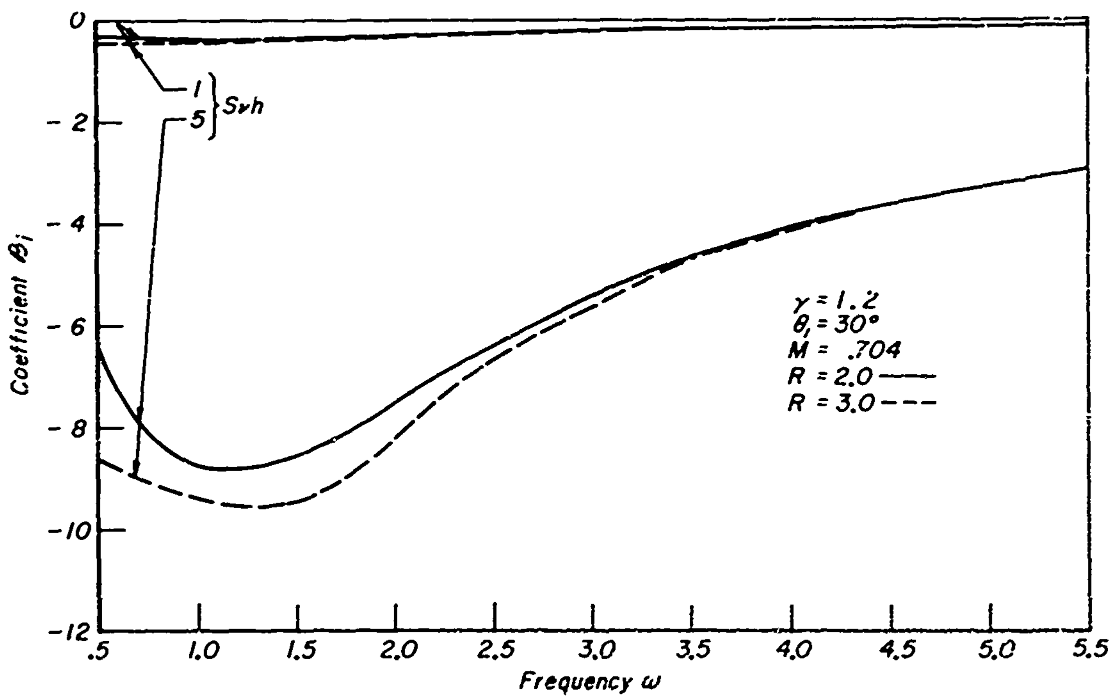


Fig. 13(b) Imaginary part of radial velocity admittance coefficient versus frequency: Effect of throat wall curvature

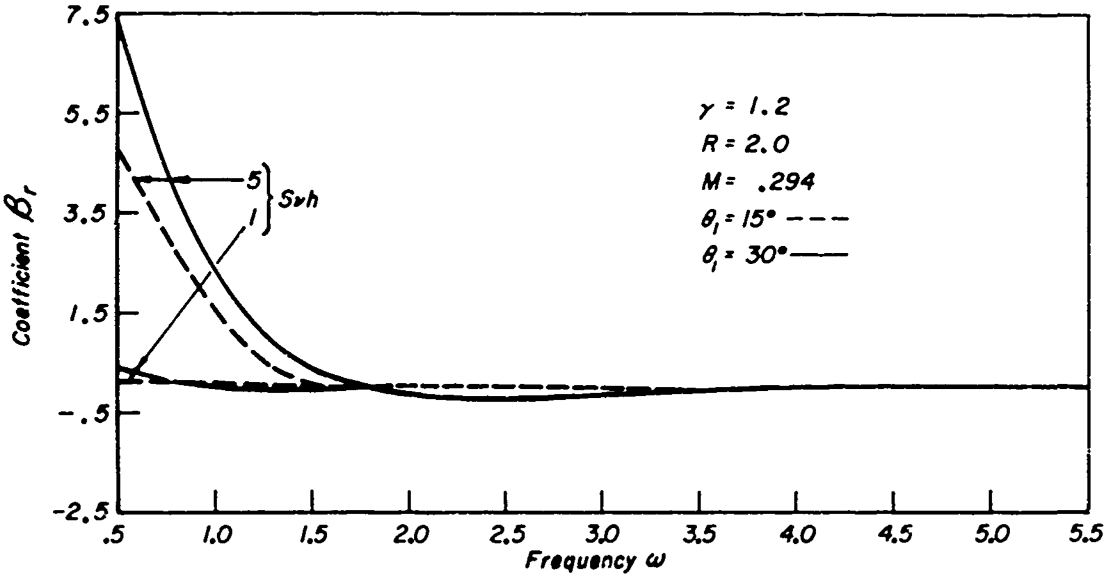


Fig. 13(c) Real part of radial velocity admittance coefficient versus frequency: Effect of cone angle

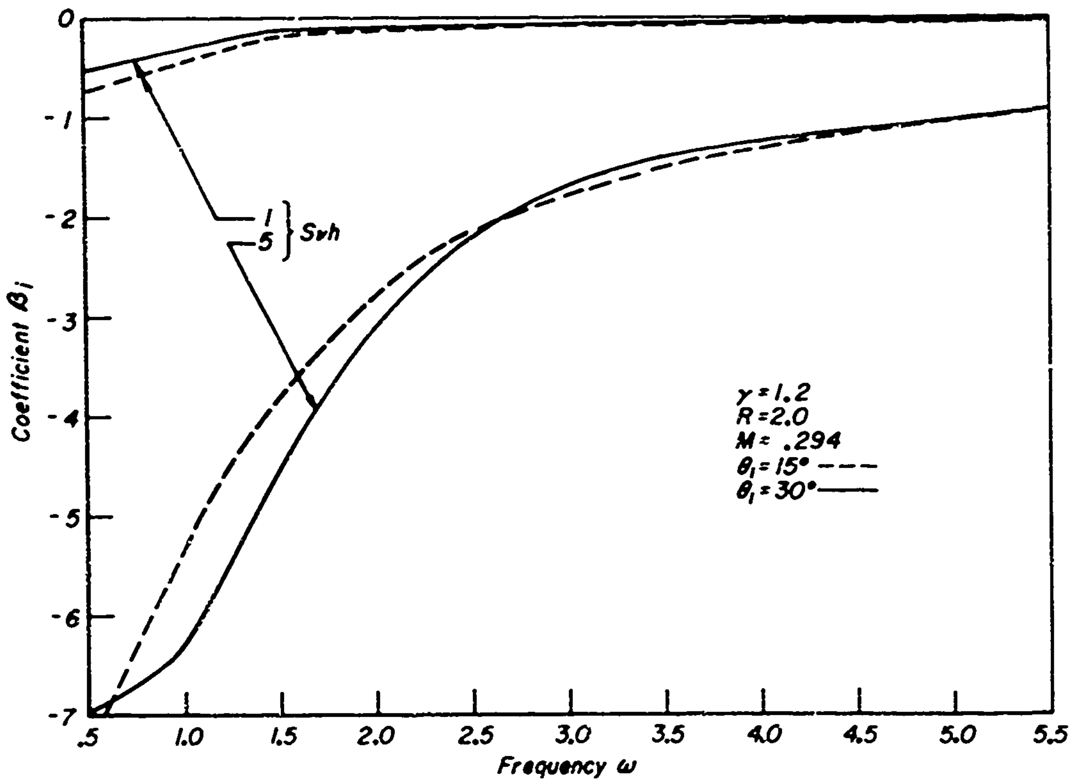


Fig. 13(d) Imaginary part of radial velocity admittance coefficient versus frequency: Effect of cone angle

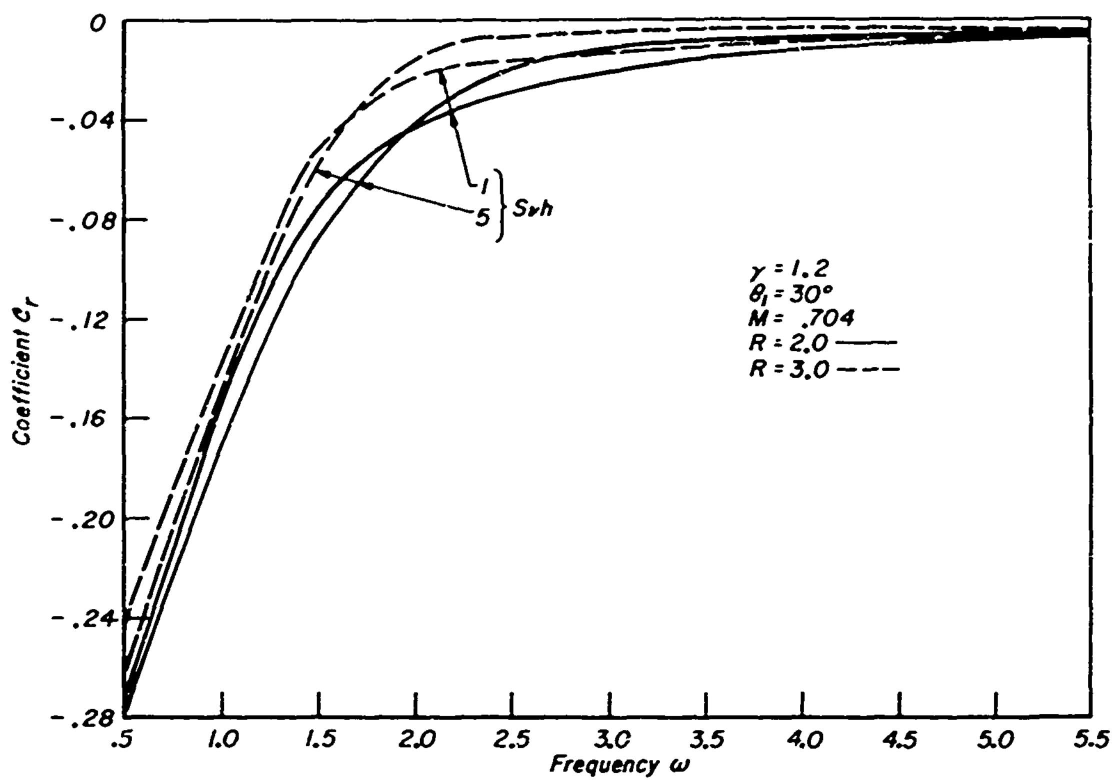


Fig. 14(a) Real part of entropy admittance coefficient versus frequency: Effect of throat wall curvature

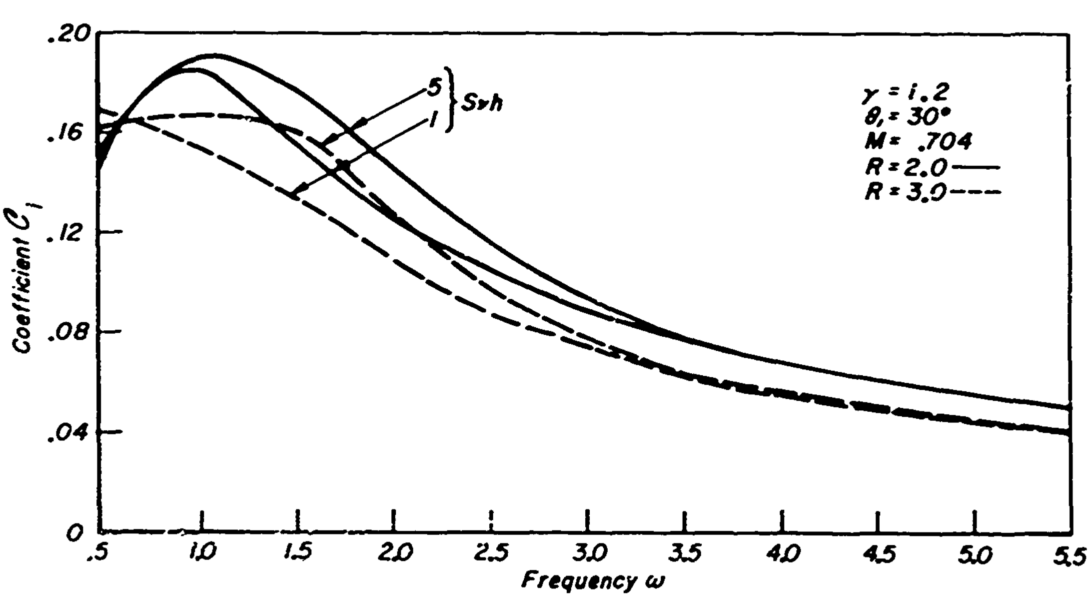


Fig. 14(b) Imaginary part of entropy admittance coefficient versus frequency: Effect of throat wall curvature

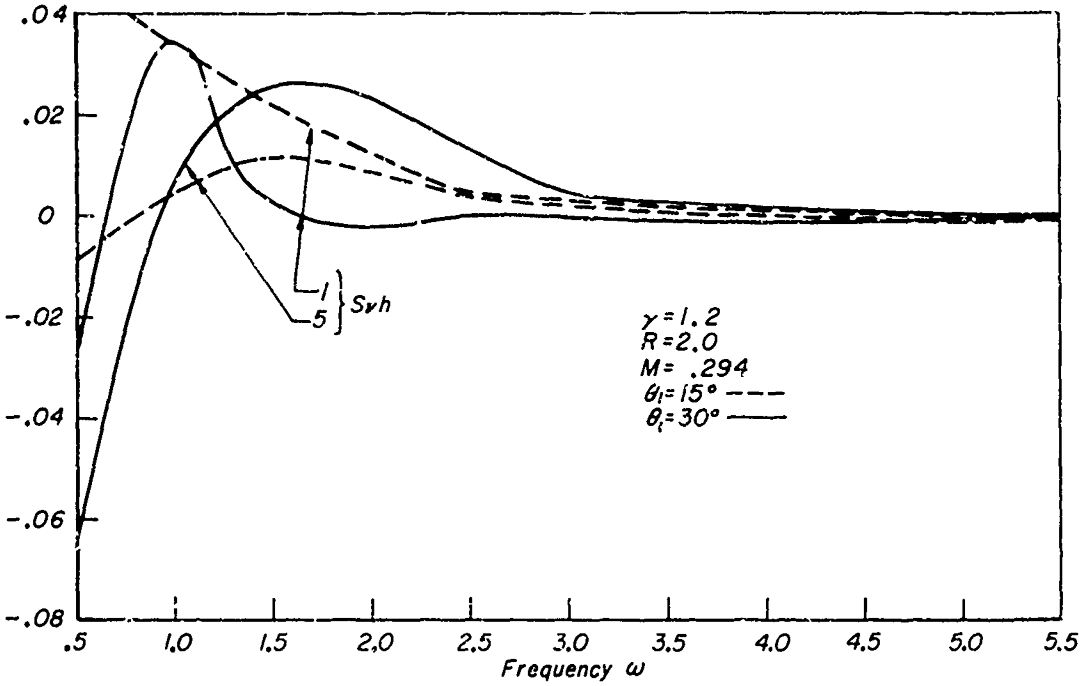


Fig. 14(c) Real part of entropy admittance coefficient versus frequency: Effect of cone angle

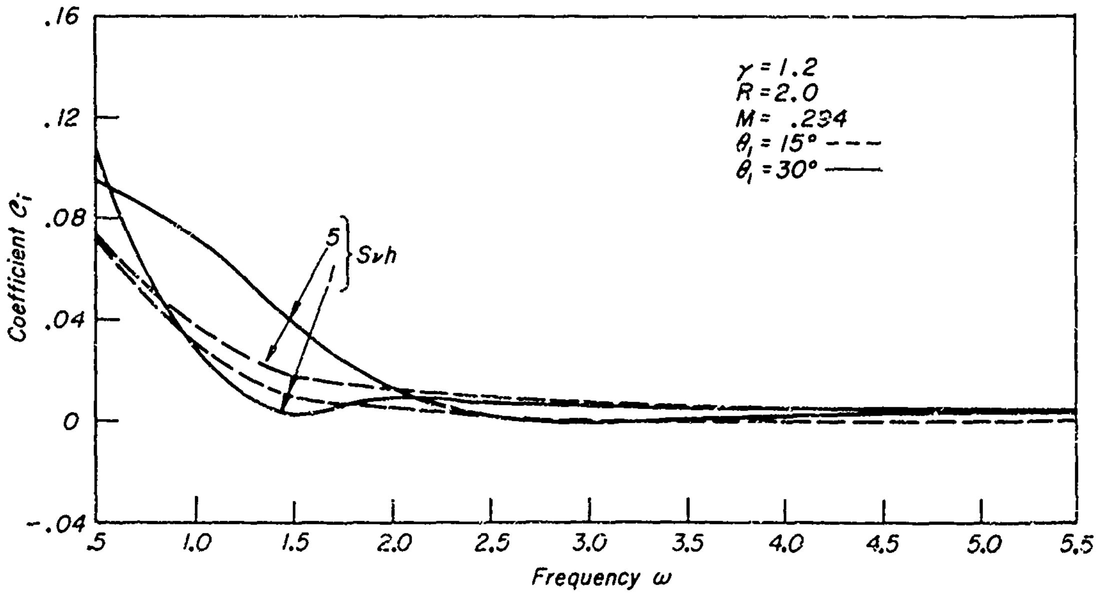


Fig. 14(d) Imaginary part of entropy admittance coefficient versus frequency: Effect of cone angle

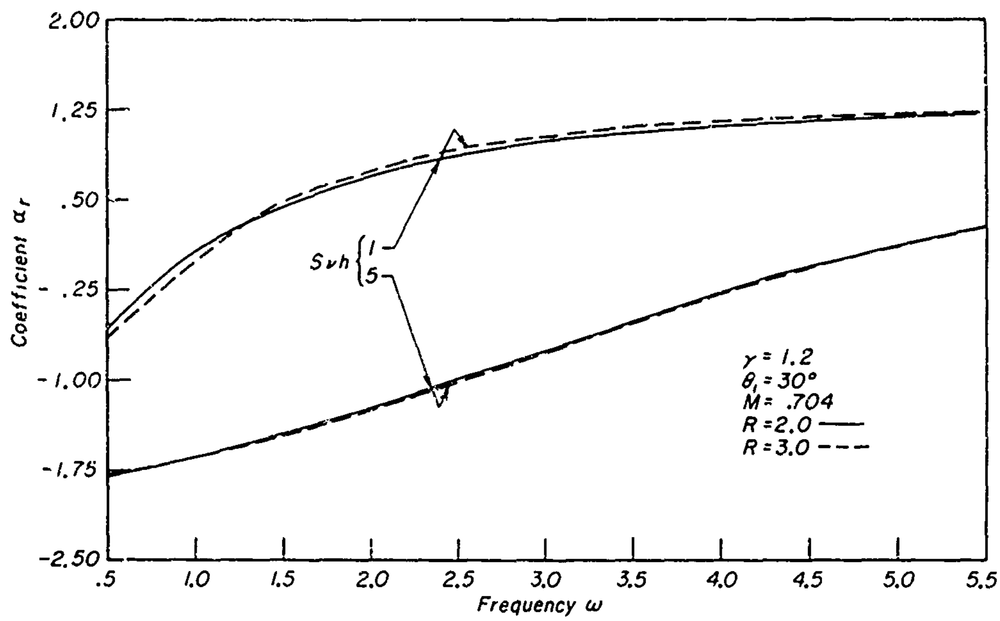


Fig. 15(a) Real part of irrotational admittance coefficient versus frequency: Effect of throat wall curvature

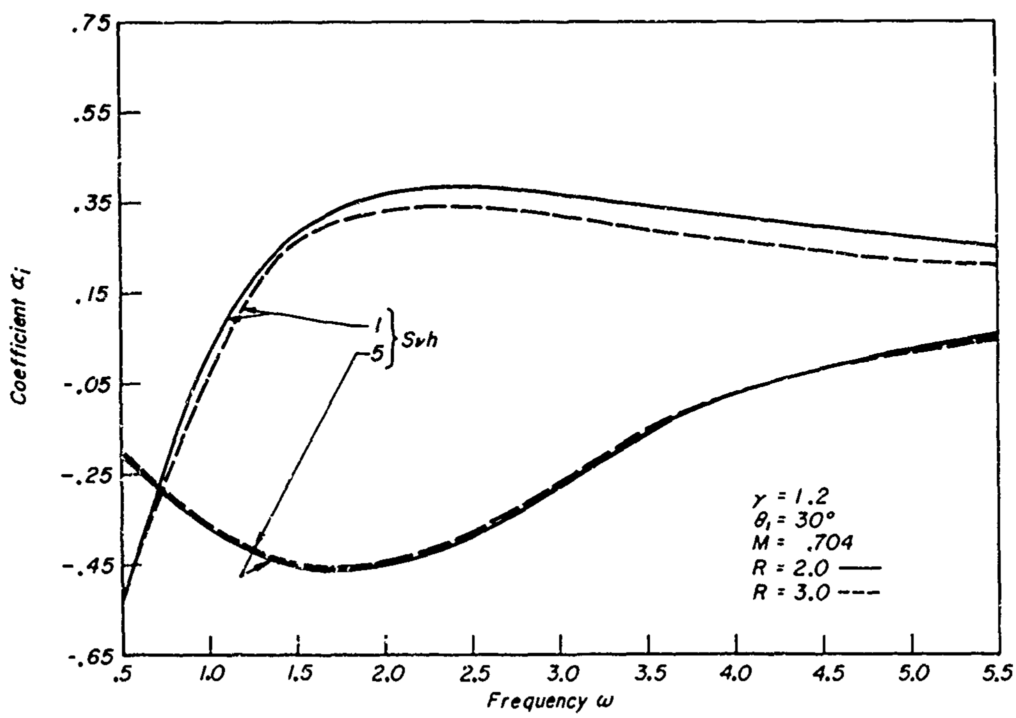


Fig. 15(b) Imaginary part of irrotational admittance coefficient versus frequency: Effect of throat wall curvature

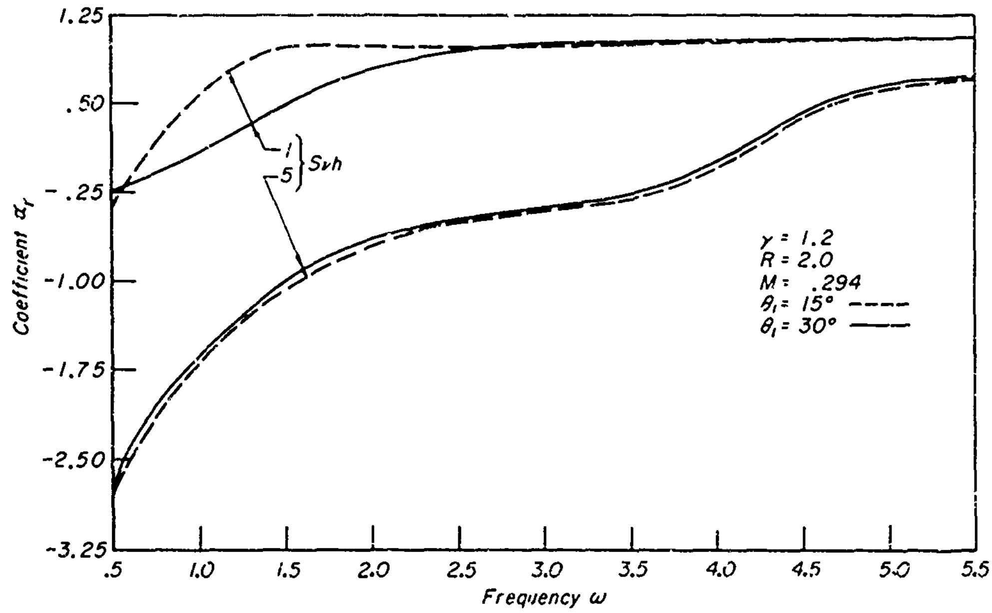


Fig. 15(c) Real part of irrotational admittance coefficient versus frequency: Effect of cone angle

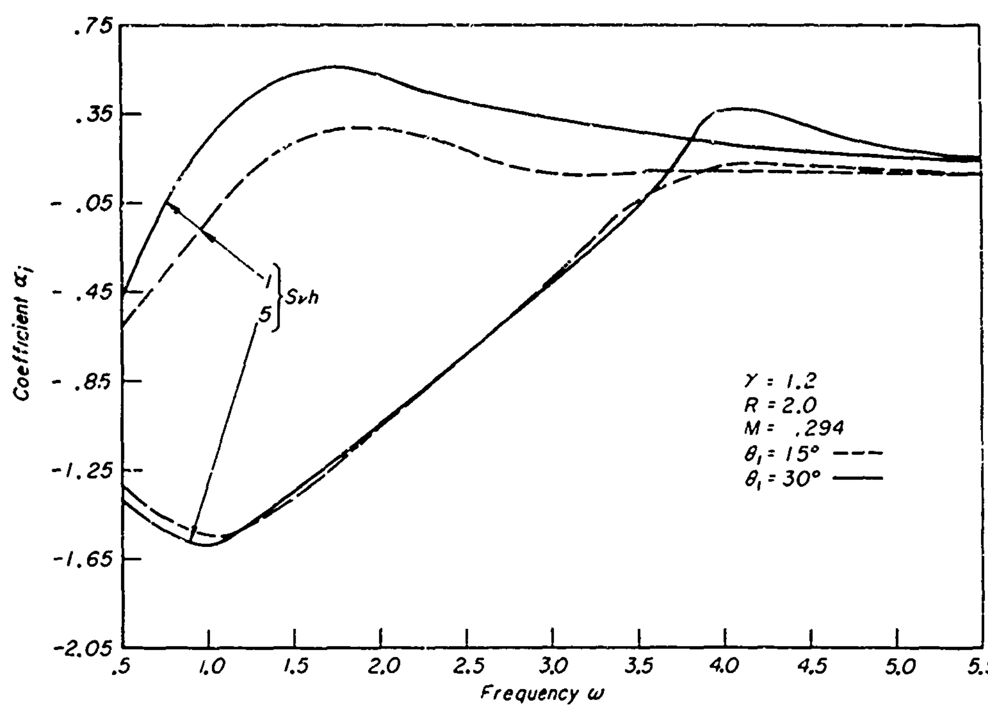


Fig. 15(d) Imaginary part of irrotational admittance coefficient versus frequency: Effect of cone angle

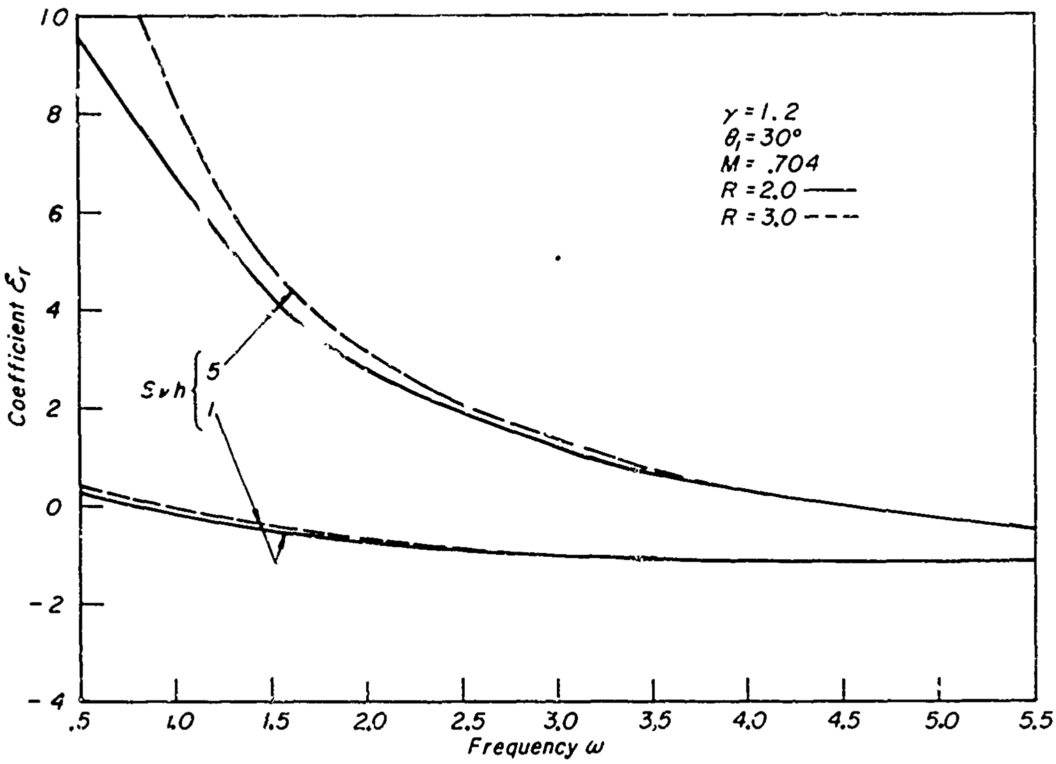


Fig. 16(a) Real part of combined admittance coefficient versus frequency: Effect of throat wall curvature

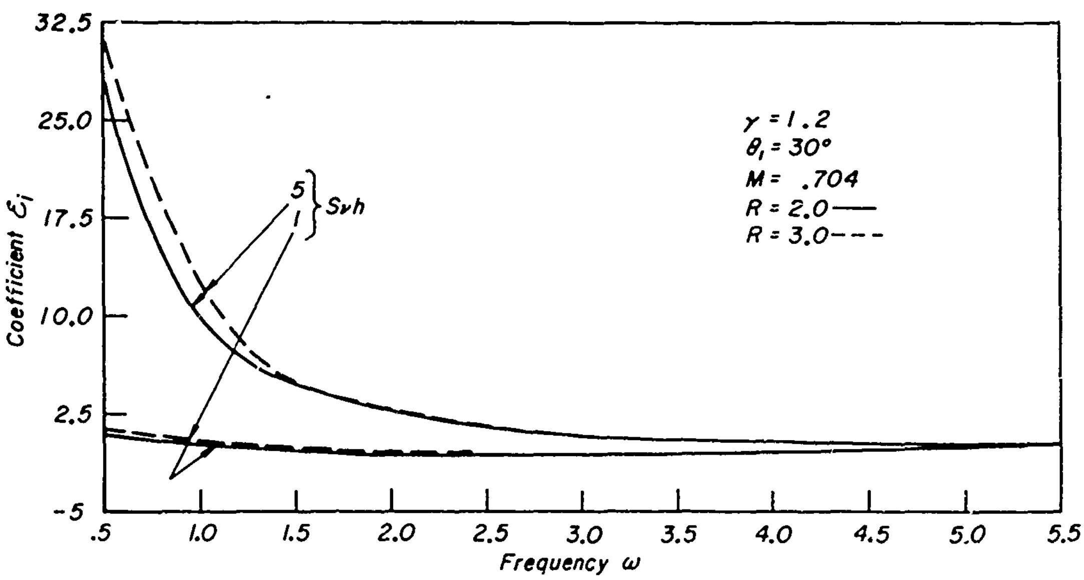


Fig. 16(b) Imaginary part of combined admittance coefficient versus frequency: Effect of throat wall curvature

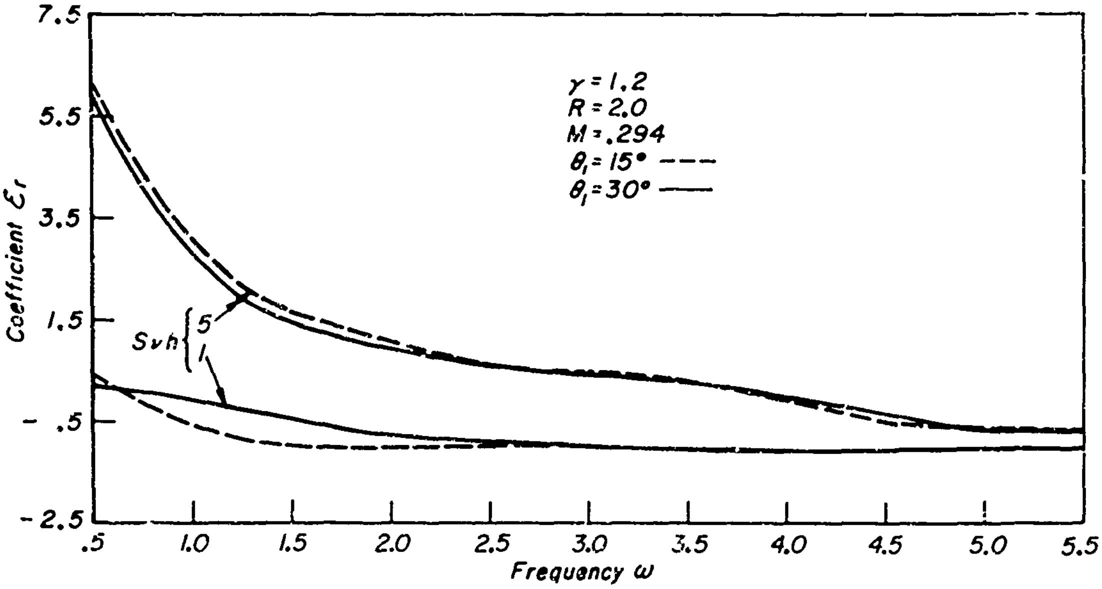


Fig. 16(c) Real part of combined admittance coefficient versus frequency: Effect of cone angle

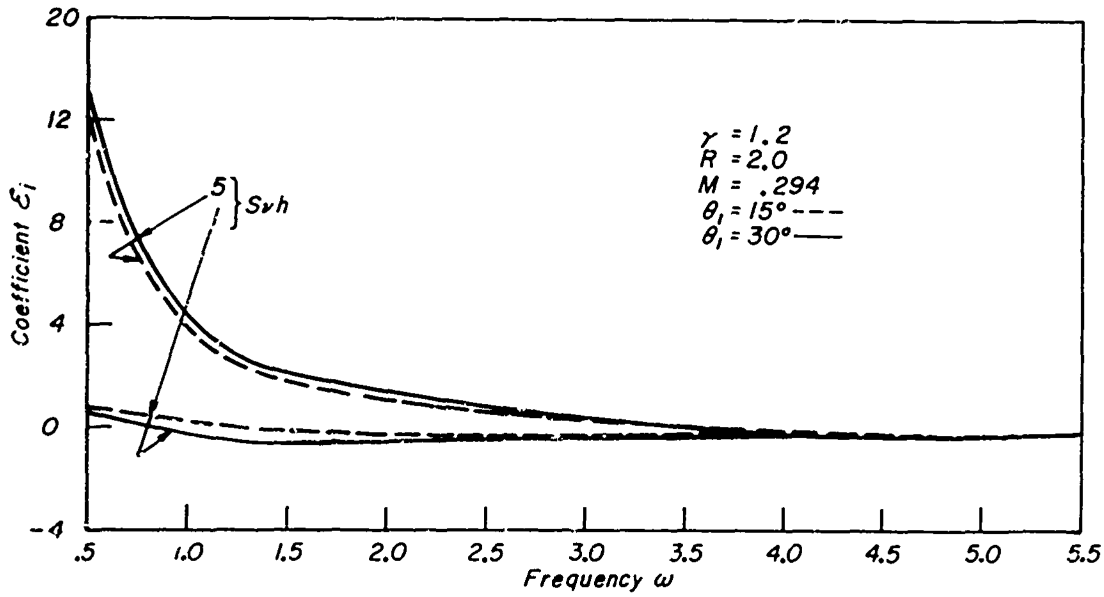


Fig. 16(d) Imaginary part of combined admittance coefficient versus frequency: Effect of cone angle

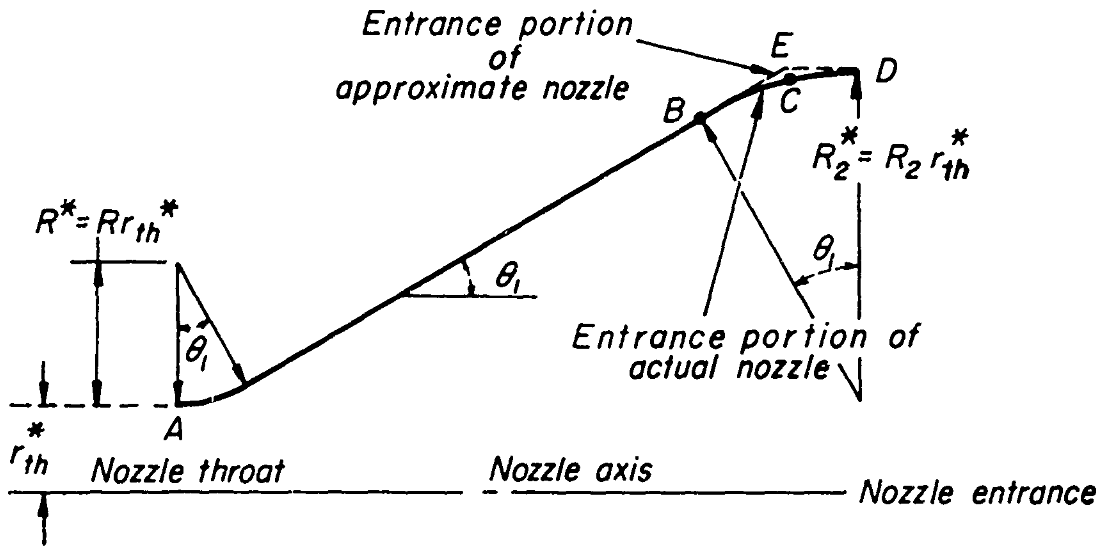


Fig. 17 Nozzle geometry and comparison of entrance portions of approximate and actual nozzle contours

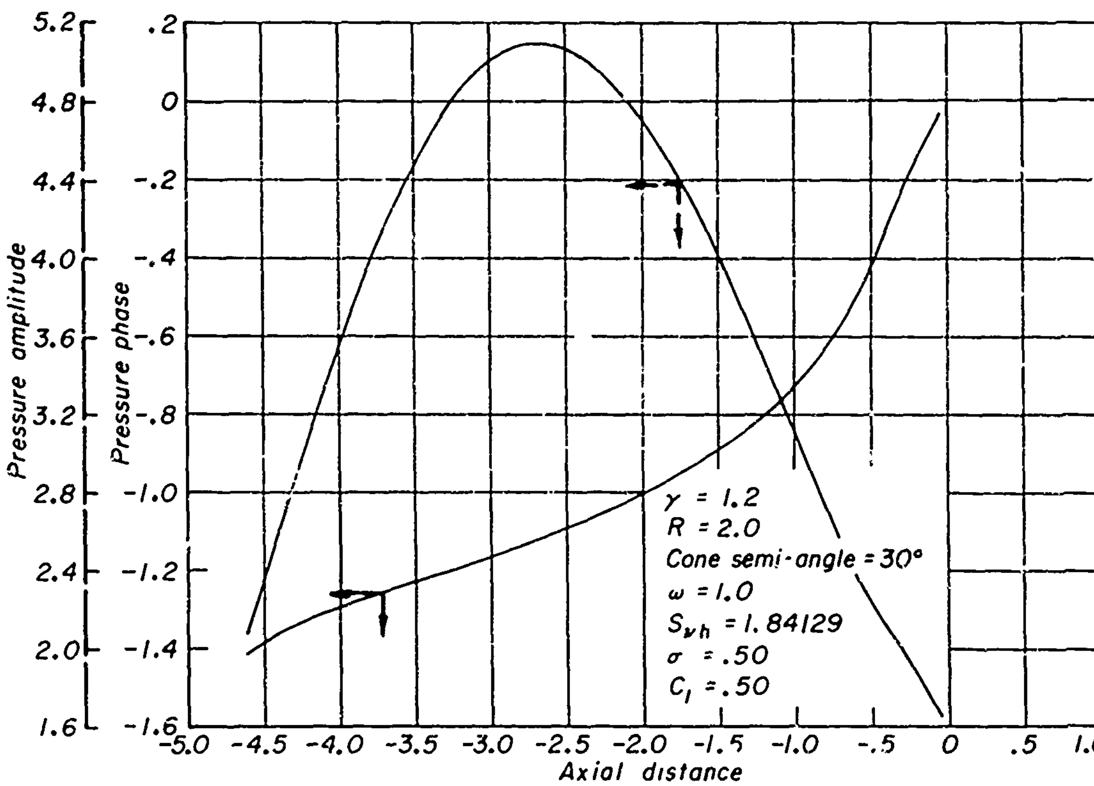


Fig. 18 Pressure perturbation versus axial distance from nozzle throat

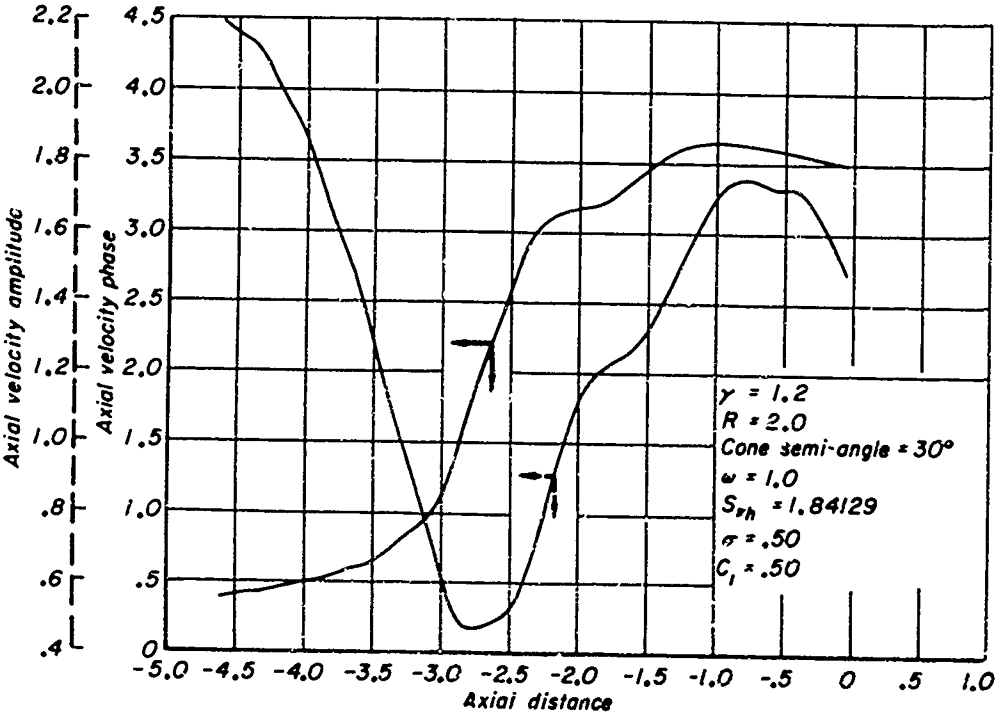


Fig. 19 Axial velocity perturbation versus axial distance from nozzle throat

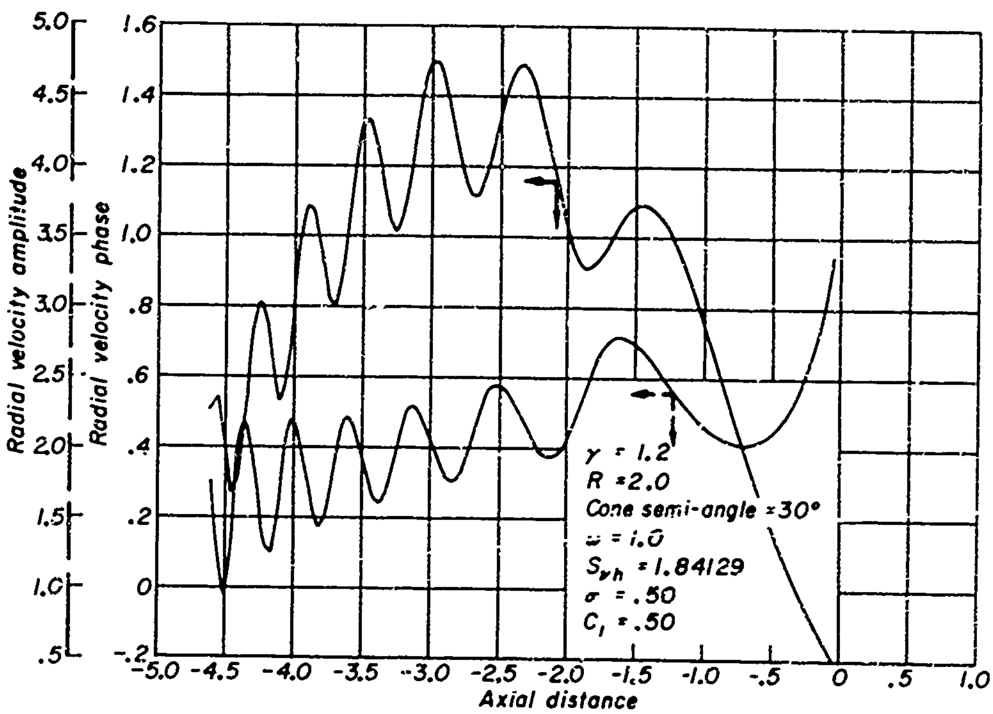


Fig. 20 Radial velocity perturbation versus axial distance from nozzle throat

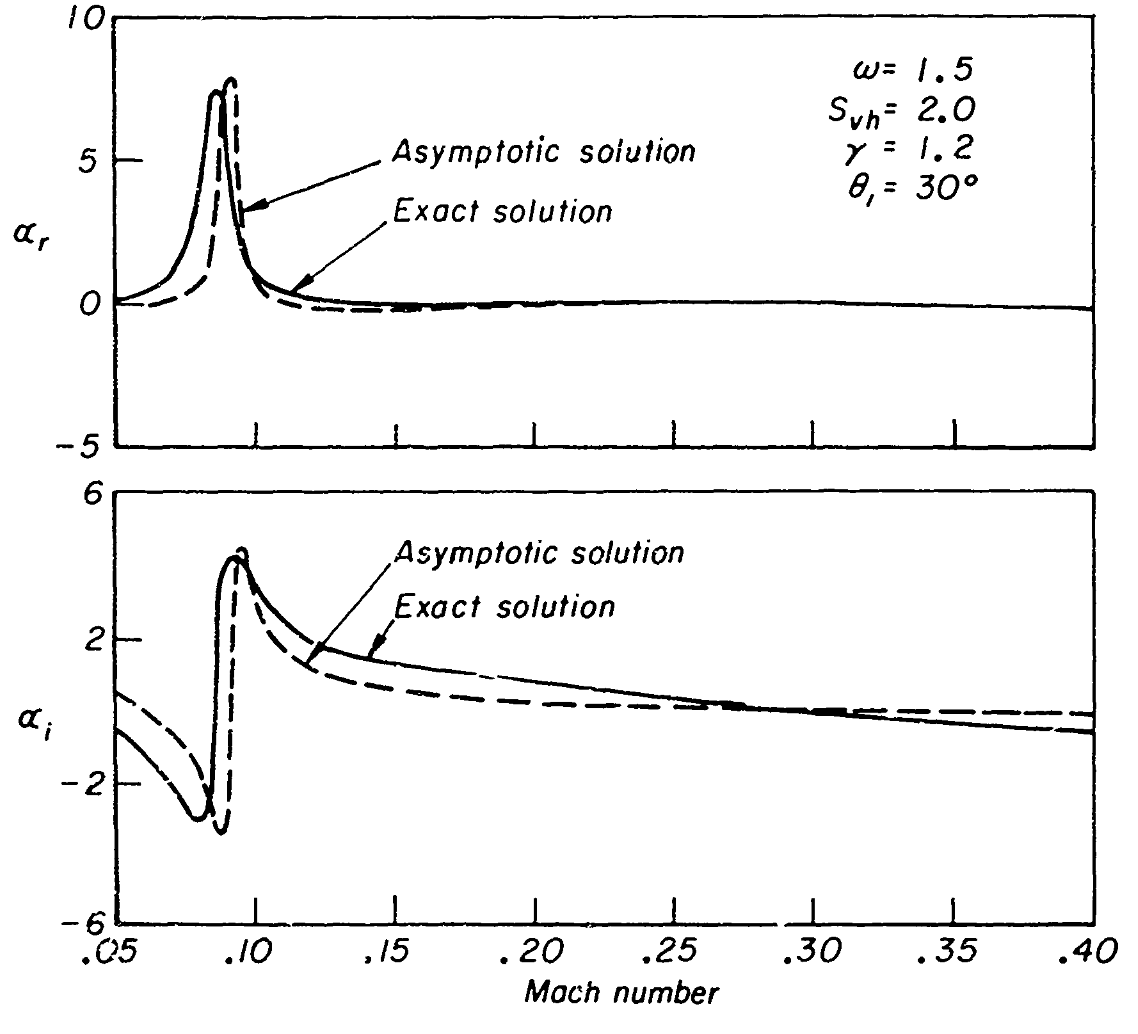


Fig. 21 Irrotational admittance coefficient: Comparison between exact and asymptotic solutions